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Brief paper

A discrete-time framework for stability analysis of nonlinear networked control systems[☆]

N. van de Wouw[a,1], D. Nešić[b], W.P.M.H. Heemels[a]

[a] Eindhoven University of Technology, Department of Mechanical Engineering, P.O. Box 513, NL 5600 MB Eindhoven, The Netherlands

[b] Department of Electrical and Electronic Engineering, University of Melbourne, Parkville VIC 3010, Australia

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ABSTRACT

In this paper we develop a prescriptive framework for the stabilising controller design based on approximate discrete-time models for nonlinear Networked Control Systems (NCSs) with time-varying sampling intervals, large time-varying delays and packet dropouts. As opposed to emulation-based approaches where the effects of sampling-and-hold and delays are ignored in the phase of controller design, we propose an approach in which the controller design is based on approximate discrete-time models constructed for a set of nominal (non-zero) sampling intervals and nominal delays while taking into account sampling-and-hold effects. Subsequently, sufficient conditions for the global exponential stability of the closed-loop NCS are provided.

1. Introduction

Networked control systems (NCSs) are control systems in which sensor data and control commands are being communicated over a wired or wireless communication network. The recent increase of interest in NCSs is motivated by the many benefits they offer such as ease of maintenance and installation, large flexibility and low cost. Moreover, NCSs are applied in a broad range of systems, such as mobile sensor networks, remote surgery, automated highway systems and unmanned aerial vehicles. However, many challenges still need to be faced before all the advantages of networked control systems can be exploited to their full extent. One of the major challenges is related to guaranteeing the robustness of stability (and performance) of the control system in the face of imperfections and constraints imposed by the communication network, such as variable sampling/transmission intervals, variable communication delays and packet dropouts caused by the unreliability of the network, so-called communication constraints caused by the sharing of the network by multiple nodes and quantization-related errors.

Most of the work on NCSs has been focussing on the stability analysis of *linear* NCSs, in which different approaches towards the modelling and stability analysis have been developed. In Gao, Chen, and Lam (2008), Naghshtabrizi, Hespanha, and Teel (2010) and van de Wouw, Naghshtabrizi, Cloosterman, and Hespanha (2010) a continuous-time modelling approach is taken leading to NCS models in terms of (impulsive) delay-differential equations (DDEs) and stability analysis results based on the Razumikhin and Lyapunov–Krasovskii functional methods. Discrete-time approaches, based on the exact discretisation of the linear plant (typically on the sampling instants) have been developed in Cloosterman et al. (2010, 2009), Fujioka (2009), Garcia-Rivera and Barreiro (2007), Hetel, Daafouz, and Iung (2006), Sala (2005), van de Wouw et al. (2010) and Zhang, Branicky, and Phillips (2001); Zhang, Shi, Chen, and Huang (2005) and many others.

Results on the stability analysis and controller design for *nonlinear* NCSs have also been obtained in the literature. In Yu, Wang, and Chu (2005), Cao, Zhong, and Hu (2008) a continuous-time approach leading to NCS models in terms of DDEs and a stability analysis based on Lyapunov–Krasovskii functionals is pursued for certain classes of nonlinear systems. Results

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E-mail addresses: N.v.d.Wouw@tue.nl (N. van de Wouw), dnesic@unimelb.edu.au (D. Nešić), W.P.M.H.Heemels@tue.nl (W.P.M.H. Heemels).

[1] Tel.: +31 40 247 3358; fax: +31 40 246 1418.

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on the stabilisation of nonlinear systems with limited-capacity communication channels (i.e. quantisation-related issues) have been reported in Liberzon and Hespanha (2005), Savkin and Cheng (2007). Model predictive control strategies for nonlinear NCSs can be found in e.g. Munoz de la Pena and Christofides (2008) and Liu, Munoz de la Pena, Christofides, and Davis (2009). In Heemels, Teel, van de Wouw, and Nesić (2010), Nesić and Teel (2004a) and Walsh, Belidman, and Bushnell (2001) a comprehensive emulation-based framework for the stability analysis of nonlinear NCSs has been developed, where the control design is based on the continuous-time plant, ignoring the effect of sampling-and-hold and the network, and stability analysis is performed on the basis of a hybrid systems model of the NCS. These results consider network-induced effects such as time-varying sampling intervals, delays, packet dropouts, communication constraints and quantisation; however, the results are limited to the case of delays smaller than the sampling interval.

Results on discrete-time approaches for nonlinear NCSs are rare. Some extensions of the discrete-time approach for sampled-data systems as developed in Nesić and Teel (2004b) and Nesić, Teel, and Kokotovic (1999) towards NCS-related problem settings have been pursued in Polushin and Marquez (2004, 2008). In Polushin and Marquez (2004), an extension towards multi-rate sampled-data systems is proposed. In Polushin and Marquez (2008), results for NCSs with time-varying sampling intervals and delays for a specific predictive control scheme and matching protocol are presented. However, in these results the delays are always assumed to be a multiple of the sampling interval and delays are artificially elongated to match a 'worst-case' delay.

