

# Variable gain motion control for transient performance improvement

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**Abstract**—In this paper, we introduce variable gain controllers for linear motion systems designed to improve transient performance. In particular, we focus on the well-known tradeoff induced by integral action, which removes steady-state errors due to constant external disturbances, but deteriorates transient performance in terms of increased overshoot. We propose a switched integral controller (SIC) and a variable gain integral controller (VGIC), which both limit the integral action if the error exceeds a certain threshold and balance this tradeoff in a more desirable manner. The resulting nonlinear controller structure consist of, on the one hand, loop-shaped linear controllers, and on the other hand, the variable gain element. This structure makes the design and tuning of the variable gain controller intuitive for motion control engineers. The results are illustrated by application of the VGIC and SIC to an experimental set-up.

## I. INTRODUCTION

Transient performance of a motion control system is often quantified in terms of step-responses of the closed-loop system. In general, the controller design aims at combining a fast response with small overshoot and zero steady-state error. It is well known that steady-state errors due to unknown constant disturbances can be removed by including integrating action in the controller. The steady-state error can be caused by constant input disturbances, in case of a plant without pure integrators, or by constant force disturbances acting on the system. However, it is also well known that integral control action increases the amount of overshoot in case of a step-change in the reference. To balance this tradeoff between steady-state accuracy in the face of disturbances and fast transients in a more desirable manner, nonlinear variable gain controllers are proposed in this paper.

The idea of variable gain control for linear motion systems to enhance the performance is not new [18], [3]. The work in [8], [7] exploits variable gain control for linear motion systems to balance the tradeoff between high-frequent noise sensitivity and low-frequent disturbance suppression in a more desirable manner, and hence focuses on the *steady-state* performance of the system in the presence of time-varying perturbations. In this paper, we focus on enhancing *transient* performance through variable gain control. Several concepts for improving the transient performance of a control system have been proposed in literature. One concept is reset control, of which the so-called Clegg integrator introduced in [5] in 1958 is one of the first examples. The Clegg integrator resets the state of the integrator to zero when the error changes sign. Generalizations include first-order reset elements (FORE) which reset part, or all, of the controller states, if certain conditions are satisfied, to attain improved transient performance [4], [14]. The concept of composite

nonlinear feedback control is employed in [13], [11], which varies the amount of damping of the controller depending on the amplitude of the error, to improve the transient step-response. A non-linear state-feedback control approach to improve the transient response of a constrained linear system is proposed in [1]. A 5-parameter nonlinear PI controller has been proposed in [15] to improve the transient performance of linear systems. In [6], potential benefits of hybrid control for linear systems has been discussed, and it has been shown that a switched integral controller can improve the transient performance for a plant consisting of an integrator. Switched integrator control schemes with resets and saturation have been considered in [12] for integrating plants. In [16], [17], the concept of conditional integrators has been introduced in a sliding mode control framework and a more general feedback control framework which uses Lyapunov redesign and saturated high-gain feedback, to obtain regulation of nonlinear systems. In these works, integral action is introduced in a boundary layer, as not to degrade the transient response of the nonlinear system.

In this paper, we focus on the performance tradeoff in overshoot and steady-state error when employing integral control action. The integrator is typically employed to achieve zero steady-state errors, but increases the amount of overshoot. Therefore, the integrating action can be switched off, or limited, if the error exceeds a certain threshold, thereby limiting the amount of overshoot the system exhibits. We will propose and study the following two strategies that are based on the above philosophy: a switched integral controller (SIC), which switches off the integrator if the error exceeds the threshold, and a variable gain integral controller (VGIC), which reduces the amount of integrating action if the error exceeds the threshold.

The SIC and VGIC introduced in this paper, in contrast to [12], [6], can be applied to any linear plant. The VGIC and SIC guarantee asymptotic stability of the set-point under easy-to-use graphical conditions, in the presence of constant disturbances while improving upon transient performance (comparing to linear controllers). Because the building blocks of the variable gain controllers are linear motion controllers, the tuning can be done using well-known loop-shaping techniques. Moreover, the controller design and stability analysis can be performed on the basis of measured frequency response data. These facts will be illustrated by application to an experimental setup. Furthermore, as opposed to reset control as in [4], [14], no controller states are reset in SIC and VGIC, which makes the closed-loop dynamics less complex, and, moreover, opposed to [16], [17], we focus on linear motion systems. Therefore, the stability and performance analysis becomes easier and the interpretation of the closed-loop response relates in a clear way to the underlying linear controller designs, which greatly enhances practical applicability.

