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Compensation-Based Control for Lossy Communication Networks

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Abstract— **In this paper we are concerned with the stability analysis and the design of stabilizing compensation-based control algorithms for networked control systems (NCSs) that exhibit packet dropouts. We propose a new type of model-based dropout compensator, which depends on the local dropout history, and we provide LMI-based conditions for their synthesis. The analysis and design framework includes stochastic models to describe the packet dropout behavior in both the sensor-to-controller and controller-to-actuator channel. Via examples we demonstrate the significantly improved robustness with respect to packet dropouts using the proposed dropout compensator, compared to using the zero strategy and the hold strategy.**

I. Introduction

Networked control systems (NCSs) are feedback control systems, in which the communication between spatially distributed components, such as sensors, actuators and controllers, occurs through a shared communication network. The use of networks offers many advantages for control systems, such as low installation and maintenance costs, reduced system wiring (in the case of wireless networks) and increased flexibility of the system. However, from a control theory point of view, the presence of a communication network also introduces several, possibly destabilizing, effects, such as packet dropout, time-varying transmission intervals and delays. In this paper we focus on packet dropouts, which can occur, for instance, if there are transmission failures or message collisions. As packet dropouts are a potential source of instability in NCSs, it is of interest to investigate measures to mitigate the influence of dropouts on the stability and also performance of a NCS.

In the literature several different strategies have been proposed to deal with packet dropouts. These strategies can be categorized into three groups: strategies for dropouts in the sensor-to-controller channel, strategies for dropouts in the controller-to-actuator channel and strategies for dropouts in both the sensor-to-controller and controller-to-actuator channel. For dropouts in the sensor-to-controller channel, typically model-based observers are used to alleviate the effect of dropouts. For dropouts in the controller-to-actuator channel, a solution proposed in [12] is the zero strategy, in which the actuator input is set to zero if a packet is dropped. The hold strategy, in which the actuator holds the last received control input instead of setting it to zero, is used in [11]. Instead of holding the previous control input or setting the control input to zero, dynamical predictive outage compensators have been presented in [9]. The latter approach is related to our approach, but considers only dropouts in the controller-to-actuator channel. An alternative scheme based on sending future predicted control values to the actuator was proposed in, e.g., [1], [2], [3]. For packet dropouts in both the controller-to-actuator channel and sensor-to-controller channel, so-called generalized hold functions, which extend the basic hold strategy have been studied in [10], where the optimal hold function is found by solving a LQG problem. The approach in [10] is based on a TCP protocol, and requires acknowledgements of successful packet transmissions. In [15] Markov chains are used to model the stochastic packet dropout behavior in both channels. This approach uses the hold strategy to compensate for packet loss in both channels, but in addition, designs a dropout-dependent controller.

In this paper, we provide systematic design methodologies for a novel dropout compensation strategy that minimizes the influence of dropouts on the stability of the NCS. This new compensation strategy applies for NCSs in which both the controller-to-actuator and the sensor-to-controller channel are subject to dropouts, and does not require any acknowledgement of successful transmissions. In modeling the dropout behavior, we consider a stochastic approach that employs stochastic information on the occurrence of dropouts, given in the form of the well-known Bernoulli or Gilbert-Elliott models [6], [7]. For these dropout models we design dropout compensators, which act as model-based, closed-loop observers if information is received and as open-loop predictors if a dropout occurs. These compensators, designed for each lossy channel, depend only on a single channel's dropout history, and hence, we require no additional information to be sent over the network. The conditions for the stability analysis and design of the compensators are given in terms of linear matrix inequalities (LMIs) and can therefore be solved efficiently. The effectiveness of the proposed compensation strategy and the design tools will be illustrated through a numerical example. In particular, we will show that the designed compensators outperform the zero strategy and the hold strategy (see [11], [12]) significantly in terms of robustness of the stability with respect to dropouts.

A. Nomenclature

The following notational conventions will be used. Let $\mathbb{R}$ and $\mathbb{N}$ denote the field of real numbers and the set of non-negative integers, respectively. For a square matrix $A \in \mathbb{R}^{n\times n}$ we write $A \succ 0$, $A \succeq 0$, $A \prec 0$ and $A \preceq 0$ when A is symmetric and, in addition, A is positive definite, positive semi-definite, negative definite and negative semi-

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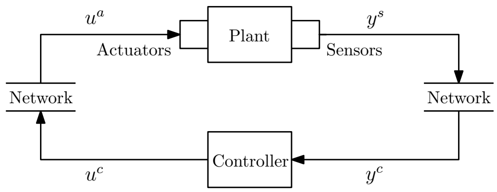

Fig. 1. Scheme of the NCS.

