Switched position-force tracking control of a manipulator interacting with a stiff environment

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Abstract-This work proposes a control law for a manipulator with the aim of realizing desired time-varying motion/force profiles in the presence of a stiff environment. In many cases, the interaction with the environment affects only one degree of freedom of the end-effector of the manipulator. Therefore, the focus is on this contact degree of freedom, and a switching position-force controller is proposed to perform the hybrid position-force tracking task. Sufficient conditions are presented to guarantee input-to-state stability of the switching closed-loop system with respect to perturbations related to the time-varying desired motion-force profile. The switching occurs when the manipulator makes or breaks contact with the environment. The analysis shows that to guarantee closed-loop stability while tracking arbitrary time-varying motion-force profiles, the controller should implement a considerable (and often unrealistic) amount of damping, resulting in inferior tracking performance. By redesigning the manipulator with a compliant wrist and employing the designed switching control strategy, stable tracking of a motion-force reference trajectory can be achieved and bouncing of the manipulator while making contact with the stiff environment can be avoided.

I. INTRODUCTION

Numerous robotic applications, such as for example bilateral teleoperation, automated assembly tasks, drilling, grinding and surface polishing, involve the interaction between a manipulator and a stiff environment. To this end, many different control architectures have been proposed for a combination of motion and force control (see Chapter 7 of [1] for an overview).

The most studied and applied control schemes include impedance and admittance control [2], [3], hybrid positionforce control [4], [5] and parallel position-force control [6]. These control architectures are designed for the free motion and contact phases separately, and stability is analyzed for these separate regions using standard Lyapunov methods. The transitions between the two phases is not included in the stability analysis, so bouncing or unstable contact behavior might still occur. From a practical point of view, the manipulator is restricted to approach the environment very slowly to prevent contact instability. Therefore, we focus on the design of a controller to also guarantee stability during the transitions. Moreover, since our main research interest is in telerobotics, we are interested in tracking of time-varying motion and force profiles.

Only a few theoretical studies have addressed directly the root cause of the bouncing instability. In [7], [8], a switched position-force controller is considered, where the controller switches from motion to force control when contact with the environment is made. Using analysis techniques for switched systems, conditions for asymptotic stability are derived for a *constant* position or force setpoint regulation problem. Hysteresis switching is considered in [9] to prevent bouncing of the manipulator against the environment. Active impedance control is proposed in [10] for "velocity regulation in free motion, impact attenuation" and tracking of a constant force setpoint in contact. Again for a force regulation task, the number of bounces is minimized in [11] by exploiting a transition controller. In [12], nonlinear damping is proposed to minimize the force overshoot without compromising the settling time. In all these publications, tracking of desired time-varying motion and force profiles, required in applications such as bilateral teleoperation or automated assembly tasks, is not considered.

In the above mentioned papers, the manipulatorenvironment interaction is modeled using a flexible springdamper contact model. The stiffness and damping properties of the environment are modeled explicitly and the impact phase has a finite time duration. This is the approach we also take in this paper. The manipulator-environment interaction can also be studied using the approach of nonsmooth mechanics [13], where the impact phase is instantaneous and a static impact map (e.g., Newton's law of restitution) is employed to characterize the interaction. Stable tracking of specific force/position profiles using such an approach has been addressed in [14], [15], but to the best of authors' knowledge, stable tracking of an arbitrary force/position profile as we consider in this work has not been solved yet.

In this work, we propose a control law for making a manipulator track a *time-varying* motion and force profile. Because in many tasks of practical interest the interaction of the robot end-effector with the environment occurs just in one direction, we derive and study the contact stability problem using a 1-DOF dynamic model. The remaining unconstrained DOFs can be controlled with standard motion control techniques (see [16]). We propose a switched motion-force tracking control strategy and include the transition from free motion to contact in the stability analysis of the closed-loop dynamics. An interesting and unexpected result of our analysis is that the controller should implement a considerable amount of damping to guarantee stability while tracking an arbitrary time-varying motion-force profile. Because an excessing amount of damping limits the track-

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ing due to a sluggish response, we propose an alternative manipulator design by including a compliant wrist. The use of such an "energy absorbing component" is mentioned in [17], but a stability analysis is not considered. The novel result in this work is the combination of the compliant wrist design with the proposed switched motion-force controller and the stability analysis that results in design guidelines for the compliant wrist and controller to guarantee stable contact while tracking arbitrary motion and force profiles. In particular, we show how bouncing of the manipulator against the stiff environment can be prevented without the need of a considerable amount of damping from the controller.