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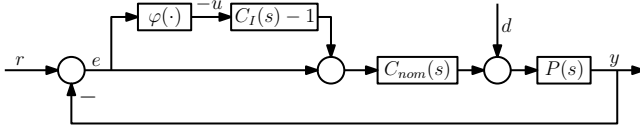


Fig. 1. Variable gain control scheme.

Concluding, the main contributions of this paper are: firstly, the design of SIC and VGIC motion controllers that, on the one hand, guarantee robustness against constant disturbances by employing integral control, and on the other hand, significantly improve transient performance, compared to linear motion controllers, and, secondly, the experimental validation of the effectiveness of these control designs.

The remainder of the paper is organized as follows. In Section II, the two variable gain control strategies, variable gain integral control (VGIC) and switched integrator control (SIC), will be introduced. Moreover, stability properties induced by the two proposed control schemes will be studied. The controllers will be applied to an experimental motion system in Section III, which illustrates the effectiveness of the proposed control strategies. Conclusions will be presented in Section IV.

## II. VARIABLE GAIN INTEGRAL CONTROL

Two variable gain control strategies, the switching integral controller (SIC) and variable gain integral controller (VGIC), will be proposed in this section. In Section II-A, a description of the motion control system will be given, followed by the design of the variable gain element in Section II-B. An example will be given in Section II-C to illustrate the main idea of the controller designs. A stability analysis of the closed-loop dynamics will be presented in Section II-D.

### A. Description of the control system

Consider the SISO closed-loop variable gain control scheme in Fig. 1, with plant  $P(s)$ ,  $s \in \mathbb{C}$ , nominal linear controller  $C_{nom}(s)$ , which does not have integral action, reference  $r$ , disturbance  $d$ , and measured output  $y$ . Additionally, we introduce the variable gain part of the controller consisting of the variable gain element  $\varphi(e)$  ( $u = -\varphi(e)$ ), depending on the error  $e$ , and a weak integrator described by transfer function

$$C_I(s) = \frac{s + \omega_i}{s}, \quad (1)$$

with  $\omega_i > 0$  the zero of the weak integrator. First consider the situation in which  $\varphi(e)$  is not a nonlinear function but a linear element and study the following two limiting cases:

- 1) If  $\varphi(e) = 0$ , we have a linear control scheme with linear controller  $C(s) := C_{nom}(s)$ ;
- 2) If  $\varphi(e) = e$ , we also have a linear control scheme, but with linear controller  $C(s) := C_{nom}(s)C_I(s)$ .

In case 1, steady-state errors due to constant disturbances cannot be removed, but the amount of overshoot is limited. In case 2, zero steady-state error can be achieved, but the overshoot is increased. By choosing the variable gain element  $\varphi(e)$  in a smart way, we can combine the best of both worlds and obtain both an improved transient response (small overshoot) and zero steady-state error.

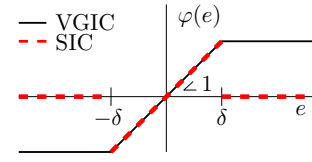


Fig. 2. Variable gain elements  $\varphi(e)$  for the VGIC and SIC, with saturation length  $\delta$ .