In this paper, we consider the problem setting of a nonlinear system being controlled by a digitally implemented (discrete-time) nonlinear controller over a communication network. In particular, we develop a prescriptive framework for the stabilising controller design based on approximate discrete-time models for NCSs with time-varying sampling intervals, potentially large (i.e. larger than the sampling interval) and time-varying delays, not being limited to multiples of the sampling interval, and packet dropouts. Although an emulation-based approach is powerful in its simplicity since, in the phase of controller design, one ignores sampled-data and network effects, an approach towards stability analysis and controller design based on approximate discrete-time models may exhibit several advantages over an emulation-based approach. Firstly, in the emulation approach one typically designs the controller for the case of fast sampling (and no delay) and subsequently investigates the robustness of the resulting closed-loop NCS with respect to uncertainties in the sampling intervals (and delays), see Heemels et al. (2010) and Nesić and Teel (2004a). In the context of networked control one generally faces the situation in which sampling intervals exhibit some level of jitter (uncertainty) around a nominal (non-zero) sampling interval and the delays exhibit some uncertainty around a nominal delay. It appeals to our intuition, which is supported by earlier results for nonlinear sampled-data systems in Laila, Nesic, and Astolfi (2006), Nesić and Teel (2004b) and Nesić et al. (1999) that it is beneficial to design a discrete-time controller based on a nominal (non-zero) sampling interval and a nominal delay. Secondly, it has been shown in Laila et al. (2006) and Nesić et al. (1999) for the case of nonlinear sampled-data systems with fixed sampling intervals (and no delays) that controllers based on approximate discrete-time models may provide superior performance (in terms of the domain of attraction and convergence speed) compared to emulation-based controllers. Finally, we would like to note that, for the case of linear NCSs, it has been shown in Donkers, Heemels, Hetel, van de Wouw, and Steinbuch (2011), that the discrete-time approach may provide less conservative bounds on sampling intervals and delays.

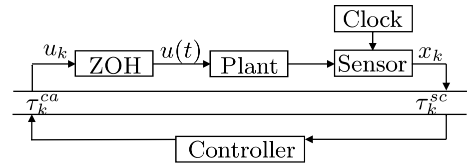

Fig. 1. Schematic of the networked control system.

The main contribution of this paper can be summarised as follows. We extend the results of Nesić et al. (1999) and Nesić and Teel (2004b) on the stabilisation of nonlinear sampled-data systems based on approximate discrete-time models to the case with time-varying and uncertain sampling intervals and delays. Based on such an extension, we develop a prescriptive framework for the design of robustly stabilising discrete-time controllers for nonlinear NCSs with time-varying sampling intervals, large time-varying delays and packet dropouts. In this sense it also extends results on the discrete-time approach for *linear* NCSs with such network-induced uncertainties, as developed in Cloosterman et al. (2010, 2009), Fujioka (2009), Garcia-Rivera and Barreiro (2007), Hetel et al. (2006), Sala (2005), van de Wouw et al. (2010) and Zhang et al. (2001, 2005) and exploiting exact discretisations of the sampled-data NCS dynamics, to the realm of *nonlinear* systems.

The outline of the paper is as follows. In Section 2, an (approximate) discrete-time modelling approach for nonlinear NCSs will be discussed. Based on the resulting approximate discrete-time models and discrete-time controllers designed to stabilise these approximate models, we propose sufficient conditions for the global exponential stability of the closed-loop sampled-data NCS in Section 3. The results are illustrated by means of an example in Section 4. Finally, concluding remarks are given in Section 5. The proofs can be found in Appendix A.

The following notational conventions will be used in this paper. $\mathbb{R}$ denotes the field of all real numbers and $\mathbb{N}$ denotes all nonnegative integers. By $|\cdot|$ we denote the Euclidean norm. A function $\alpha: [0, \infty) \rightarrow [0, \infty)$ is said to be of class-$\mathcal{K}$ if it is continuous, zero at zero and strictly increasing. It is of class-$\mathcal{K}_\infty$ if it is of class-$\mathcal{K}$ and unbounded. For a locally Lipschitz function $f(x)$, $\partial f(x)$ denotes the generalised differential of Clarke.