The proofs of all lemmas and theorems in this article are omitted due to space limitations, but can be found in [18].

II. SYSTEM MODELING AND CONTROLLER DESIGN

Our primary goal is to design a controller for making a manipulator track a desired motion-force profile. As explained in the introduction, we focus on a 1-DOF modeling of the manipulator-environment interaction.

The decoupled contact DOF is modeled as

$$M\ddot{x} + b\dot{x} = F_c - F_e,\tag{1}$$

where x represents the manipulator position, M > 0 the equivalent mass of the manipulator, b > 0 the viscous friction in the joint, F_c the control force and F_e the force exerted by the manipulator on the environment. The environment is modeled as a static wall at x = 0 and, without loss of generality, the manipulator is in contact with the environment for x > 0. In [7], [8], the environment is modeled as a piecewise linear spring. We consider, similarly to [9], the Kelvin-Voigt contact model

$$F_e(x, \dot{x}) = \begin{cases} 0 & \text{for } x \le 0\\ k_e x + b_e \dot{x} & \text{for } x > 0 \end{cases}$$
(2)

with $k_e, b_e > 0$ the stiffness and damping properties of the environment, respectively. This model is nonlinear and non-smooth due to the potentially abrupt change in F_e at x = 0.

In free motion, the manipulator is required to follow a bounded desired motion profile $x_d(t)$, whereas in contact, a desired force profile $F_d(t)$ should be applied to the environment. With impedance controllers the contact force is controlled indirectly. Instead, we propose the following switched motion-force controller that switches between a resolved acceleration controller in free motion and a force controller in the contact phase:

$$F_{c} = \begin{cases} M_{a}\ddot{x}_{d}(t) + k_{d}(\dot{x}_{d}(t) - \dot{x}) + k_{p}(x_{d}(t) - x), \ \forall x \leq 0, \ (3a) \\ F_{d}(t) + k_{f}(F_{d}(t) - F_{e}) - b_{f}\dot{x}, \qquad \forall x > 0, \ (3b) \end{cases}$$

such that both motion and force are controlled directly. Here, $k_p > 0$ and $k_d > 0$ are the proportional and derivative gains of the motion controller, respectively. The estimated mass of the manipulator $M_a > 0$ might differ from the actual mass M due to uncertainties in the model parameter identification. The gain $k_f > 0$ represents the proportional term of the force controller and $b_f > 0$ is the damping gain, dissipating energy during the contact phase. For the

controller (3), it is assumed that the contact force F_e , position x and velocity \dot{x} can be measured. Note that the controller switches based on x instead of the contact force F_e . However, for stiff environments, $k_e \gg b_e$, such that switching based on either x > 0 or $F_e > 0$ can be considered similar. A priori knowledge of the location of the environment is not required for the implementation of the controller (3).

In order to analyze stability of the system described by (1)-(3), we reformulate the closed-loop dynamics as a switching state-space model. A key idea for the stability analysis, detailed in Section III, is to express the force tracking error $F_d(t) - F_e$ in terms of the motion tracking error $x_d(t) - x$, such that both in free motion and in contact the goal is to make the tracking error $x_d(t) - x$ small. In contact, $x_d(t)$ represents the 'virtual' desired trajectory, corresponding to the desired contact force $F_d(t)$. For the relationship between $F_d(t)$ and $x_d(t)$ during contact, $x \to x_d(t)$ should also imply $F_e \to F_d(t)$. To this end, we consider the following relationship to deduce $x_d(t)$ from $F_d(t)$ in the contact phase

$$F_d(t) = \hat{k}_e x_d(t) + \hat{b}_e \dot{x}_d(t), \quad \text{for } F_d(t) > 0,$$
 (4)

where \hat{k}_e and \hat{b}_e are available estimates of k_e and b_e . When these estimates are exact, $x - x_d(t) \rightarrow 0$ indeed implies $F_e - F_d(t) \rightarrow 0$.