### B. Design of the variable gain element $\varphi(e)$

Consider the two choices for  $\varphi(e)$  depicted in Fig. 2, corresponding to the following two variable gain controller designs:

- The switched integral controller (SIC), which uses the switching nonlinearity (dashed), is the most straightforward choice. The controller induces full integral control action when the error  $|e| \leq \delta$  (i.e.  $C_{nom}(s)C_I(s)$  is active, to achieve zero steady-state error), but *completely switches off* the integrating action when the error exceeds  $\delta$  (i.e.  $C_{nom}(s)$  is active if  $|e| > \delta$  to limit the amount of overshoot).
- The variable gain integral controller (VGIC), which uses the saturation nonlinearity (solid), only *limits* the integrating action when the error  $|e|$  exceeds the saturation length  $\delta$ . Thereby, it limits the amount of overshoot, while inducing full integral control when the error satisfies  $|e| \leq \delta$ , and hence removes steady-state errors. Note that essentially, if  $|e| \gg \delta$ , the linear controller  $C_{nom}(s)$  is active, while if  $|e| \leq \delta$  the linear controller  $C_{nom}(s)C_I(s)$  is active;

Note that other choices for the nonlinearities  $\varphi(e)$  are also possible. The VGIC and SIC nonlinearity are chosen as in Fig. 2 because they result in the desired closed-loop behavior (see Sections II-C and III) and are relatively simple to tune since both can be parameterized by only one parameter  $\delta$ . We will show in Section II-D and Section III that the choice for the VGIC should be preferred over the SIC because it has beneficial stability properties.

**Remark II.1** Note that the amount of integral action induced can be increased (decreased) by increasing (decreasing) the frequency of the zero  $\omega_i$  of the weak integrator (1).

### C. Illustrative example

To illustrate the effectiveness and main idea behind the proposed control strategies, consider the motion system depicted in Fig. 3, with  $m = 0.01$  kg,  $b = 0.03$  Ns/m,  $k = 1$  N/m, and control input  $F$  (an experimental study with more complex dynamics will be treated in detail in Section III). A nominal controller  $C_{nom}(s)$  without integrator and a controller  $C_{nom}(s)C_I(s)$  with integrator have been designed using loop-shaping techniques to control the system to the reference  $r = 1$  (note that the force disturbance  $d = 0$  in this case).  $C_{nom}(s) = k_p(s + \omega_z)/(s + \omega_p)$  is a lead-filter with the zero at  $\omega_z = 10$  rad/s, the pole at  $\omega_p = 100$  rad/s,  $k_p = 100$ , and the integrator is given by (1), with  $\omega_i = 6$  rad/s. The transient unit step response simulations are depicted in Fig. 4. As can be concluded from the figure, the

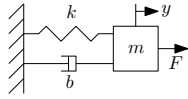


Fig. 3. Schematic representation of a simple motion system.

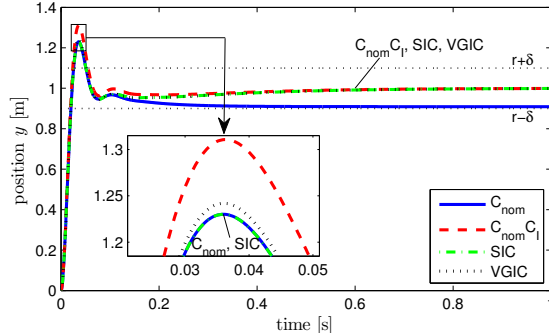


Fig. 4. Illustrative example of the controlled motion system in Fig. 3 with  $\delta = 0.1$ .

controller without integrator ( $C_{nom}(s)$ ) has the least amount of overshoot, but is not capable of achieving zero steady-state error. The controller with integrator ( $C_{nom}(s)C_I(s)$ ) is capable of removing the steady-state error, but exhibits the negative effect of increased overshoot. Clearly, the variable gain controllers (with  $\delta = 0.1$ ) combine the small overshoot characteristics with zero steady-state error responses, and SIC has the smallest overshoot of the nonlinear controllers.

The stability properties of the closed-loop variable gain control system will be discussed in the following section.

**Remark II.2** Note that the trade-off induced by the integral action not only occurs in case of a step in reference. Namely, if, for example, a certain force-disturbance pushes the error to a significant level, the same trade-off will become apparent.