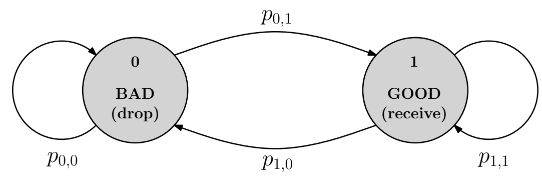

Fig. 2. Gilbert-Elliott model of a lossy network link.

definite, respectively. For $x \in \mathbb{R}^n$ we denote the Euclidean norm as $\|x\|_2 := \sqrt{x^T x}$. For a matrix $A \in \mathbb{R}^{n\times m}$ we denote its transpose by A^T. For the sake of brevity, we sometimes write symmetric matrices of the form $\left[\begin{smallmatrix} A & B^T \\ B & C \end{smallmatrix}\right]$ as $\left[\begin{smallmatrix} A & \star \\ B & C \end{smallmatrix}\right]$. We use $\operatorname{diag}(A_1, A_2, \ldots)$ to indicate a block diagonal matrix with matrices $A_1, A_2, \ldots$ on its diagonal. With some abuse of notation, we will use both $(z_0, z_1, \ldots)$ and $\{z_l\}_{l\in\mathbb{N}}$ with $z_l \in \mathbb{R}^n, l \in \mathbb{N}$, to denote a sequence of vectors in $\mathbb{R}^n$. Let X and Y be random variables. We denote by $\mathbb{P}(X = x)$ the probability of the event $X = x$ occurring. The expected value of X is denoted by $\mathbb{E}(X)$. The (conditional) probability of event $X = x$ occurring, given event $Y = y$, is denoted by $\mathbb{P}(X = x|Y = y)$. The conditional expectation of X given the event $Y = y$ is denoted $\mathbb{E}(X\,|\,Y = y)$.

II. Problem Formulation

This section has the following outline. In Section II-A, we define the NCS with the lossy communication links. In Section II-B, we present stochastic dropout models for the lossy communication links. In Section II-C, we discuss and develop the novel dropout compensation strategy. Finally, in Section II-D we define the problem considered in this paper.

A. Description of the NCS

In this paper, we consider a NCS consisting of a plant and a controller communicating over a network, see Fig. 1. The plant is given by a discrete-time linear time-invariant system of the form

$$\mathcal{P} : \begin{cases} x_{k+1} &= Ax_k + Bu_k^a, \\ y_k^s &= Cx_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k^a \in \mathbb{R}^m$ is the input to the actuator and $y_k^s \in \mathbb{R}^p$ is the output measured by the sensor, at discrete time $k \in \mathbb{N}$. The controller is given by a discrete-time static output feedback law

$$\mathcal{C} : \quad u_k^c = K y_k^c, \tag{2}$$

where y_k^c is the information of the plant output available at the controller and u_k^c is the desired actuator command computed by the controller, at time $k \in \mathbb{N}$. The reason for introducing both y^s and y^c, and both u^c and u^a, is the fact that, due to a non-ideal communication network, y^s and y^c (and u^c and u^a) are typically not equal. Therefore, we sometimes call y^c the networked version of y^s and u^a the networked version of u^c. In this paper, we are interested in the situation where the differences between y^s and y^c, and u^c and u^a, are caused by the fact that the network links between the controller and the actuator, and between the sensor and the controller, are lossy, meaning that packet loss can occur. To model packet loss, we introduce the binary variables $\delta_k \in \{0,1\}$ and $\Delta_k \in \{0,1\}$, $k \in \mathbb{N}$. In case of a successful transmission in the sensor-to-controller channel at time $k \in \mathbb{N}$, $\Delta_k = 1$, and otherwise $\Delta_k = 0$. Similarly in case of a successful transmission in the controller-to-actuator channel, $\delta_k = 1$, and otherwise $\delta_k = 0$.

Using the binary variables δ_k and Δ_k, $k \in \mathbb{N}$, we can now relate y^c to y^s, and u^a to u^c. If a transmission over a channel is successful at time $k \in \mathbb{N}$, the networked version of a signal will be equal to the original signal, i.e., $y_k^c = y_k^s$ in case $\Delta_k = 1$ and $u_k^a = u_k^c$ in case $\delta_k = 1$. If however, the transmission fails at time k, there are multiple strategies for selecting the values y_k^c and u_k^a. Some basic and existing strategies, such as the "zero" strategy and the "hold" strategy, are used in [11], [12]. In Section II-C, we will propose a novel "compensation-based" strategy. The latter strategy employs observer-like compensators on both sides of the network to mitigate the effect of packet loss on the stability of the NCS as much as possible. However, before doing so, first we provide models for the dropout behavior.

B. Dropout Models

In describing packet dropouts in both network links, stochastic models are used. The simplest stochastic model of random packet losses over each of the network channels is to describe the packet loss as a Bernoulli process [12]. In this case, a packet sent over the network from controller to actuator can be lost with probability $p^a \in [0,1]$ and can arrive with probability $1 - p^a$, i.e., $\mathbb{P}(\delta_k = 0) = p^a$ and $\mathbb{P}(\delta_k = 1) = 1 - p^a$, $k \in \mathbb{N}$. Similarly for the packets sent from sensor to controller, we have $\mathbb{P}(\Delta_k = 0) = p^c$, $p^c \in [0,1]$ and $\mathbb{P}(\Delta_k = 1) = 1 - p^c$, $k \in \mathbb{N}$. This setup models a memoryless channel, since the probability of dropouts at a certain time instant is independent of the channel's dropout history.