Assumption 1: The desired trajectory $x_d(t)$ in (3a) and (4) is bounded and twice differentiable.

Assumption 1 implies that $\ddot{x}_d(t)$ is also bounded (and defined almost everywhere). For desired motion and force profiles provided by a user, the satisfaction of Assumption 1 can be guaranteed by applying the method proposed in [18].

In terms of the exact environment parameters k_e and b_e , (4) can be rewritten as

$$F_d(t) = k_e x_d(t) + b_e \dot{x}_d(t) + w_f(t), \text{ for } F_d(t) > 0,$$
 (5)

with $w_f(t) := (\hat{k}_e - k_e)x_d(t) + (\hat{b}_e - b_e)\dot{x}_d(t)$ a bounded perturbation due to Assumption 1. The tracking error

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} := \begin{bmatrix} x_d(t) - x \\ \dot{x}_d(t) - \dot{x} \end{bmatrix},\tag{6}$$

can be used to rewrite the closed-loop system dynamics (1)-(3) and (5) as the perturbed switched system

$$\Sigma^{p}: \quad \dot{z} = A_{i}z + Nw_{i}(t) = \begin{bmatrix} 0 & 1\\ -K_{i} & -B_{i} \end{bmatrix} z + Nw_{i}(t),$$
$$\forall t, z \in \Omega_{i}(t), i \in \{1, 2\}, \tag{7}$$

where $N = [0, 1]^T$,

$$K_1 := \frac{k_p}{M}, \quad B_1 := \frac{k_d + b}{M}, \tag{8a}$$

$$K_2 := \frac{(1+k_f)k_e}{M}, \quad B_2 := \frac{(1+k_f)b_e + b_f + b}{M},$$
(8b)

$$w_1(t) := \frac{M - M_a}{M} \ddot{x}_d(t) + \frac{b}{M} \dot{x}_d(t), \tag{8c}$$

$$w_2(t) := \ddot{x}_d(t) + \frac{b_f + b}{M} \dot{x}_d(t) - \frac{1}{M} w_f(t).$$
 (8d)

The perturbations $w_i(t)$, $i = \{1, 2\}$ are bounded, because by Assumption 1, $x_d(t)$, $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ are bounded. All system parameters are positive, so $K_i, B_i > 0$ and A_i in (7) is Hurwitz for $i = \{1, 2\}$. The environment is located at x = 0, so switching occurs at $x = x_d(t) - z_1 = 0$. Expressed in the z-coordinates, the free motion and contact subspaces, respectively denoted by Ω_1 and Ω_2 , are timevarying: $\Omega_1(t) := \{z \in \mathbb{R}^2 | x_d(t) - z_1 \le 0\}$ and $\Omega_2(t) :=$ $\{z \in \mathbb{R}^2 | x_d(t) - z_1 > 0\}$. Note that for all t, $\Omega_1(t) \cup \Omega_2(t) =$ \mathbb{R}^2 and $\Omega_1(t) \cap \Omega_2(t) = \emptyset$.

In practice, the environment stiffness k_e is typically extremely higher than the control gain k_p . The true value of k_e and b_e are usually unknown, so the control parameters generically cannot be selected to result in $K_1 = K_2$ and $B_1 = B_2$. Thus, in general, Σ^p represents a switched system, as $K_1 \neq K_2$ and $B_1 \neq B_2$. The stability of Σ^p does not follow from the stability of each of the two continuous subsystems (corresponding to free motion and contact) taken separately, as shown, e.g., in [4], [6]. The switching between the two subsystems, corresponding to making and breaking contact, must also be taken into account [19].

III. STABILITY ANALYSIS

In this section, we provide sufficient conditions to make (7) input-to-state stable (ISS) with respect to the input $w_i(t)$, $i = \{1, 2\}$. Note that $w_i(t)$ depends $x_d(t)$, thus encoding the information of $F_d(t)$ during the contact phases.

The following definitions, taken from [20], are required for the stability analysis.

Definition 1: Consider a region $\mathcal{T}_i \subset \mathbb{R}^2$. If $z \in \mathcal{T}_i$ implies $cz \in \mathcal{T}_i$, $\forall c \in (0, \infty)$ and $\mathcal{T}_i \setminus \{0\}$ is connected, then \mathcal{T}_i is a **cone**.