#### D. Stability analysis

In order to perform stability analysis of the nonlinear variable gain control schemes, we firstly observe from Fig. 1 that the system belongs to the class of Lur'e-type systems, consisting of a linear dynamical system

$$G_{eu}(s) = \frac{\omega_i}{s} \frac{P(s)C_{nom}(s)}{1 + P(s)C_{nom}(s)}, \quad (2)$$

denoting the transfer function between input  $u$  and output  $e$ , with a nonlinearity  $\varphi(e)$  in the feedback loop. Note that  $G_{eu}(s)$  has a simple pole at  $s = 0$ . A minimal realization of the closed-loop dynamics can be described in state-space form as follows:

$$\dot{x} = Ax + Bu + B_r r + B_d d \quad (3)$$

$$e = Cx + D_r r + D_d d \quad (4)$$

$$u = -\varphi(e), \quad (5)$$

with state  $x$  and  $G_{eu}(s) := C(sI - A)^{-1}B$ .

Let us adopt the following two assumptions, which are both natural in a motion control setting:

**Assumption II.3** The complementary sensitivity, given by transfer function

$$T(s) = \frac{P(s)C_{nom}(s)}{1 + P(s)C_{nom}(s)}, \quad (6)$$

has all poles in the open LHP.

**Assumption II.4** The complementary sensitivity  $T(s)$  in (6) satisfies  $T(0) \geq 0$ .

**Remark II.5** Note that Assumptions II.3 and II.4 are very mild assumptions. The poles of  $T(s)$  will lie in the open LHP by design of an asymptotically stabilizing controller  $C_{nom}(s)$  and the condition that  $T(0) \geq 0$  is usually satisfied because the complementary sensitivity generally equals +1 for  $\omega \rightarrow 0$ .

Below,  $x^*$  is defined as the equilibrium point of system (3)-(5) satisfying  $e = 0$ . Note that  $x^*$  is the only equilibrium point satisfying  $e = 0$ , due to observability of the minimal state-space realization (3)-(5) (the observability matrix has full rank such that the equations  $e = 0, \frac{de}{dt} = 0, \dots, \frac{d^{n-1}e}{dt^{n-1}} = 0$ , exhibit a unique solution  $x^*$ , for  $e = 0$ ).

The following theorem poses conditions under which certain stability properties of the reference can be guaranteed for the SIC and VGIC control system.

**Theorem 1** Consider system (3)-(5), with constant references  $r$  and constant disturbances  $d$ . If Assumptions II.3 and II.4 hold, and transfer function  $G_{eu}(s)$  in (2) satisfies

$$\operatorname{Re}(G_{eu}(j\omega)) \geq -1 \quad \forall \omega \in \mathbb{R}, \quad (7)$$

then the VGIC renders the equilibrium point  $x^*$  globally asymptotically stable (GAS), and the SIC renders the equilibrium point  $x^*$  locally asymptotically stable.

*Proof:* Note that integrator (1) has dynamics that can be described by (with state  $x_I$ , input  $\varphi(e)$ , and output  $y_I$ )

$$\dot{x}_I = \varphi(e) \quad (8)$$

$$y_I = \omega_i x_I + \varphi(e), \quad (9)$$

such that there exists an equilibrium point  $x^*$  (satisfying  $\dot{x}_I = \varphi(e) = 0$ ) for both the VGIC and SIC system, that satisfies  $e = 0$ . Namely, both for the SIC and VGIC schemes  $\varphi(0) = 0$ . Because we consider constant (step) references in  $r$  and constant disturbances  $d$  in order to assess transient performance, we can employ a coordinate transformation  $z = x - x^*$  to study stability of the equilibrium  $x^*$  of the closed-loop system (3)-(5). The transformed dynamics can be written as

$$\dot{z} = Az + Bu \quad (10)$$

$$e = Cz \quad (11)$$

$$u = -\varphi(e). \quad (12)$$

Note that the nonlinearities  $\varphi(e)$ , see Fig. 2, lie in the sector  $\varphi(e) \in [0, 1]$  (i.e.  $0 \leq \varphi(e) \leq e \quad \forall e \in \mathbb{R}$ ), for both the SIC and VGIC. Because

- 1) Due to Assumption II.3, the poles of  $G_{eu}(s) = \omega_i T(s)/s$ , see (2) and (6), all lie in the open LHP, except for the simple pole at  $s = 0$ ;