The situation in which packet losses occur in bursts can not be captured with this memoryless model [7]. Therefore, in this paper we also consider the packet losses in each of the two channels being governed by different two-state Markov chains, as depicted in Fig. 2. This model is known as the Gilbert-Elliott model for fading channels and consists of a good and a bad network state, see e.g., [6], [7]. The probability of packet loss at a certain time instant now depends on the success or failure at the previous transmission instant, i.e., for $k \in \mathbb{N}$,

$$\begin{aligned} \mathbb{P}(\delta_{k+1} = \delta \mid \delta_k = \delta^-) &= p^a_{\delta^-,\delta}, \\ \mathbb{P}(\Delta_{k+1} = \Delta \mid \Delta_k = \Delta^-) &= p^c_{\Delta^-,\Delta}, \end{aligned} \tag{3}$$

where $p^a_{\delta^-,\delta}$ and $p^c_{\Delta^-,\Delta}$ denote the transition probabilities in the controller-to-actuator and sensor-to-controller channel, respectively, for $\delta, \delta^-, \Delta, \Delta^- \in \{0,1\}$. Obviously, $p^a_{\delta^-,0} +$

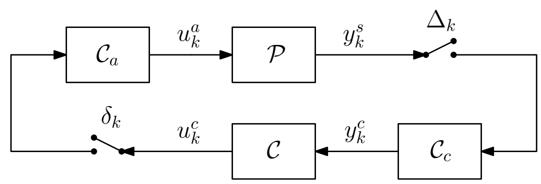

Fig. 3. Scheme of the compensation-based strategy NCS.

$p^a_{\delta^-,1} = 1$ and $p^c_{\Delta^-,0} + p^c_{\Delta^-,1} = 1$ for all $\delta^-, \Delta^- \in \{0, 1\}$. As for each channel the packet loss is modeled by a separate Gilbert-Elliott model, we can use that

$$\begin{aligned} p_{\delta^-,\Delta^-,\delta,\Delta} &:= \mathbb{P}(\delta_{k+1}{=}\delta \wedge \Delta_{k+1}{=}\Delta \mid \delta_k{=}\delta^- \wedge \Delta_k{=}\Delta^-) \\ &= \mathbb{P}(\delta_{k+1}{=}\delta \mid \delta_k{=}\delta^-)\mathbb{P}(\Delta_{k+1}{=}\Delta \mid \Delta_k{=}\Delta^-) \\ &= p^a_{\delta^-,\delta} p^c_{\Delta^-,\Delta}, \end{aligned} \tag{4}$$

where $\delta, \delta^-, \Delta, \Delta^- \in \{0, 1\}$.

C. Compensation-Based Strategy

In addition to the existing basic dropout compensation strategies such as the zero strategy and the hold strategy, in this paper we propose a new compensation-based strategy consisting of two packet loss compensators situated before the controller and the actuator, denoted by $\mathcal{C}_c$ and $\mathcal{C}_a$, respectively (see Fig. 3). The main idea behind the functioning of the compensator is that if a packet arrives, the compensator just forwards the packet and, additionally, acts as a model-based closed-loop observer, i.e., the received signal information is also used to innovate the compensator's estimate of the state of the plant. In case of a packet drop, the compensator acts as an open-loop predictor and, additionally, forwards its best prediction of y^s_k or u^c_k, based on its estimate of the plant state. To formalize this idea, we propose to give the compensators $\mathcal{C}_c$ and $\mathcal{C}_a$ the following structures:

$$\mathcal{C}_c : \begin{cases} x^c_{k+1} \;=\; Ax^c_k + Bu^c_k + \Delta_k L^c_{\tilde{j}_{k-1}} (y^s_k - Cx^c_k) \\ \;\;y^c_k \;\;=\; \begin{cases} Cx_k\,(= y^s_k) & \text{if} \quad \Delta_k = 1 \\ Cx^c_k & \text{if} \quad \Delta_k = 0\,, \end{cases} \end{cases} \tag{5}$$

$$\mathcal{C}_a : \begin{cases} x^a_{k+1} \;=\; Ax^a_k + Bu^a_k + \delta_k L^a_{\tilde{i}_{k-1}} (u^c_k - KCx^a_k) \\ \;\;u^a_k \;\;=\; \begin{cases} Ky^c_k\,(= u^c_k) & \text{if} \quad \delta_k = 1 \\ KCx^a_k & \text{if} \quad \delta_k = 0\,. \end{cases} \end{cases} \tag{6}$$

In (6), we use the fact that the compensator $\mathcal{C}_a$ is collocated with the actuators and, hence, has access to the true implemented control signal u^a_k, which is beneficial for the closed-loop observer design. This is not the case for compensator $\mathcal{C}_c$, which is collocated with the controller $\mathcal{C}$, and, consequently, can only employ the controller output u^c_k at time $k \in \mathbb{N}$. Note that u^c_k is typically not equal to the true control signal u^a_k that is implemented at the actuators at time $k \in \mathbb{N}$. This complicates the closed-loop observer design considerably. The gains $L^c_{\tilde{j}_{k-1}}$ and $L^a_{\tilde{i}_{k-1}}$ are designed to improve the robustness of the stability of the NCS in the presence of dropouts. Note that in (5) and (6) these gains are only effective (i.e. innovation is applied) at time $k \in \mathbb{N}$, if a packet is received, i.e., $\Delta_k = 1$ or $\delta_k = 1$. Moreover, these compensator gains depend on the counters $\tilde{i}_{k-1}$ and $\tilde{j}_{k-1}$, which are related to the number of successive dropouts that occurred just before and including time $k-1$, in the controller-to-actuator channel (i_{k-1}) and sensor-to-controller channel (j_{k-1}), respectively. More specifically, the cumulative dropout counters after the latest successful transmission are defined as