2. Discrete-time modelling of nonlinear NCSs

Consider a NCS as depicted schematically in Fig. 1. The NCS consists of a nonlinear continuous-time plant

$$\dot{x} = f(x, u), \tag{1}$$

where $f(0, 0) = 0$ and f is globally Lipschitz in x and u, $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the continuous-time control input, and a discrete-time static time-invariant controller, which are connected over a communication network that induces delays (τ^{sc} and τ^{ca}). The state measurements of the plant are being sampled by a time-driven sampler at the sampling instants s_k, $k \in \mathbb{N}$, with $s_0 = 0$. The related sampling intervals $h_k = s_{k+1} - s_k$ are time-varying and satisfy $h_k \in [\underline{h}, \overline{h}]$, $k \in \mathbb{N}$, with $0 < \underline{h} \leq \overline{h}$. We denote $x_k := x(s_k)$. Moreover, u_k denotes the discrete-time controller command corresponding to x_k. In the model, both the varying computation time (τ_k^c), needed to evaluate the controller, and the time-varying network-induced delays, i.e. the sensor-to-controller delay (τ_k^{sc}) and the controller-to-actuator delay (τ_k^{ca}), are taken into account. As stated above, the sensor acts in a time-driven fashion and we assume that both the controller and the actuator act in an event-driven fashion (i.e. they respond instantaneously to newly arrived data). Under these assumptions and given the fact that the controller is static and time-invariant, all three delays can be captured by a single delay $\tau_k := \tau_k^{sc} + \tau_k^c + \tau_k^{ca}$

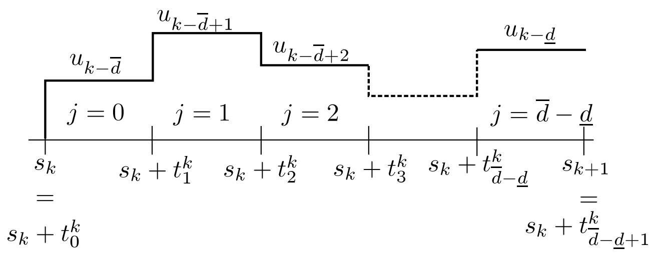

Fig. 2. Graphical illustration of t_j^k.

(Zhang et al., 2001). Furthermore, we model the occurrence of message rejection, i.e. the effect that older data is neglected because more recent control data is available before the older data. We assume that the time-varying delays are bounded according to $\tau_k \in [\underline{\tau}, \overline{\tau}]$, $k \in \mathbb{N}$, with $0 \le \underline{\tau} \le \overline{\tau}$. Note that the delays may be both smaller and larger than the sampling interval. Define $\underline{d} := \lfloor \underline{\tau}/\bar{h} \rfloor$, the largest integer smaller than or equal to $\underline{\tau}/\bar{h}$ and $\bar{d} := \lceil \frac{\overline{\tau}}{\underline{h}} \rceil$, the smallest integer larger than or equal to $\overline{\tau}/\underline{h}$. Finally, the zero-order-hold (ZOH) function (in Fig. 1) is applied to transform the discrete-time control inputs u_k, $k \in \mathbb{N}$, to a continuous-time control input $u(t) = u_{k^*(t)}$, where $k^*(t) := \max\{k \in \mathbb{N} | s_k + \tau_k \le t\}$. More explicitly, in the sampling interval $[s_k, s_{k+1})$, $u(t)$ can be described by

$$u(t) = u_{k+j-\bar{d}} \quad \text{for } t \in [s_k + t_j^k, s_k + t_{j+1}^k), \tag{2}$$

where the actuation update instants $t_j^k \in [0, h_k]$ are defined as, see Cloosterman et al. (2010):

$$\begin{aligned} t_j^k = \min \Bigg\{ & \max \Bigg\{ 0, \tau_{k+j-\bar{d}} - \sum_{l=k+j-\bar{d}}^{k-1} h_l \Bigg\}, \\ & \max \Bigg\{ 0, \tau_{k+j-\bar{d}+1} - \sum_{l=k+j+1-\bar{d}}^{k-1} h_l \Bigg\}, \\ & \ldots, \max \Bigg\{ 0, \tau_{k-\underline{d}} - \sum_{l=k-\underline{d}}^{k-1} h_l \Bigg\}, h_k \Bigg\} \end{aligned} \tag{3}$$

with $t_j^k \le t_{j+1}^k$ and $j \in \{0, 1, \ldots, \bar{d} - \underline{d}\}$. Moreover, $0 = t_0^k \le t_1^k \le \cdots \le t_{\bar{d}-\underline{d}}^k \le t_{\bar{d}-\underline{d}+1}^k := h_k$. See Fig. 2 for a graphical explanation of the meaning of the control update instants t_j^k. Note that the expression for the continuous-time control input in (2) and (3) accounts for possible out-of-order packet arrivals and message rejection. Let us define the vector $\psi_j^k = [\tau_{k-\bar{d}+j}\ \tau_{k-\bar{d}+j+1} \cdots \tau_{k-\underline{d}}\ h_{k-\bar{d}+j}\ h_{k-\bar{d}+j+1}\ \cdots\ h_k]^T$ containing all past delays and sampling intervals defining t_j^k, i.e. we can write (3) as $t_j^k = t_j^k(\psi_j^k)$. Note that $\psi_j^k \in \Psi_j := [\underline{\tau}, \overline{\tau}]^{\bar{d}-\underline{d}-j+1} \times [\underline{h}, \bar{h}]^{\bar{d}-j+1}$ for all $k \in \mathbb{N}$ and $j \in \{0, 1, \ldots, \bar{d} - \underline{d}\}$.