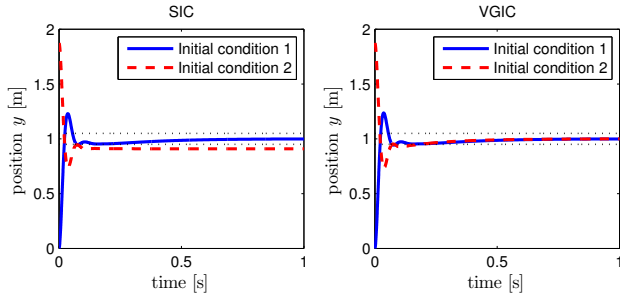


Fig. 5. Local asymptotic stability for the SIC vs. global asymptotic stability for the VGIC.

- 2) Due to Assumption II.4, the residue of the simple pole  $\text{res}(G_{eu}(s)) = \lim_{s \rightarrow 0} s G_{eu}(s) = \omega_i T(0) \geq 0$ ;
- 3) Due to the condition in the theorem,  $\text{Re}(G_{eu}(j\omega)) \geq -1 \forall \omega \in \mathbb{R}$ ,

the transfer function  $G_{eu}(s)$  is positive real [9] (not *strictly* positive real, due to the simple pole at  $s = 0$ ). Following similar lines as the proof of the circle criterion in section 7.1 in [9], we can show with Lyapunov function candidate  $V(z) = z^T P z$ , with  $P$  positive definite (and coming from the Kalman-Yakubovich-Popov lemma [9]), that  $\dot{V} \leq 0$ , and that  $\dot{V} = 0$  only if  $\varphi(e) = 0$ . Using a LaSalle-type argument (and observability of the minimal state-space realization) it then follows for the VGIC that  $z = 0$  ( $x = x^*$ , with  $e = 0$ ) is GAS, because  $\varphi(e) = 0$  has only one unique zero at  $e = 0$ , see Fig. 2 (see also [2]). For the SIC, we can only conclude local asymptotic stability of  $z = 0$  ( $x = x^*$ , with  $e = 0$ ) because  $\varphi(e) = 0$  also for  $|e| > \delta$ . ■

The difference between global asymptotic stability of  $x^*$  for the VGIC, and local asymptotic stability of  $x^*$  for the SIC, can intuitively easily be understood: the saturation nonlinearity used in the VGIC always (i.e., also outside the band  $[-\delta, \delta]$ ) applies a certain amount of integrating action (see Fig. 2), such that the error is always forced to zero. For the switched nonlinearity used in the SIC, however, the integrator is switched off completely if  $|e| > \delta$ . On the one hand, for certain initial conditions, if the error settles outside this band, the integrator will not be active and the steady-state error will remain. On the other hand, if the error settles inside the band, the steady-state error will converge to zero. This is a drawback of the SIC compared to the VGIC and this lack of *global* asymptotic stability is illustrated by simulations in Fig. 5. Herein, the response of the controlled motion system of Fig. 3 is shown for different initial conditions.

### III. EXPERIMENTAL RESULTS

In this section, the proposed integral control strategies will be applied to an experimental setup and will be compared to the linear controllers with and without integrator. The experimental setup under study consists of two rotating inertias interconnected by a flexible shaft, see Fig. 6. The system is controlled in a non-collocated manner: the rotation  $y$  of the left inertia is measured at mass 2 and is to be controlled to the reference, but the actuation takes place at the right inertia, which resembles a realistic industrial setting where the position of the load can be measured but actuation at the load is in general not possible.

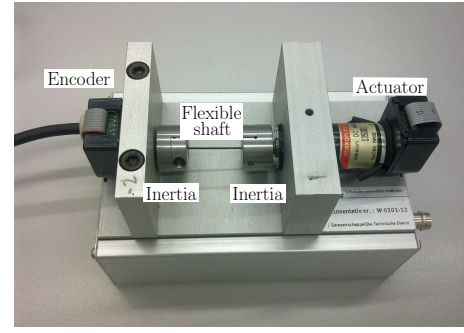


Fig. 6. Experimental setup.