$$\begin{aligned} i_k &:= \min\{l^a \in \mathbb{N} \mid \delta_{k-l^a} = 1, \;\; k - l^a \geq -1\}, \\ j_k &:= \min\{l^c \in \mathbb{N} \mid \Delta_{k-l^c} = 1, \;\; k - l^c \geq -1\}, \end{aligned} \tag{7}$$

for $k \in \mathbb{N}$, where we set $\delta_{-1} := 1$ and $\Delta_{-1} := 1$. However, the gains $L^c_{\tilde{j}_{k-1}}$ and $L^a_{\tilde{i}_{k-1}}$ do not depend directly on i_{k-1} and j_{k-1}, but on "saturated" versions of i_{k-1} and j_{k-1} denoted by $\tilde{i}_{k-1}$ and $\tilde{j}_{k-1}$, respectively. We will explain next the reasons why and introduce $\tilde{i}_k$ and $\tilde{j}_k$ formally.

The adopted Gilbert-Elliott models allow for the occurrence of an infinite number of successive dropouts in the controller-to-actuator channel if $p^a_{0,0} \neq 0$, and in the sensor-to-controller channel if $p^c_{0,0} \neq 0$. Having compensators depending on the counters i_k and j_k would lead to designing infinitely many compensator gains L^a_i and L^c_j, $i \in \mathbb{N}$, $j \in \mathbb{N}$. Clearly, for practical reasons it is desirable to have a finite number of compensator gains, and therefore we choose the compensators to depend on saturated dropout counters $\tilde{i}_k$ and $\tilde{j}_k$ subject to the saturation levels $\tilde{\delta}$ and $\tilde{\Delta}$, respectively, i.e.,

$$\begin{aligned} \tilde{i}_k &= \min(i_k, \tilde{\delta}), \\ \tilde{j}_k &= \min(j_k, \tilde{\Delta}), \end{aligned} \tag{8}$$

for $k \in \mathbb{N}$, where i_k and j_k are defined as in (7). The number of compensators gains, $L^a_{\tilde{i}}$, $\tilde{i} \in \{0, \ldots, \tilde{\delta}\}$, and $L^c_{\tilde{j}}$, $\tilde{j} \in \{0, \ldots, \tilde{\Delta}\}$, to be designed for each channel is now finite. The exact number can be chosen freely by selecting $\tilde{\delta}$ and $\tilde{\Delta}$ in a desirable manner. A direct consequence of these choices is that for all $i_k \geq \tilde{\delta}$, $k \in \mathbb{N}$ we apply the same gain $L^a_{\tilde{\delta}}$ in (6). Similarly, for all $j_k \geq \tilde{\Delta}$, $k \in \mathbb{N}$ we apply the same gain $L^c_{\tilde{\Delta}}$ in (5). Increasing $\tilde{\delta}$ and $\tilde{\Delta}$ increases flexibility of the compensators, however, the complexity of the synthesis problem also increases.

Remark 1: Note that the compensator proposed in (6) is more complex than the controller. If the actuator has enough computational power to run the compensator, the controller could also be collocated with the actuator, thereby effectively removing the controller-to-actuator channel. As a consequence, packet loss between the controller and the actuator is eliminated. In this paper we discuss the general case of a non-collocated controller, i.e., the link between the controller and actuator is subject to packet loss. The situation where a controller collocated with the actuator is used is a simpler problem, which fits as a special case in the general framework and analysis described in this paper.

D. Problem Formulation

The main objectives of this paper are to study the stability properties of the NCS with the compensation-based strategy, as presented in Section II-C, for the stochastic dropout models, as presented in Section II-B. In addition, we aim at deriving effective design conditions for the compensator gains $L^a_{\tilde{i}}$ and $L^c_{\tilde{j}}$ leading to the largest regions of stability in terms of the largest dropout probabilities that can be allowed while still guaranteeing stability.

$$A^{cb}_{\delta,\Delta,i,j} = \begin{bmatrix} A+BKC & -(1-\delta)BKC & -\delta(1-\Delta)BKC \\ O_{n\times n} & A-\delta L^a_i KC & \delta(1-\Delta)L^a_i KC \\ O_{n\times n} & -(1-\delta)BKC & A+(1-\delta)(1-\Delta)BKC-\Delta L^c_j C \end{bmatrix} \tag{10}$$

III. Markov Jump Linear System Model

In this section, we combine the NCS (Section II-A), the compensation based strategy (Section II-C) and the dropout models (Section II-B) to obtain a Markov Jump Linear System (MJLS), see, e.g., [5].