Definition 2: Let $\dot{z} = A_i z$ be the dynamics on an open cone $\mathcal{T}_i \subset \mathbb{R}^2$, i = 1, ..., m. An eigenvector of A_i is visible if it lies in $\overline{\mathcal{T}}_i$, the closure of \mathcal{T}_i .

As a stepping stone towards proving ISS of (7), we provide sufficient conditions for the global uniform exponential stability (GUES) of the origin of Σ^p when $w_i \equiv 0$. This corresponds to studying the unperturbed system

$$\Sigma^{u}: \quad \dot{z} = A_{i}z \quad \forall z \in \Omega_{i}(t). \tag{9}$$

The GUES of Σ^u for any $x_d(t)$ satisfying Assumption 1 can be concluded by considering the worst-case switching sequence [19]. In this way, we obtain the time-invariant system Σ^w , defined below, with state-based switching, that represents the worst-case switching sequence for Σ^u in (9). The worst-case switching sequence is defined as the sequence that results in the slowest convergence of the solution of Σ^u towards the origin. The solution of Σ^u starting from z_0 at t_0 will be written as $z(t) = \Phi_u(t, t_0; \sigma) z_0$, with $\Phi_u(t, t_0; \sigma)$ denoting the state transition matrix associated with the switching sequence $\sigma(t) : \mathbb{R} \to \{1, 2\}$. For $K_2 > K_1$, representing a manipulator interacting with a stiff environment, the worst-case dynamical system Σ^w is characterized by the following lemma.

Lemma 1: Consider

$$\Sigma^w: \quad \dot{z} = A_i z, \qquad \forall z \in \mathcal{S}_i, \tag{10}$$

with A_1 and A_2 as in (7), assume $K_2 > K_1$ and let

$$S_1 = \{ z \in \mathbb{R}^2 | z_2((K_1 - K_2)z_1 + (B_1 - B_2)z_2) \le 0 \}, S_2 = \{ z \in \mathbb{R}^2 | z_2((K_1 - K_2)z_1 + (B_1 - B_2)z_2) > 0 \}.$$



Fig. 1. Switching surfaces and domains of Σ^w for $K_2 > K_1$ and $B_2 > B_1$. The vectors v_1^1 and v_2^1 represent an example of real eigenvectors of S_1 . Here, only v_2^1 lies in the closure of S_1 , so only v_2^1 is visible.

For the solution of Σ^u in (9) corresponding to an arbitrary switching signal $\sigma(t)$ and initial condition z_0 , $\|\Phi_u(t,t_0;\sigma)z_0\| \leq \|\Phi_w(t,t_0)z_0\|$ for $t \geq t_0$, where Φ_w denotes the state transition matrix of Σ^w in (10). We will refer to $\Phi_w(t,t_0)z_0$, $t \geq t_0$, as the worst-case response of Σ^u with initial condition z_0 .

In the following Theorem 1, necessary and sufficient conditions for the global uniform asymptotic stability (GUAS) of Σ^w are given. We then show in Lemma 2, that GUAS of Σ^w implies GUES of Σ^u and this in turn implies ISS of Σ^p w.r.t. w_i for an arbitrary $x_d(t)$ satisfying Assumption 1. This result is given in Theorem 2 at the end of this section and is the main result of this paper.

From the definition of S_1 and S_2 given in Lemma 1, we obtain the two switching surfaces $z_2 = 0$ and $(K_1 - K_2)z_1 + (B_1 - B_2)z_2 = 0$ that characterize the worst-case switching. These switching surfaces and the subsystems of Σ^w that are active between the switching surfaces are visualized in Fig. 1 for $K_2 > K_1$ and $B_2 > B_1$. We refer the interested reader to the appendix for further details about the background material used to obtain the following results.