Remark 1. Packet dropouts can be directly incorporated in the above model as well, see Cloosterman et al. (2010) for the modified expressions for t_j^k in the case of packet dropouts (replacing (3)) assuming that there exists a bound on the maximal number of subsequent packet dropouts.

Next, let us consider the exact discretisation of (1)–(3) at the sampling instants s_k:

$$\begin{aligned} x_{k+1} &= x_k + \int_{s_k}^{s_{k+1}} f(x(s), u(s)) ds \\ &= x_k + \sum_{j=0}^{\bar{d}-\underline{d}} \int_{s_k+t_j^k}^{s_k+t_{j+1}^k} f(x(s), u_{k+j-\bar{d}}) ds \\ &=: F_{\theta_k}^e(x_k, \bar{u}_k, u_k) \end{aligned} \tag{4}$$

with $\theta_k := \begin{bmatrix} h_k & t_1^k & t_2^k & \cdots & t_{\bar{d}-\underline{d}}^k \end{bmatrix}^T \in \mathbb{R}^{\bar{d}-\underline{d}+1}$, $k \in \mathbb{N}$, the vector of uncertainty parameters consisting of the sampling interval h_k and the control update instants within the interval $[s_k, s_{k+1}]$. Moreover, $\bar{u}_k := \begin{bmatrix} u_{k-1}^T & u_{k-2}^T & \cdots, u_{k-\bar{d}}^T \end{bmatrix}^T$ represents a vector containing past control inputs. The uncertain parameter vector θ_k is taken from the uncertainty set Θ with

$$\begin{aligned} \Theta = \Theta(\underline{h}, \bar{h}, \underline{\tau}, \overline{\tau}) = \{ & \theta \in \mathbb{R}^{\bar{d}-\underline{d}+1} \,|\, h \in [\underline{h}, \bar{h}], t_j \in [\underline{t}_j, \bar{t}_j], \\ & 1 \le j \le \bar{d} - \underline{d}, 0 \le t_1 \le \cdots \le t_{\bar{d}-\underline{d}} \le h \}, \end{aligned} \tag{5}$$

where $\underline{t}_j$ and $\bar{t}_j$ denote the minimum and maximum values of t_j^k, $j = 1, 2, \ldots, \bar{d} - \underline{d}$, respectively, given by

$$\underline{t}_j = \min_{\psi_j \in \Psi_j} t_j(\psi_j), \quad \text{and} \quad \bar{t}_j = \max_{\psi_j \in \Psi_j} t_j(\psi_j), \tag{6}$$

for $1 \le j < \bar{d} - \underline{d}$. Explicit expressions for $\underline{t}_j$ and $\bar{t}_j$ are given in Cloosterman et al. (2010): $\underline{t}_j = \min\{\underline{\tau} - \underline{d}\bar{h}, \underline{h}\}$ for $j = \bar{d} - \underline{d}$, $\underline{t}_j = 0$ for $1 \le j < \bar{d} - \underline{d}$, and $\bar{t}_j = \min\{\overline{\tau} - (\bar{d} - j)\underline{h}, \bar{h}\}$ for $1 \le j \le \bar{d} - \underline{d}$. Additionally, $t_0^k := 0$ and $t_{\bar{d}-\underline{d}+1}^k := h_k$, which implies $t_{\bar{d}-\underline{d}+1}^k \in [\underline{h}, \bar{h}]$, $k \in \mathbb{N}$.

Let us now introduce the extended (augmented) state vector $\xi_k := \begin{bmatrix} x_k^T & u_{k-1}^T & u_{k-2}^T & \cdots & u_{k-\bar{d}}^T \end{bmatrix}^T = \begin{bmatrix} x_k^T & \bar{u}_k^T \end{bmatrix}^T \in \mathbb{R}^{n+\bar{d}m}$. Then, the exact discrete-time plant model can be written as:

$$\begin{aligned} \xi_{k+1} &= \begin{bmatrix} x_{k+1}^T & u_k^T & u_{k-1}^T & \cdots & u_{k-\bar{d}+1}^T \end{bmatrix}^T \\ &= \begin{bmatrix} F_{\theta_k}^{eT}(x_k, \bar{u}_k, u_k) & u_k^T & u_{k-1}^T & \cdots & u_{k-\bar{d}+1}^T \end{bmatrix}^T \\ &=: \bar{F}_{\theta_k}^e(\xi_k, u_k). \end{aligned} \tag{7}$$