To obtain a closed-loop model for the control system including the compensators, we denote the estimation errors at time $k \in \mathbb{N}$ corresponding to the compensators $\mathcal{C}_c$ and $\mathcal{C}_a$ as $e^c_k := x_k - x^c_k$ and $e^a_k := x_k - x^a_k$, respectively, and define the augmented state $\xi_k := [\, x_k^T \quad (e^a_k)^T \quad (e^c_k)^T \,]^T$. The closed-loop dynamics for the compensation-based strategy can then be given by

$$\xi_{k+1} = A^{cb}_{\delta_k,\Delta_k,\tilde{i}_{k-1},\tilde{j}_{k-1}}\xi_k, \tag{9}$$

with $A^{cb}_{\delta,\Delta,i,j}$ as in (10), for $\delta \in \{0,1\}$, $\Delta \in \{0,1\}$, $i \in \{0,\ldots,\tilde{\delta}\}$ and $j \in \{0,\ldots,\tilde{\Delta}\}$. We also define

$$\tilde{\mu}_k := (\delta_k, \Delta_k, \tilde{i}_{k-1}, \tilde{j}_{k-1}) \in \tilde{\mathcal{M}}, \tag{11}$$

where $\tilde{\mathcal{M}} = \{0,1\}^2 \times \{0,\ldots,\tilde{\delta}\} \times \{0,\ldots,\tilde{\Delta}\}$. This allows a compact representation of (9) as

$$\xi_{k+1} = A^{cb}_{\tilde{\mu}_k}\xi_k. \tag{12}$$

Consider now the closed-loop representation (12) and note that not all transitions from $\tilde{\mu}_k \in \tilde{\mathcal{M}}$ to $\tilde{\mu}_{k+1} \in \tilde{\mathcal{M}}$ are possible. In fact, for $k \in \mathbb{N}$, it holds that

$$\tilde{i}_{k+1} = \tilde{g}_{\tilde{\delta}}(\tilde{i}_k, \delta_k), \qquad \tilde{j}_{k+1} = \tilde{g}_{\tilde{\Delta}}(\tilde{j}_k, \Delta_k) \tag{13}$$

$$\delta_{k+1} \in \{0,1\}, \qquad \Delta_{k+1} \in \{0,1\}, \tag{14}$$

where the parameterized set-valued map
$\tilde{g}_r : \{0,\ldots,r\} \times \{0,1\} \rightrightarrows \{0,\ldots,r\}$ for $r \in \mathbb{N}$ is given by

$$\tilde{g}_r(s_1,s_2) := \begin{cases} 0 &, s_2 = 1 \\ s_1+1 &, s_2 = 0,\ s_1 \in \{0,\ldots,r-1\}, \\ s_1 &, s_2 = 0,\ s_1 = r. \end{cases} \tag{15}$$

We combine the maps in (13) and (14) to obtain

$$\tilde{\mu}_{k+1} \in G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}_k) \tag{16}$$

for all $k \in \mathbb{N}$, where the set-valued map $G_{\tilde{\delta},\tilde{\Delta}} : \tilde{\mathcal{M}} \rightrightarrows \tilde{\mathcal{M}}$ is defined for $\tilde{\mu} = (\delta,\Delta,\tilde{i},\tilde{j}) \in \tilde{\mathcal{M}}$ as

$$G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}) := \{0,1\}^2 \times \left\{\tilde{g}_{\tilde{\delta}}(\tilde{i},\delta)\right\} \times \left\{\tilde{g}_{\tilde{\Delta}}(\tilde{j},\Delta)\right\}. \tag{17}$$

A final step to model the complete NCS with the compensation-based strategy using the compensators $\mathcal{C}_c$ and $\mathcal{C}_a$, as defined in (5) and (6), respectively, is to include the transition probabilities from $\tilde{\mu}_k$ to $\tilde{\mu}_{k+1} \in G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}_k)$ based on the Gilbert-Elliott models for the dropout behavior in each channel. These probabilities combined with (12) and (16) will lead to a MJLS model. To obtain these probabilities, observe that the probability of going from $\tilde{\mu}_k = (\delta_k, \Delta_k, \tilde{i}_{k-1}, \tilde{j}_{k-1})$ to $\tilde{\mu}_{k+1} = (\delta_{k+1}, \Delta_{k+1}, \tilde{i}_k, \tilde{j}_k)$ is completely determined by the probability of going from δ_k to δ_{k+1} and Δ_k to Δ_{k+1} as already expressed in (4). As a consequence, the probability $p_{\tilde{\mu}^-,\tilde{\mu}}$ of going from $\tilde{\mu}^- = (\delta^-, \Delta^-, i^-, j^-) \in \tilde{\mathcal{M}}$ to $\tilde{\mu} = (\delta,\Delta,i,j) \in G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}^-)$ is given by $p^a_{\delta^-,\delta} p^c_{\Delta^-,\Delta}$, and thus we obtain the transition probabilities

$$p_{\tilde{\mu}^-,\tilde{\mu}} = \begin{cases} p^a_{\delta^-,\delta} p^c_{\Delta^-,\Delta}, & \text{when } \tilde{\mu}^- \in \tilde{\mathcal{M}},\ \tilde{\mu} \in G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}^-), \\ 0, & \text{when } \tilde{\mu}^- \in \tilde{\mathcal{M}},\ \tilde{\mu} \in \tilde{\mathcal{M}} \setminus G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}^-). \end{cases} \tag{18}$$

Note that with these probabilities a Markov chain with state $\tilde{\mu} \in \tilde{\mathcal{M}}$ is obtained. The discrete-time system (12) with $A^{cb}_{\tilde{\mu}}$ as in (10) combined with the Markov chain (18) forms the overall model of the NCS in the form of a MJLS, with initial conditions $\xi_0 \in \mathbb{R}^{3n}$ and $\mu_0 \in \tilde{\mathcal{M}}$. We denote this MJLS for brevity by Σ_{MJLS}.

IV. Stability Analysis

In this section, we provide conditions to analyse stability of Σ_{MJLS}.