Theorem 1: Let $K_i, B_i > 0$, $\Delta K := K_1 - K_2 < 0$ and $\Delta B := B_1 - B_2$. The origin of the unperturbed, conewise linear system Σ^w is GUAS if at least one of the following conditions is satisfied:

- *i.* Σ^w has a visible eigenvector associated with an eigenvalue $\lambda < 0$; in other words, one of the following two conditions is satisfied:
 - a) a visible eigenvector exists in S_1 , i.e.,

$$B_1^2 \ge 4K_1 \text{ and } \frac{\Delta K}{\Delta B} < \frac{2K_1}{B_1 - \sqrt{B_1^2 - 4K_1}}.$$
 (11)

b) a visible eigenvector exists in S_2 , i.e., $B_2^2 \ge 4K_2$ and one of the following conditions is satisfied:

1)
$$\Delta B < 0$$
 and $\frac{\Delta K}{\Delta B} > \frac{2K_2}{B_2 + \sqrt{B_2^2 - 4K_2}}$, or
2) $\Delta B \ge 0$.

ii. Σ^w has no visible eigenvectors and Λ := Λ₁Λ₂ < 1, where Λ_i, i = {1,2}, are given by:
1) if B_i² < 4K_i,

$$\Lambda_i = \left(\frac{K_i}{\omega_i} \left(\frac{(\Delta K)^2}{L^2} + \frac{Q^2}{4\omega_i^2 L^2}\right)^{-1/2}\right)^{(-1)^i} e^{-\frac{B_i}{2\omega_i}\varphi_i}$$
(12)

with
$$\varphi_i := \mod\left(-\arctan\left(\frac{(-1)^i 2\omega_i \Delta K}{Q}\right), \pi\right),$$

 $Q := B_i \Delta K - 2K_i \Delta B, \ \omega_i := \frac{1}{2}\sqrt{4K_i - B_i^2} \text{ and }$
 $L := \sqrt{(\Delta K)^2 + (\Delta B)^2}.$
2) if $B_i^2 = 4K_i,$
 $\Lambda_i = \left|\frac{B_i L}{2\Delta K - B_i \Delta B}\right| e^{\left((-1)^i \frac{2\Delta K}{2\Delta K - B_i \Delta B}\right)}.$ (13)
3) if $B_i^2 > 4K_i,$
 $\Lambda_i = \left|\frac{\Delta K \lambda_{bi} + K_i \Delta B}{K L}\right|^{\phi_1} \left|\frac{\Delta K \lambda_{ai} + K_i \Delta B}{K L}\right|^{\phi_2}$ (14)

with
$$\phi_1 := \frac{(-1)^i \lambda_{ai}}{\lambda_{bi} - \lambda_{ai}}$$
, $\phi_2 := \frac{(-1)^i \lambda_{bi}}{\lambda_{ai} - \lambda_{bi}}$,
 $\lambda_{ai} := \frac{-B_i - \sqrt{B_i^2 - 4K_i}}{2}$, and $\lambda_{bi} := \frac{-B_i + \sqrt{B_i^2 - 4K_i}}{2}$.

This proposition can be interpreted as follows. If the system Σ^{w} does not have a visible eigenvector, the response spirals around the origin and visits the regions S_1 and S_2 infinitely many times. The worst-case system Σ^w switches between free motion and contact, but if $\Lambda < 1$ the resulting bouncing behavior is asymptotically stable, implying that the amplitude of the oscillation decays over time. Furthermore, since the trajectory leaves each cone in *finite* time (see Lemma 3 in [18]), the time between two switches is fixed and finite, implying that Zeno behavior (infinitely many switches in finite time) of Σ^w is excluded. If Σ^w does have a visible eigenvector with $\lambda < 0$, the response converges to the origin exponentially without leaving the cone (see Lemma 4 in [18]). The system does not switch between free motion and contact and bouncing of the manipulator against the environment does not occur.

Lemma 2: If Σ^w in (10) is GUAS, then the origin of Σ^u in (9) is GUES for arbitrary $x_d(t)$ satisfying Assumption 1.

From Lemma 2 it follows that Σ^u is GUES if Σ^w GUAS, which is guaranteed when one of the conditions in Theorem 1 holds. Using Lemma 2, the following proposition provides conditions for ISS of the perturbed system Σ^p in (7).