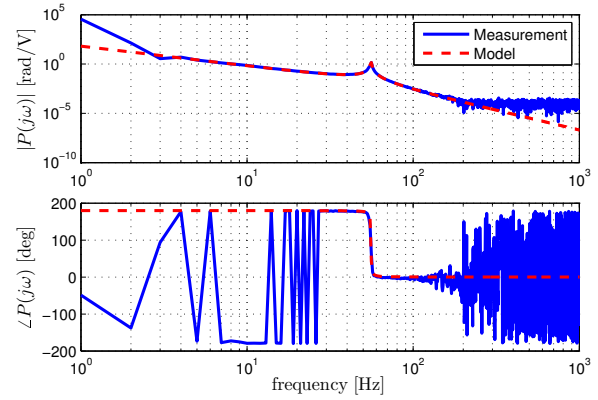


Fig. 7. Measured frequency response function and 4th-order model fit.

The plant dynamics are obtained by a frequency response measurement and is depicted in Fig. 7, together with a 4th-order linear model fitted to the measurement:

$$P(s) = \frac{3.176 \cdot 10^8}{s^4 + 3.466s^3 + 1.2248 \cdot 10^5 s^2}, \quad s \in \mathbb{C}. \quad (13)$$

#### A. Controller design

This model is used for the controller design. Using manual loop-shaping techniques, a nominal stabilizing controller  $C_{nom}(s)$  without integral action is designed, consisting of a lead-filter, a notch-filter, and a 2nd-order low-pass filter:

$$C_{nom}(s) = \frac{1.096 \cdot 10^4 s^3 + 2.679 \cdot 10^5 s^2}{0.005305 s^5 + 7.666 s^4 + 5864 s^3} \cdots \frac{+1.34 \cdot 10^9 s + 2.805 \cdot 10^{10}}{+2.321 \cdot 10^6 s^2 + 5.715 \cdot 10^8 s + 5.61 \cdot 10^{10}}, \quad s \in \mathbb{C}. \quad (14)$$

The integral part  $C_I(s)$  in (1) is chosen with the integrator zero  $\omega_i$  at 1 Hz, so that that two open-loop transfer functions

$$O_l(s) = P(s)C_{nom}(s) \quad (15)$$

$$O_l(s) = P(s)C_{nom}(s)C_I(s), \quad (16)$$

are obtained with both a 10 Hz bandwidth, see Fig. 8. Note that the integrator  $C_I(s)$  is also used in both the SIC and VGIC setting, see the closed-loop control scheme in Fig. 1. Because these controllers are nonlinear, they cannot be visualized by a frequency response function.

It is easily verified that Assumptions II.3 and II.4 are satisfied, see also Remark II.5, and the frequency domain

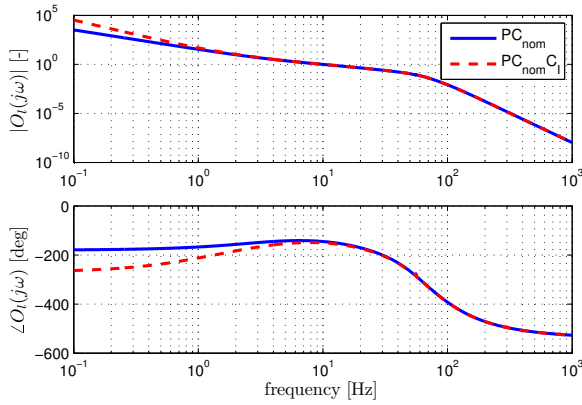


Fig. 8. Open-loop transfer functions with and without integrator.

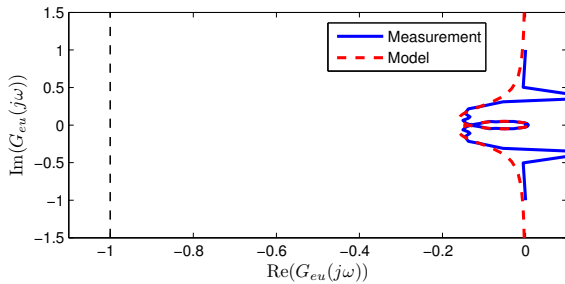


Fig. 9. Nyquist plot of  $G_{eu}(s)$  showing  $\text{Re}(G_{eu}(j\omega)) \ge -1 \forall \omega \in \mathbb{R}$ .

condition  $\text{Re}(G_{eu}(j\omega)) \ge -1 \forall \omega \in \mathbb{R}$  is verified, as can be concluded from the Nyquist plots of the model and measurement data in Fig. 9. Therefore, all the conditions of Theorem 1 are satisfied, such the equilibrium point of the VGIC system (3)-(5) is GAS; hence, the steady-state error will converge to zero for any initial condition. Note that for the SIC we can only guarantee local asymptotic stability because it can exhibit multiple equilibria.