Definition 1 ([4], [13]): The MJLS given by Σ_{MJLS} is:

1) mean-square stable (MSS) if for every initial state $(\xi_0, \tilde{\mu}_0)$, $\lim_{k\to\infty} \mathbb{E}\big(\|\xi_k\|_2^2 \,|\, \xi_0, \tilde{\mu}_0\big) = 0$;
2) stochastically stable (SS) if for every initial state $(\xi_0, \tilde{\mu}_0)$, $\mathbb{E}\big(\sum_{k=0}^{\infty} \|\xi_k\|_2^2 \,|\, \xi_0, \tilde{\mu}_0\big) < \infty$;
3) exponentially mean square stable (EMSS) if for every initial state $(\xi_0, \tilde{\mu}_0)$, there exist constants $0 \le \alpha < 1$ and $\beta \ge 0$ such that for all $k \ge 0$, $\mathbb{E}\big(\|\xi_k\|_2^2 \,|\, \xi_0, \tilde{\mu}_0\big) \le \beta\alpha^k \|\xi_0\|_2^2$;
4) uniformly exponentially mean square stable (UEMSS), if it is EMSS with α and β independent of ξ_0 and $\tilde{\mu}_0$;
5) almost surely stable (ASS) if for every initial state $(\xi_0, \tilde{\mu}_0)$, we have that $\mathbb{P}\left(\lim_{k\to\infty} \|\xi_k\| = 0\right) = 1$.

It is shown in [4] that the first four stability properties in Definition 1 are equivalent and any one implies almost-sure stability, i.e.,

$$MSS \Leftrightarrow SS \Leftrightarrow EMSS \Leftrightarrow UEMSS \Rightarrow ASS. \tag{19}$$

In the remainder of this section, we present conditions under which Σ_{MJLS} is EMSS. To do so, we observe that in the closed-loop description of the resulting NCS, as given in (12) with $A^{cb}_{\tilde{\mu}}$ given in (10) for $\tilde{\mu} = (\delta,\Delta,\tilde{i},\tilde{j}) \in \tilde{\mathcal{M}}$, the states e^a_{k+1} and e^c_{k+1} are independent of x_k. Therefore, we can split (12) in two subsystems, one related to x_k, the other to $e_k := \left[(e^a_k)^T \quad (e^c_k)^T\right]^T$, for $k \in \mathbb{N}$. This yields the two subsystems

$$x_{k+1} = \bar{A}x_k + w_k, \tag{20a}$$

$$e_{k+1} = E_{\delta_k,\Delta_k,\tilde{i}_{k-1},\tilde{j}_{k-1}} e_k, \tag{20b}$$

$$E_{\delta,\Delta,i,j} = \begin{bmatrix} A - \delta L_i^a KC & \delta(1-\Delta) L_i^a KC \\ -(1-\delta) BKC & A + (1-\delta)(1-\Delta) BKC - \Delta L_j^c C \end{bmatrix} \quad (23)$$

$$\Omega_{\delta,\Delta,i,j} = \begin{bmatrix} P^a_{\delta,i} A - \delta R_i^a KC & \delta(1-\Delta) R_i^a KC \\ -(1-\delta) P^c_{\Delta,j} BKC & P^c_{\Delta,j} A + (1-\delta)(1-\Delta) P^c_{\Delta,j} BKC - \Delta R_j^c C \end{bmatrix} \quad (27)$$

where $w_k := \bar{B}_{\delta_k,\Delta_k} e_k$, $k \in \mathbb{N}$,

$$\bar{A} := A + BKC, \quad (21)$$

$$\bar{B}_{\delta,\Delta} := \begin{bmatrix} -(1-\delta) BKC & -\delta(1-\Delta) BKC \end{bmatrix}, \quad (22)$$
$$\Delta \in \{0,1\}, \delta \in \{0,1\},$$

and where $E_{\tilde{\mu}}$ is given in (23) for $\tilde{\mu} \in \tilde{\mathcal{M}}$. To prove that Σ_{MJLS} is EMSS, we use that (20) is in the form of a cascaded system. In Theorem 1, we will provide a result that can be used to conclude that if $\bar{A} = A + BKC$ is a Schur matrix and if the e-system (20b) with (18) is EMSS, then the system Σ_{MJLS} given by (20) with (18) is EMSS. Note that all stability properties in Definition 1 can be defined similarly for (20b) with (18). In Theorem 2, we will present necessary and sufficient matrix inequality conditions for EMSS of the e-system (20b) with (18), which are proven in [4]. Combining Theorems 1 and 2 will result in EMSS of Σ_{MJLS} as will be formulated in Theorem 3.

Theorem 1 ([8]): Consider system (20a) where $\{w_k\}_{k\in\mathbb{N}}$ is a sequence of random variables such that for some $c_1 \geq 0$ and $0 \leq \rho < 1$ it holds that, for any $w_0 \in \mathbb{R}^{2n}$, $\mathbb{E}(\|w_k\|_2^2) \leq c_1 \rho^k \|w_0\|_2^2$, $k \in \mathbb{N}$. If $\bar{A} = A + BKC$ is a Schur matrix, then there exist $c_2 \geq 0$, $c_3 \geq 0$ and $0 \leq r < 1$ such that

$$\mathbb{E}\big(\|x_k\|_2^2 \,|\, x_0\big) \leq c_2 r^k \|x_0\|_2^2 + c_3 r^k \|w_0\|_2^2 \quad (24)$$

for all x_0, w_0, $k \in \mathbb{N}$.