Theorem 2: Consider the perturbed system Σ^p in (7), with piecewise-continuous, bounded input $w_i(t)$. If the origin of the unperturbed system Σ^u in (9) is GUES for arbitrary $x_d(t)$ satisfying Assumption 1, which is guaranteed if the conditions in Lemma 2 hold, then Σ^p is ISS w.r.t. $x_d(t)$.

This proposition can be interpreted as follows. If $Nw_i(t) \equiv 0$, the response of Σ^p is equivalent to the response of Σ^u , whose origin is GUES. Due to (5), $x_d(t)$ encodes the information of $F_d(t)$ during the contact phase, so $x \rightarrow x_d(t)$ and $F_e \rightarrow F_d(t)$ exponentially. If $Nw_i(t) \neq 0$, the response of Σ^p deviates from the response of Σ^u , (i.e. x and F_e only converge to neighbourhoods of $x_d(t)$ and $F_d(t)$, respectively), but due to the ISS property the response of Σ^p is bounded and the bound on the error norm ||z||, with z defined in (6), depends on the norm of the perturbation Nw_i .

IV. A STIFF ENVIRONMENT EXAMPLE

We now illustrate the use of the theory developed by means of simulations and show the implications of satisfying Theorem 2 on the controller design. Consider a manipulator with M = 1 kg and b = 0 Ns/m (i.e. no viscous friction



Fig. 2. Simulation results with $b_f = 5$. The grey area indicates the contact phase.

is present in the manipulator to help dissipate energy), interacting with an environment with $k_e = 10^6$ N/m and $b_e = 10$ Ns/m. For the control parameters we choose $M_a = 0.8 \, \mathrm{kg}, \, k_p = 4000, \, k_d = 80, \, k_f = 1 \, \mathrm{and}$ $b_f = 5$. For this parameter set, the eigenvectors of A_2 in (7) are complex, such that no visible eigenvectors exist in the contact phase (see Definition 2). The eigenvectors of A_1 in (7) are real, but not visible. The response of the system is shown in Fig. 2. Although $x_d(t)$ and $F_d(t)$ used for the simulation in Fig. 2 are not necessarily worst-case inputs, the value $\Lambda = 10.16$ indicates that the system is *potentially* unstable (as the worst-case system Σ^w is unstable, see Theorem 1). Clearly, the controller tracks $x_d(t)$ in free motion, but due to the stiff environment and nonzero impact velocity, a large peak force occurs (see bottom plot in Fig. 2). The manipulator bounces back from the environment and breaks contact. During the 0.15 s of intended contact, the manipulator continues to bounce and is not able (see Fig. 2) to track the desired contact force $F_d(t)$, which has a maximum of 7 N. Around 0.27 s the motion controller is no longer able to bring the manipulator in contact with the environment due to the relatively large negative derivative term in (3a). The amplitude of the bouncing does decay over time, but Fig. 2 clearly illustrates an undesired response. The problem is the lack of damping in contact. Increasing the damping level in the force controller to $b_f = 9000$ results in $\Lambda = 0.98$, such that the origin of Σ^{w} is GUAS (see Theorem 1) and the system Σ^p is ISS, for any $x_d(t), F_d(t)$ (see Theorem 2) satisfying Assumption 1. With $b_f = 9000$, the manipulator does not bounce against the environment (see Fig. 3) and, after the peak impact force, the contact force F_e approximately tracks $F_d(t)$. For the parameter values of this example, it is not sufficient to change only b_f to satisfy one of the conditions *i.b*) in Theorem 1 and obtain a visible eigenvector of the system Σ^w in the contact phase S_2 .

V. COMPLIANT MANIPULATOR DESIGN

This section discusses the motivation and design of a compliant manipulator, and shows how Theorem 2 can be used to assign parameter values to the introduced compliancy.

A. Motivation and design

A drawback of the high damping gain b_f used in the simulation in Fig. 3 is that it results in a lag in tracking



Fig. 3. Simulation results with $b_f = 9000$. The grey area indicates the contact phase.



Fig. 4. Manipulator with compliant wrist.

 $F_d(t)$ for $t \in [0.17, 0.28]$ (sluggish response). In practice, most manipulators are not equipped with velocity sensors, so the velocity signal \dot{x} , used in (3b), must be obtained from the position measurements. Due to measurement noise, encoder quantization and a finite sample interval, a high damping value b_f in (3b) to compensate the lack of damping of the contact case is not desired/impossible to be implemented in practice. An observer to estimate \dot{x} is not considered as a practical solution due to the required accuracy of the high frequency of the impact oscillations.