In general, we can not explicitly compute the exact discrete-time model as in (7) since the plant is nonlinear. In order to design a stabilising discrete-time controller, we construct an approximate discrete-time plant model (using a discretisation scheme) based on a nominal choice θ^* for the uncertain parameters θ_k given by $\theta^* = \begin{bmatrix} h^* & t_1^* & t_2^* & \cdots & t_{\bar{d}-\underline{d}}^* \end{bmatrix}^T \in \Theta \subset \mathbb{R}^{\bar{d}-\underline{d}+1}$, where $h^* \in (\underline{h}, \bar{h}]$ is a nominal sampling interval and $t_j^* \in [\underline{t}_j, \bar{t}_j]$, $j \in \{1, 2, \ldots, \bar{d} - \underline{d}\}$, are nominal control update instants. Note that arbitrarily choosing the nominal parameter vector $\theta^* = \begin{bmatrix} h^* & t_1^* & t_2^* & \ldots & t_{\bar{d}-\underline{d}}^* \end{bmatrix}^T \in \Theta \subset \mathbb{R}^{\bar{d}-\underline{d}+1}$, such that $h^* \in (\underline{h}, \bar{h}]$ and $t_j^* \in [\underline{t}_j, \bar{t}_j]$, $j \in \{1, 2, \ldots, \bar{d} - \underline{d}\}$, may lead to sequences of control update instants that, when repeated for each sampling interval, represent unfeasible sequences of control updates for the real NCS. Therefore, we will choose θ^* in a particular way. Let us define

$$\theta^* := \begin{bmatrix} h^* & t_1^* & t_2^* & \ldots & t_{\bar{d}-\underline{d}}^* \end{bmatrix}^T \in \mathbb{R}^{\bar{d}-\underline{d}+1} \tag{8}$$

with $h^* > 0$ chosen arbitrarily and

$$t_j^* := \begin{cases} 0, & j \in \{0, 1, \ldots, \bar{d} - \underline{d}^* - 1\} \\ \tau^* - \underline{d}^* h^*, & j = \bar{d} - \underline{d}^* \\ h^*, & j \in \{\bar{d} - \underline{d}^* + 1, \ldots, \bar{d} - \underline{d} + 1\}, \end{cases} \tag{9}$$

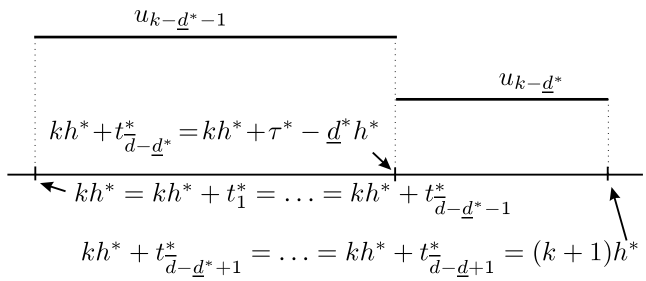

Fig. 3. Graphical interpretation of t_j^*.

where $\tau^* = \eta(h^*) \in [\underline{d}h^*, \overline{d}h^*]$, in which $\eta(\cdot)$ expresses some continuous function from the nominal sampling interval h^* to the nominal delay τ^*, and $\underline{d}^* := \lfloor \tau^*/h^* \rfloor$. Note that θ^* now only depends on two nominal parameters; namely h^* and $\tau^* = \eta(h^*)$. Hence, the nominal control update instants t_j^* correspond to this nominal sampling interval h^* and nominal delay τ^*, see Fig. 3.

By exploiting a discretisation scheme[2] we can now formulate the approximate discrete-time plant model as: $x_{k+1} = F^a_{\theta^*}(x_k, \bar{u}_k, u_k)$, which leads to

$$\xi_{k+1} = \begin{bmatrix} F^{a\,T}_{\theta^*}(x_k, \bar{u}_k, u_k) & u_k^T & u_{k-1}^T & \cdots & u^T_{k-\overline{d}+1} \end{bmatrix}^T =: \bar{F}^a_{\theta^*}(\xi_k, u_k) \tag{10}$$

and corresponds to the nominal parameter vector θ^* defined in (8) and (9). Next, we design a controller of the form

$$u_k = u_{\theta^*}(\xi_k) \tag{11}$$

to stabilise the nominal approximate discrete-time plant model (10) for a nominal distribution of the (past) control inputs over the sampling interval $[s_k, s_{k+1})$ corresponding to the nominal parameter vector θ^* defined in (8) and (9). In fact, since θ^* only depends on h^* and τ^*, $u_{\theta^*}(\xi)$ in (11) represents a controller that is designed to stabilise the system for the nominal sampling interval h^* and nominal delay τ^*. Let us now define the set of possible nominal parameters θ^*:

$$\begin{aligned} \Theta_0^* &= \Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot)) \\ &= \Big\{ \theta^* \in \mathbb{R}^{\overline{d}-\underline{d}+1} \,|\, h^* \in (0, \overline{h}^*], \\ &\quad t_j^* := 0, \ \text{for } j \in \{0, 1, \ldots, \overline{d} - \underline{d}^* - 1\}, \\ &\quad t_j^* := \tau^* - \underline{d}^* h^*, \ \text{for } j = \overline{d} - \underline{d}^*, \\ &\quad t_j^* := h^*, \ \text{for } j \in \{\overline{d} - \underline{d}^* + 1, \ldots, \overline{d} - \underline{d} + 1\}, \\ &\quad \text{with } \tau^* = \eta(h^*) \Big\} \end{aligned} \tag{12}$$

with $\eta(h^*) \in [\underline{d}h^*, \overline{d}h^*]$ $\forall h^* \in (0, \overline{h}^*]$, where $\overline{h}^*$ represents the maximal nominal sampling interval for which we aim to design stabilising controllers (stabilising the approximate discrete-time plant (10)).