Theorem 2 ([4]): The MJLS given by (20b) with (18), is EMSS if and only if there exists a set $\{P_{\tilde{\mu}} | \tilde{\mu} \in \tilde{\mathcal{M}}\}$ of positive definite matrices satisfying

$$P_{\tilde{\mu}^-} - \sum_{\tilde{\mu} \in G_{\tilde{\delta},\tilde{\Delta}}(\tilde{\mu}^-)} p_{\tilde{\mu}^-,\tilde{\mu}} E_{\tilde{\mu}}^T P_{\tilde{\mu}} E_{\tilde{\mu}} \succ 0, \ \tilde{\mu}^- \in \tilde{\mathcal{M}}. \quad (25)$$

We now combine Theorem 1 and Theorem 2 to obtain the main result of this section, which formulates conditions under which Σ_{MJLS} is EMSS. For the proof see [8].

Theorem 3 ([8]): Consider system Σ_{MJLS} given by (20) with (18). System Σ_{MJLS} is EMSS if and only if there exists a set $\{P_{\tilde{\mu}} | \tilde{\mu} \in \tilde{\mathcal{M}}\}$ of positive definite matrices satisfying (25) and $\bar{A} = A + BKC$ is a Schur matrix.

V. Compensator Synthesis

Using Theorem 3, one can *analyse* stability of Σ_{MJLS} for *given* compensator gains $L^a_{\tilde{i}}$ and $L^c_{\tilde{j}}$, $\tilde{i} \in \{0,\ldots,\tilde{\delta}\}$, $\tilde{j} \in \{0,\ldots,\tilde{\Delta}\}$. Since we are interested in designing $L^a_{\tilde{i}}$ and $L^c_{\tilde{j}}$ to obtain stability with a large robustness with respect to dropouts, Theorem 4 will state LMI-based conditions for the *synthesis* of $L^a_{\tilde{i}}$ and $L^c_{\tilde{j}}$, based on Theorem 3.

Theorem 4 ([8]): Consider the system Σ_{MJLS} given by (20) with (18). Suppose $\bar{A} = A + BKC$ is a Schur matrix, and there exist a set $\{P_{\tilde{\mu}} | \tilde{\mu} \in \tilde{\mathcal{M}}\}$ of symmetric matrices, with $P_{\tilde{\mu}}$ of the form $P_{\tilde{\mu}} = \operatorname{diag}\left(P^a_{\delta,\tilde{i}},\ P^c_{\Delta,\tilde{j}}\right)$, $\tilde{\mu} \in \tilde{\mathcal{M}}$ and a set $\{R_{\tilde{\mu}} | \tilde{\mu} \in \tilde{\mathcal{M}}\}$ of matrices, with $R_{\tilde{\mu}}$ of the form $R_{\tilde{\mu}} = \operatorname{diag}\left(R^a_{\tilde{i}},\ R^c_{\tilde{j}}\right)$, $\tilde{\mu} \in \tilde{\mathcal{M}}$ satisfying

$$\begin{bmatrix} P_{\tilde{\mu}^-} & \star \\ \Xi_1(\tilde{\mu}^-) & \Xi_2(\tilde{\mu}^-) \end{bmatrix} \succ 0, \quad \tilde{\mu}^- \in \tilde{\mathcal{M}} \quad (26)$$

with for $\tilde{\mu}^- = (\delta^-, \Delta^-, \tilde{i}^-, \tilde{j}^-)$

$$\Xi_1(\tilde{\mu}^-) := \begin{bmatrix} \sqrt{p^a_{\delta^-,0} p^c_{\Delta^-,0}}\ \Omega_{0,0,\tilde{i},\tilde{j}} \\ \sqrt{p^a_{\delta^-,0} p^c_{\Delta^-,1}}\ \Omega_{0,1,\tilde{i},\tilde{j}} \\ \sqrt{p^a_{\delta^-,1} p^c_{\Delta^-,0}}\ \Omega_{1,0,\tilde{i},\tilde{j}} \\ \sqrt{p^a_{\delta^-,1} p^c_{\Delta^-,1}}\ \Omega_{1,1,\tilde{i},\tilde{j}} \end{bmatrix},$$

$$\Xi_2(\tilde{\mu}^-) := \operatorname{diag}(P_{0,0,\tilde{i},\tilde{j}},\ P_{0,1,\tilde{i},\tilde{j}},\ P_{1,0,\tilde{i},\tilde{j}},\ P_{1,1,\tilde{i},\tilde{j}}),$$

where $\tilde{i} = \tilde{g}_{\tilde{\delta}}(\tilde{i}^-, \delta^-)$, $\tilde{j} = \tilde{g}_{\tilde{\Delta}}(\tilde{j}^-, \Delta^-)$ and $\Omega_{\tilde{\mu}}$ as in (27). Then Σ_{MJLS} is EMSS for the compensator gains $L^a_{\tilde{i}}$ and $L^c_{\tilde{j}}$ given by

$$\begin{aligned} L^a_{\tilde{i}} &= (P^a_{1,\tilde{i}})^{-1} R^a_{\tilde{i}}, & \tilde{i} &= 0,\ldots,\tilde{\delta}, \\ L^c_{\tilde{j}} &= (P^c_{1,\tilde{j}})^{-1} R^c_{\tilde{j}}, & \tilde{j} &= 0,\ldots,\tilde{\Delta}. \end{aligned} \quad (28)$$

Remark 2: In case dropouts only occur in the sensor-to-controller channel (see also Remark 1) the above result can be improved as there is no need to enforce structure on $P_{\tilde{\mu}}$ and $R_{\tilde{\mu}}$, $\tilde{\mu} \in \{0,1\} \times \{0,\ldots,\tilde{\Delta}\}$.