Inspired by the favorable properties of the skin around a human finger, we propose, as a more practical alternative, to design the manipulator by including passive compliance in the connection between the arm and the end-effector (wrist) as shown in Fig. 4. Indicating with x_t the position of the end-effector, with k_t and b_t , respectively, the stiffness and damping coefficient of the wrist and F_t the internal force, the dynamics of this system are given by

$$M\ddot{x} + b\dot{x} = F_c - F_t,\tag{15a}$$

$$M_t \ddot{x}_t = F_t - F_e(x_t, \dot{x}_t), \tag{15b}$$

$$F_t = k_t(x - x_t) + b_t(\dot{x} - \dot{x}_t).$$
 (15c)

The environment model and controller design are again given by (2) and (3), respectively. Due to the *passive* compliance, only x in (15) is actuated and (3) controls x to $x_d(t)$.

The compliant wrist and end-effector are designed to improve the response during and after the impact phase. So, we consider a design where the mass M_t is smaller than Mto reduce the kinetic energy of M_t engaged at impact. The damping b_t is larger than b_e to help dissipate the impact energy and provide more damping in the contact phase. The stiffness $k_t \ll k_e$ (k_e is much larger than all other parameters) to reduce the eigenfrequency and increase the damping ratio of the contact phase. Hence, we assume that

$$M_t \ll M, \ k_t \ll k_e, \ b_t \gg b_e, \ \text{and} \ b_t \ll k_e.$$
 (16)

B. Model of reduced order

The stability results of Section III only apply to twodimensional systems. The dynamics of the 2-DOF compliant manipulator of (15) is 4-dimensional, so Theorem 1 cannot be applied directly. However, when (16) is satisfied, the compliant 2-DOF manipulator (15) exhibits a clear separation between fast and slow dynamics. In free motion, the fast dynamics are related to $x - x_t$, and, in contact, to the endeffector position x_t . The time-scale of the (exponentially stable) fast dynamics is very small compared to the timescale of interest, so the slow dynamics can be considered as the dominant dynamics describing the response x of the compliant manipulator to the control input $F_c(t)$.

Consider the 2-DOF compliant manipulator (15), (2) with $M \sim 10^0$, $b \sim 10^0$, $M_t \sim 10^{-2}$, $k_t \sim 10^4$, $b_t \sim 10^2$, $k_e \sim 10^6$ and $b_e \sim 10^1$. The model reduction analysis in [18] shows that the slow time-scale response of this system in free motion and contact considered separately can be approximated by the following model of reduced (2nd)order:

$$M\ddot{x} + b\dot{x} = F_c - \bar{F}_e(x, \dot{x}), \qquad (17)$$

$$\bar{F}_e(x, \dot{x}) = \begin{cases} 0 & \text{for } x \le 0\\ \bar{b}_e \dot{x} + \bar{k}_e x & \text{for } x > 0 \end{cases}$$
(18)

with $\bar{b}_e := b_t \frac{k_e}{k_t + k_e}$ and $\bar{k}_e := k_t \frac{k_e}{k_t + k_e}$. The fraction $\frac{k_e}{k_t + k_e} \approx 1$ for $k_t \ll k_e$, so k_t and b_t directly influence the perceived environment damping and stiffness by the mass M.

The reduced-order dynamics (17), (18) are obtained separately for the free motion and contact case. During free motion to contact transitions, the high-frequency dynamics of (15), (2), which are not captured in (17), (18), might still be excited. However, the simulations provided in [18] indicate that the response of (17), (18) accurately approximates the response of (15), (2), subject to (16) and controlled by (3). Hence, the reduced-order model (17), (18) can be used to analyze stability of (15), (2), in closed-loop with (3).