The problem considered in the paper can now be formulated as follows. Given a nonlinear plant and a (family of) discrete-time controller(s), parametrised by and designed for a range of nominal sampling intervals h^* and nominal delays $\tau^* = \eta(h^*)$, we aim to provide sufficient conditions for the robust stability of the resulting sampled-data NCS in the face of (time-varying) uncertainties in the sampling interval and delays. In other words for each nominal parameter θ^* (related to a pair (h^*, τ^*)) we aim to determine the bounds $\underline{h}$, $\overline{h}$, $\underline{\tau}$ and $\overline{\tau}$ for which robust stability of the exact discrete-time closed-loop system (7), (11) (and of the sampled-data NCS (1)–(3), (11)) can be guaranteed. In order to tackle this problem, we will require, in Section 3, the approximate discrete-time plant model $\bar{F}^a_{\theta^*}(\xi, u)$,[3] the controller $u_{\theta^*}(\xi)$ and the resulting approximate discrete-time closed-loop system $\bar{F}^a_{\theta^*}(\xi, u_{\theta^*}(\xi))$ to exhibit certain properties for $\theta^* \in \Theta^* \subseteq \Theta_0^*$ that will be used to guarantee certain stability properties for the exact uncertain discrete-time closed-loop system $\bar{F}^e_\theta(\xi, u_{\theta^*}(\xi))$ as in (7) and the sampled-data NCS (1)–(3), (11).

3. Global exponential stability of the NCS

In Section 3.1, we present a Lyapunov characterisation of GES for a class of uncertain discrete-time nonlinear systems. We exploit such a characterisation in formulating conditions under which the closed-loop sampled-data system (1)–(3), (11) is globally exponentially stable (GES) in Section 3.2.

3.1. Lyapunov characterisation of global exponential stability

Here, we formulate a Lyapunov-based characterisation of global exponential stability for a parametrised family of uncertain discrete-time nonlinear closed-loop systems $\xi_{k+1} = F_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k))$, $\theta_k \in \Theta(\theta^*)$, $k \in \mathbb{N}$, $\theta^* \in \Theta^* \subseteq \Theta_0^*$, with $F_\theta(0, 0) = 0$, $\forall \theta \in \Theta(\theta^*)$ and $u_{\theta^*}(0) = 0$, for all $\theta^* \in \Theta^*$, based on a Lyapunov function $V_{\theta^*}(\xi_k)$ that is parametrised by a nominal parameter vector $\theta^* \in \Theta^* \subseteq \Theta_0^*$, with Θ_0^* as in (12). For the results presented in Theorem 1, the meaning of θ, θ^*, Θ^*, Θ_0^* and Θ is as set forth in Section 2. For the sake of brevity, we write $\Theta(\theta^*)$ instead of $\Theta(\underline{h}(\theta^*), \overline{h}(\theta^*), \underline{\tau}(\theta^*), \overline{\tau}(\theta^*))$.

Theorem 1. *Consider a parametrised family of uncertain discrete-time systems (parametrised by θ^*)*

$$\xi_{k+1} = F_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k)), \quad \theta_k \in \Theta(\theta^*), \ \forall k \in \mathbb{N}, \tag{13}$$

with $\theta^* \in \Theta^* \subseteq \Theta_0^*$, Θ_0^* as in (12) and $\Theta(\theta^*)$ as defined in (5), where $\underline{h}$, $\overline{h}$, $\underline{\tau}$ and $\overline{\tau}$ may depend on θ^* and $0 < \underline{h} < h^* \le \overline{h}$, $0 \le \underline{\tau} \le \tau^* \le \overline{\tau}$. If there exist a family of Lyapunov functions $V_{\theta^*}(\xi)$, with $\theta^* \in \Theta^*$, and $a_i > 0$, $i = 1, 2, 3$, such that the following conditions hold for some $1 \le p < \infty$:

$$a_1|\xi|^p \le V_{\theta^*}(\xi) \le a_2|\xi|^p \quad \text{and} \\ \frac{V_{\theta^*}(F_\theta(\xi, u_{\theta^*}(\xi))) - V_{\theta^*}(\xi)}{h} \le -a_3|\xi|^p, \tag{14}$$

for all $\xi \in \mathbb{R}^{n+\overline{d}m}$, $\theta \in \Theta(\theta^*)$, $\theta^* \in \Theta^*$, then there exist $c, \lambda > 0$ such that the solutions of the family of systems (13) satisfy $|\xi_k| \le c|\xi_0|e^{-\lambda s_k} \le c|\xi_0|e^{-\lambda k\underline{h}}$, $\forall k \in \mathbb{N}$, $\forall \xi_0 \in \mathbb{R}^{n+\overline{d}m}$ and for all $\theta^* \in \Theta^*$. In other words, the family of systems (13) is globally exponentially stable, uniformly for all $\theta^* \in \Theta^*$ and $\theta_k \in \Theta(\theta^*)$, $\forall k \in \mathbb{N}$.