Note that, for the VGIC scheme, the global asymptotic stability of the reference does not depend on the choice of the saturation length  $\delta$ . This implies that  $\delta$  is thus a purely performance-based variable and is fully stability-invariant. Clearly, for the SIC scheme, the local asymptotic stability property is also guaranteed independent of  $\delta$ ; however, the region of attraction of  $e = 0$  is influenced by  $\delta$ .

### B. Performance analysis

A step-reference  $r$  of 1 rad is applied to the system at  $t = 1$  s, and a step-disturbance  $d$  of amplitude 0.1 V acts on the system at  $t = 5$  s. First, a simulation study is performed using the estimated model (13) and the VGIC, in order to gain more insight into the influence of the saturation length  $\delta$  on the closed-loop performance.

To quantify the performance of the different controllers more specifically, we consider the following two performance measures:

- Percentage overshoot;
- Integral of the Squared Error (ISE):  $\text{ISE} = \int_0^T e^2(t) dt$ .

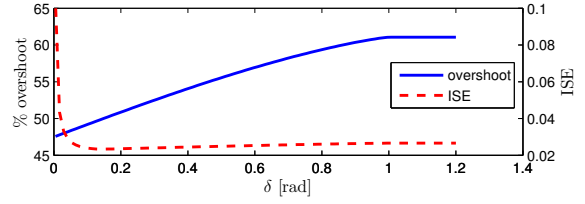


Fig. 10. Study of overshoot and ISE as a function of  $\delta$  for the VGIC.

The saturation length  $\delta$  has been varied between  $\delta = 0$  and  $\delta = 1.1$ . The calculated overshoot and Integral of the Squared Error (ISE, with  $T = 10$  s) are shown in Fig. 10 as a function of  $\delta$ . Note that the case  $\delta = 0$  corresponds to the linear case *without* integral control ( $C_{nom}(s)$ ). Because for this particular simulation study the amplitude of the error  $e$  is always smaller than 1 (as we will see in comparable experimental results in Fig. 11), the overshoot and ISE do not change for  $\delta \geq 1$ . Therefore,  $\delta \geq 1$  corresponds to the linear case *with* integral control ( $C_{nom}(s)C_I(s)$ ). From the performance curves in Fig. 10, it can easily be concluded that the VGIC with a well-chosen saturation length  $\delta$  combines reduced overshoot with improved ISE performance.

Based on Fig. 10,  $\delta = 0.1$  is used in the experiments carried out on the setup. The measured response of the system to the step-reference  $r$  of 1 rad at  $t = 1$  s and the step-disturbance  $d$  of 0.1 V at  $t = 5$  s, is shown in Fig. 11.

First, consider the linear controllers. Clearly, the inclusion of the integrator increases the overshoot of the system's closed-loop response. However, the controller without integrator, in contrast to the controller with integrator, is not capable of removing the steady-state error due to the constant force-disturbance applied at  $t = 5$  s.

Secondly, consider the nonlinear controllers. The SIC only applies integral control if the error  $|e| \leq \delta$  and the VGIC limits the integral control if  $|e| \geq \delta$ , see Fig. 2. Because the integrator is switched off, or limited, if the error is larger in amplitude than  $\delta = 0.1$ , the overshoot is reduced, see Fig. 11. Since the error due to the force-disturbance  $d$  equals  $0.18 > 0.1$ , the SIC is not capable of forcing the error to zero, because the integrator is switched off. Note that for this reason the SIC closed-loop system does not exhibit a GAS equilibrium point, see Section II-D. The VGIC does exhibit a GAS equilibrium point, and is therefore always capable of forcing the error to zero. Hence, the VGIC combines the small overshoot of the linear controller without integrator with the suppression of constant disturbances of the linear controller with integrator. Although the SIC can also force the steady-state error to zero, this depends on the initial conditions and the choice of  $\delta$ , see also Fig. 5.