C. Stability of the reduced-order model

Since (17), (18) has exactly the same structure as (1), (2), we employ the same stability analysis as in Section III to design the parameters of the controller in (3). In contact, we use a similar expression to relate $F_d(t)$ to $x_d(t)$, namely

$$F_d(t) = \bar{k}_e x_d(t) + \bar{b}_e \dot{x}_d(t) + \bar{w}_f(t), \quad \text{for } F_d(t) > 0$$
 (19)

with $\bar{w}_f(t) := (\bar{k}_e - \bar{k}_e)x_d(t) + (\bar{b}_e - \bar{b}_e)\dot{x}_d(t)$, and \bar{k}_e and \bar{b}_e available estimates of \bar{k}_e and \bar{b}_e , respectively. The design of the desired trajectories such that $x_d(t)$ is bounded and twice differentiable is discussed in [18].

The system described by (17), (18), (3) and (19) can be expressed in the form Σ^p of (7), with (8a), (8c), (8d) and

$$K_2 := \frac{(1+k_f)\bar{k}_e}{M}, \quad B_2 := \frac{(1+k_f)\bar{b}_e + b_f + b}{M}.$$
 (20)

As a result, ISS can be concluded from Theorem 2 for arbitrary $x_d(t)$ satisfying Assumption 1 if the conditions of Theorem 1 are satisfied. Compared to the system without compliant wrist, we now have more flexibility to tune the parameters for stability and performance. From Theorem 1 we can compute the required values of the design parameters



Fig. 5. Simulation results of compliant manipulator described by (15). The grey area indicates the contact phase.

 k_t and b_t to meet design specifications such as the existence of a visible eigenvector with a stable eigenvalue (implying bounceless impact) or an upper bound on Λ in Theorem 1.

D. Compliant manipulator example

The following example illustrates how to design the compliant wrist parameters M_t , b_t and k_t to improve the performance compared to the simulation results of Fig. 2. For the design of the end-effector, consider $M_t = 0.05$ kg and $k_t = 5 \cdot 10^4$ N/m ($k_t \ll k_e$, but still large to minimize the spring-travel in the wrist). With $b_f = 5$ Ns/m, we require $b_t > 170$ Ns/m to guarantee that $\Lambda < 1$, such that one of the conditions of Theorem 1 is satisfied. Fig. 5 shows the response of the *unreduced* compliant system (15), (3) and (2), with $b_t = 171$ Ns/m. Compared to Fig. 2, the peak impact force is reduced. During the first 20 ms of intended contact, the tip makes and breaks contact due to the fast dynamics of (15). After 20 ms the fast dynamics of (15) damp out, the slow dynamics become dominant and the response of (15) converges to that of (17). Hence, F_e tracks the desired trajectory $F_d(t)$ (without a sluggish response as in Fig. 3).

E. Discussion

From the expressions \bar{k}_e and \bar{b}_e in (18) and the results in Fig. 5, we see that the compliance in the manipulator can contribute to guarantee stability and improve the tracking performance during free motion to contact transitions. Due to the compliance, we can lower the stiffness and increase the damping of the perceived manipulator-environment connection in contact. As a result, the controllers (3a) and (3b) can be tuned separately for optimal performance in free motion and contact respectively, rather than a trade-off to guarantee stability during transitions in case of a rigid manipulator. With $b_t \gg b_e$, the end-effector acts as a vibration-absorber, dissipating the kinetic energy present at impact.

VI. CONCLUSION

We consider the position-force control of a manipulator in contact with a stiff environment. Since most contacts affect only one DOF of the end-effector of the manipulator, we focus on this DOF only. For this decoupled direction, we have proposed a novel switching controller that, if tuned properly, ensures tracking of time-varying motion and force profiles. Moreover, we have proposed sufficient conditions for the input-to-state stability (ISS) of the closed-loop tracking error dynamics with respect to perturbations related to the time-varying desired motion-force profile. With a numerical example, the stability analysis shows that, in contact, a high level of controller damping is required to guarantee stability of the closed-loop system when tracking a desired positionforce profile. Such high-gain velocity feedback is undesirable in practice and probably not physically realizable. Therefore, we have proposed a mechanical design of the manipulator with a compliant wrist, in combination with the proposed switching controller, as a favorable alternative. We illustrated how to design the damping and stiffness of this compliant wrist and the control parameters to guarantee stability and even prevent persistent bouncing of the manipulator against the environment for arbitrary desired motion-force profiles.

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