Proof. The proof is a slight adaptation of the proof of Proposition 1.2 in Laila et al. (2006). □

3.2. Sufficient conditions for GES

Let us adopt the following assumptions for a set of nominal parameters Θ^* satisfying $\Theta^* \subseteq \Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$ with $\Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$ as in (12) for given $\overline{h}^*$, $\underline{d}$, $\overline{d}$ and $\eta(\cdot)$.

[2] Conditions on the approximate discrete-time plant model and, hence, implicitly on the discretisation scheme used to construct it, will be formulated later in Assumption 3.

[3] For the sake of brevity, we call $\bar{F}(\xi, u)$ a plant model by which we indicate the discrete-time dynamics $\xi_{k+1} = \bar{F}(\xi_k, u_k)$.

Assumption 1. There exist a parametrised family of functions $V_{\theta^*}(\xi)$, a parametrised family of controllers $u_{\theta^*}(\xi)$, and $a_i > 0$, $i = 1, 2, 3$, such that the following inequalities hold for some $1 \le p < \infty$:

$$\frac{V_{\theta^*}(\bar{F}^a_{\theta^*}(\xi, u_{\theta^*}(\xi))) - V_{\theta^*}(\xi)}{h^*} \le -a_3|\xi|^p,$$
$$a_1|\xi|^p \le V_{\theta^*}(\xi) \le a_2|\xi|^p,\ \forall \xi \in \mathbb{R}^{n+\bar{d}m}, \quad \forall \theta^* \in \Theta^*. \tag{15}$$

This assumption requires that the control law $u_{\theta^*}(\xi)$ globally exponentially stabilises, uniformly for all $\theta^* \in \Theta^*$, the *approximate* discrete-time plant (10) (formulated for the nominal parameter set θ^*), see Theorem 1. Note that this assumption does not guarantee the stability of the *exact* closed-loop plant model (7), (11) for time-varying $\theta_k \in \Theta$ (not even for fixed $\theta_k \in \Theta^*$).

Assumption 2. The parametrised family of functions $V_{\theta^*}(\xi)$ is locally Lipschitz and satisfies the following condition uniformly over $\theta^* \in \Theta^*$: there exists an $L_v > 0$, such that $\sup_{\zeta \in \partial V_{\theta^*}(\xi)} |\zeta| \le L_v|\xi|^{p-1}$, $\forall \xi \in \mathbb{R}^{n+\bar{d}m}$, and $\forall \theta^* \in \Theta^*$, with p in accordance with Assumption 1.

Assumption 3. The parametrised family of approximate nominal discrete-time plant models $\bar{F}^a_{\theta^*}(\xi, u)$ is one-step consistent with the parametrised family of exact nominal discrete-time plant models $\bar{F}^e_{\theta^*}(\xi, u)$ uniformly over $\theta^* \in \Theta^*$, i.e. there exists $\hat{\rho} \in \mathcal{K}_\infty$ such that $|\bar{F}^a_{\theta^*}(\xi, u) - \bar{F}^e_{\theta^*}(\xi, u)| \le h^*\hat{\rho}(h^*)(|\xi| + |u|)$, $\forall \xi \in \mathbb{R}^{n+\bar{d}m}$, $u \in \mathbb{R}^m$ and $\forall \theta^* \in \Theta^*$.

The notion of consistency is commonly used in the numerical analysis literature, see e.g. Stuart and Humphries (1996), to address the closeness of solutions of families of models (obtained by numerical integration). Moreover, the notion of one-step consistency has been used before in the scope of the stabilisation of nonlinear sampled-data systems based on approximate discrete-time models (Nesić & Teel, 2004b; Nesić et al., 1999). One-step consistent integration schemes with which approximate discrete-time plant models satisfying Assumption 3 are available, see van de Wouw, Nesic, and Heemels (2010).

Assumption 4. The right-hand side $f(x, u)$ of the continuous-time plant model is globally Lipschitz, i.e. there exists $L_f > 0$ such that $|f(x_1, u_1) - f(x_2, u_2)| \le L_f(|x_1 - x_2| + |u_1 - u_2|)$, $\forall x_1, x_2 \in \mathbb{R}^n$, $u_1, u_2 \in \mathbb{R}^m$.