The measured performance measures, percentage overshoot and ISE, are depicted in Table I. We note that the rise-time and settling time are hardly influenced by the different controllers considered, see Fig. 11. Moreover, similar integral performance measures (such as Integral of the Absolute Error (IAE) or Integral of Time multiplied by Absolute Error (ITAE), see for example [10]) show the same qualitative characteristics as the ISE, but are omitted here for the sake of brevity.



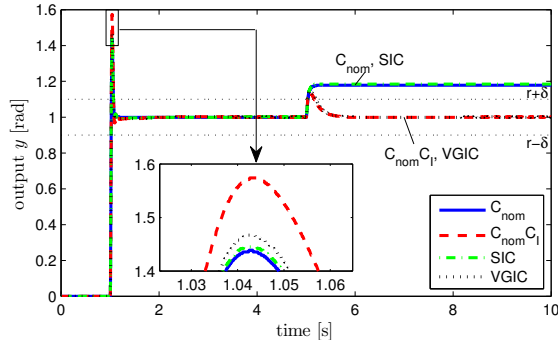


Fig. 11. Experimental comparison different controllers.

TABLE I  
PERFORMANCE MEASURES.

	overshoot	ISE = $\int_0^{10} e^2(t)dt$
$C_{nom}$	43.9 %	0.1750
$C_{nom}C_I$	57.4 %	0.0254
SIC	44.5 %	0.1870
VGIC	46.7 %	0.0230

It is worthwhile to stress the ease of implementation and intuitive design of the proposed nonlinear controllers. The two linear controller limits, with and without integral controller, can be designed using well-known frequency-domain loop-shaping arguments, and form the basis for the nonlinear controller designs. Note that there are no restrictions to the order of the linear plant or nominal controller considered and that only output measurements are used. Moreover, for the VGIC, global asymptotic stability of the equilibrium point can be guaranteed by checking easy-to-use graphical conditions. These graphical conditions also give some insight in the robustness of the problem to changes in parameters. The choice for the saturation length  $\delta$  can be made intuitively using knowledge on the size of the step-reference and the level of force disturbances.

Another aspect, worthwhile mentioning, is the essential difference of the proposed control strategies with reset control schemes in the literature. In reset control, some or all of the controller states are reset. This reset has the capability of improving the transient response of the system, as is illustrated in certain examples in literature [4], [14]. Note, however, that the reset controller drastically changes the dynamics of the closed-loop system by resetting instantaneously some of the controller states to zero. The variable gain controllers as considered in this paper, are very closely related to the linear controllers with and without integrator, which makes the change in dynamics less drastic and more intuitive. From a practical implementation and industrial acceptance point of view, these may be considered beneficial properties.

#### IV. CONCLUSIONS

In this paper, we have focused on a nonlinear variable gain control strategy for transient performance improvement of linear motion systems. In particular, we focused on the trade-off between overshoot and steady-state errors due to constant disturbances, and proposed a switching integral controller

(SIC) and a variable gain integral controller (VGIC) (both without resets) to balance this tradeoff in a more desirable manner. By limiting the amount of integral control action if the error is large, and only switching on the integral part if the error is small, these nonlinear controllers combine the desired effect of small overshoot with constant disturbance suppression. For the VGIC, it is shown that for constant disturbances, the setpoint is globally asymptotically stable if certain easy-to-check graphical conditions are satisfied. The SIC, however, does not exhibit this global stability property and only allows for local asymptotic stability, which is less favorable.

The proposed nonlinear variable gain control strategies have been implemented on an experimental motion control setup. The controllers can be designed using classical linear design techniques, which makes the implementation and design intuitive. It is shown that beneficial transient responses can be obtained in terms of overshoot and integral of the squared error, compared to the linear controllers with and without integrator.

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