VI. Numerical Examples

In this section, we illustrate the presented theory using a well-known benchmark example in the NCS literature consisting of an unstable batch reactor [14]. Here, we will assume that the full state can be measured. We sample the unstable batch reactor as presented in [14] at 100 Hz to obtain a discrete-time plant of the form (1) with

$$A = \begin{bmatrix} 1.0142 & -0.0018 & 0.0651 & -0.0546 \\ -0.0057 & 0.9582 & -0.0001 & 0.0067 \\ 0.0103 & 0.0417 & 0.9363 & 0.0563 \\ 0.0004 & 0.0417 & 0.0129 & 0.9797 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0000 & -0.0010 \\ 0.0458 & 0.0000 \\ 0.0123 & -0.0304 \\ 0.0123 & -0.0002 \end{bmatrix}, \qquad C = I_4.$$

In our analysis, based on Section IV, we will assume that the state feedback gain K in (2) is designed a priori, more specifically, K is designed such that all the eigenvalues of $A + BKC$ are 0.9.

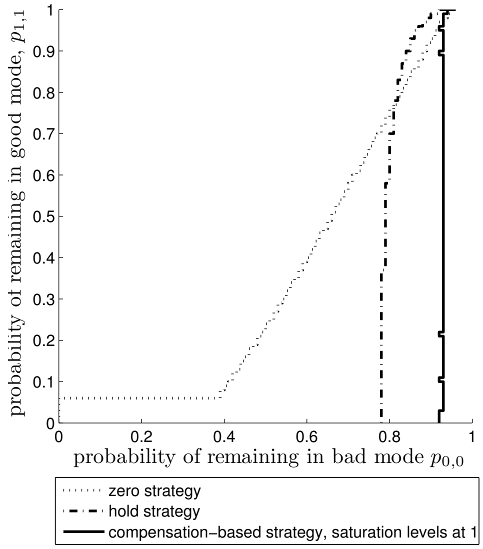

Fig. 4. Results for various dropout compensation strategies where K is designed such that all eigenvalues of $A + BKC$ are placed at 0.9.

We assume that the dropouts in the sensor-to-controller and controller-to-actuator channels are governed by Gilbert-Elliott models. For illustrative purposes, we assume that $p^a_{s^-,s} = p^c_{s^-,s} =: p_{s^-,s}$, for $s, s^- \in \{0, 1\}$. To obtain maximal robustness of the stability property for the compensation-based strategy, we design the compensator gains based on Theorem 4 for various values of $p_{s^-,s},\ s, s^- \in \{0, 1\}$. If we satisfy Theorem 4 for certain $p_{s^-,s}$, then the NCS can be rendered stable by the compensator gains as provided in (28). Note that Theorem 4 provides sufficient conditions for the existence of stabilizing compensator gains due to the imposed structure on the Lyapunov function. We compare the obtained results for the zero strategy and the hold strategy to the compensation-based strategy for counter saturation levels $\tilde{\delta} = \tilde{\Delta} = 1$. To compute the stability regions of the zero strategy and the hold strategy we apply a result similar to Theorem 2, which provides necessary and sufficient LMI-based conditions for stability, see, e.g., [13]. This leads to Fig. 4, in which we compare the region for which stability can be proven for the different strategies. The results are based on analysing an equidistant grid of $p_{s^-,s},\ s, s^- \in \{0, 1\}$, i.e, $p_{0,0} \in \{0, 0.01, \ldots, 0.99, 1\}$, $p_{1,1} \in \{0, 0.01, \ldots, 0.99, 1\}$, $p_{0,1} = 1 - p_{0,0}$ and $p_{1,0} = 1 - p_{1,1}$. Closed-loop stability is guaranteed for all the grid points to the left of each line. Even though the results for the compensation-based strategy are based on sufficient conditions, we observe that the region for which stability can be guaranteed is (much) larger than the regions for the zero strategy and the hold strategy. Hence, for this example it is clear that the compensation-based strategy is providing more robustness against packet dropouts (at the price of higher computational requirements for the implementation).

VII. CONCLUSIONS

In this paper, we presented a new compensation-based strategy for the stabilization of a networked control system (NCS) with packet dropouts. The main rationale behind the novel dropout compensators is that they act as model-based, closed-loop observers if information is received and as open-loop predictors if a dropout occurs. These compensators were considered for Bernoulli and Gilbert-Elliott models describing the dropout behavior. For these stochastic dropout models, we derived necessary and sufficient conditions for (exponential) mean square stability of the closed-loop NCS. In addition, we developed LMI-based conditions for the synthesis of the compensator gains that result in a robustly stable closed-loop system. By means of a numerical example, the significant improvements in robustness of stability with respect to packet dropouts for the compensation-based strategy compared to the zero strategy and the hold strategy were demonstrated.

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