Assumption 5. The parametrised family of discrete-time control laws $u_{\theta^*}(\xi)$ is linearly bounded uniformly over $\theta^* \in \Theta^*$, i.e. there exists $L_u > 0$, such that $|u_{\theta^*}(\xi)| \le L_u|\xi|$, $\forall \xi \in \mathbb{R}^{n+\bar{d}m}$, and $\forall \theta^* \in \Theta^*$.

We note that these assumptions are natural extensions of the assumptions used in the scope of the stabilisation of nonlinear sampled-data systems (with constant sampling intervals and no delays), see Nesić et al. (1999). Assumption 3 bounds the difference between the approximate and exact nominal discrete-time plant models. Assumption 4 is typically needed to bound the intersample behaviour, which, in turn, is needed to bound the difference between the nominal and uncertain exact discrete-time plant models. Moreover, the satisfaction of Assumption 1 guarantees GES of the approximate discrete-time plant model, for any fixed $\theta^* \in \Theta^*$, and avoids non-uniform bounds on the overshoot and non-uniform convergence rates for the solutions of the approximate nominal discrete-time plant model, whereas Assumption 5 avoids non-uniform bounds on the controls. Finally, Assumption 2 guarantees continuity of the Lyapunov function. It has been shown in Nesić and Teel (2004b); Nesić et al. (1999) that if Assumptions 1, 2 and 5 are not satisfied then the approximate closed-loop discrete-time system does not exhibit sufficient robustness to account for the mismatch between the approximate and exact discrete-time models.

Based on these assumptions we can formulate sufficient conditions under which the closed-loop uncertain exact discrete-time system (7), (11) is GES. Hereto, consider the following definition:

$$L_a := \left(2 + L_u + (1 + \max(1, L_u))\left(e^{L_f\bar{h}} - 1\right)\right) + h^*\hat{\rho}(h^*)(1 + L_u). \tag{16}$$

Theorem 2. *Consider the exact discrete-time plant model (7) with $\theta_k \in \mathbb{R}^{\bar{d}-\underline{d}+1}$, $\forall k \in \mathbb{N}$. Moreover, consider the discrete-time controller (11), parametrised by $\theta^* \in \Theta^*_0(\bar{h}^*, \underline{d}, \bar{d}, \eta(\cdot))$, and the set $\Theta^*_0(\bar{h}^*, \underline{d}, \bar{d}, \eta(\cdot))$ of nominal parameter vectors as in (12) for given $\bar{h}^*$, $\underline{d}$, $\bar{d}$ and $\eta(\cdot)$. Furthermore, consider lower and upper bounds on the sampling interval and delay such that $0 < \underline{h} < h^* \le \bar{h}$, $0 \le \underline{\tau} \le \tau^* = \eta(h^*) \le \bar{\tau}$, $\lfloor \underline{\tau}/\bar{h} \rfloor = \underline{d}$ and $\lceil \bar{\tau}/\underline{h} \rceil = \bar{d}$.*

If Assumptions 1–5 are satisfied for $\Theta^* = \{\theta^*\}$, for some $\theta^* \in \Theta^*_0(\bar{h}^*, \underline{d}, \bar{d}, \eta(\cdot))$, and if there exists $0 < \beta < 1$ such that the inequality (17)

$$\frac{L_v(L_a)^{p-1}}{h^*}\left(h^*\hat{\rho}(h^*)(1 + L_u) + \rho_\theta\left(h^*, M_h, M_{t_1}, \dots, M_{t_{\bar{d}-\underline{d}}}\right)\right) \le (1 - \beta)a_3 \tag{17}$$

is satisfied where the function $\hat{\rho}$ follows from Assumption 3 and ρ_θ is defined in (18)

$$\rho_\theta\left(h^*, M_h, M_{t_1}, \dots, M_{t_{\bar{d}-\underline{d}}}\right) := e^{L_f h^*}\left((1 + \max(1, L_u))\left(e^{L_f M_h} - 1\right) + 2L_f \max(1, L_u)\sum_{j=1}^{\bar{d}-\underline{d}} M_{t_j}\right) \tag{18}$$

with $M_h := \max_{h \in [\underline{h}, \bar{h}]} |h - h^*|$, $M_{t_j} := \max_{t_j \in [\underline{t}_j, \bar{t}_j]} |t_j - t_j^*|$, $j = 1, 2, \dots, \bar{d} - \underline{d}$, and $\underline{t}_j$ and $\bar{t}_j$ defined in (6), then the closed-loop uncertain exact discrete-time system (7), (11) is globally exponentially stable for $\theta_k \in \Theta(\underline{h}, \bar{h}, \underline{\tau}, \bar{\tau})$, $\forall k \in \mathbb{N}$, with $\Theta(\underline{h}, \bar{h}, \underline{\tau}, \bar{\tau})$ as in (5).

Proof. The proof is given in Appendix A.1. □

This theorem can be interpreted as follows. If Assumptions 1–5 hold for a fixed $\theta^* \in \Theta^*$ (i.e. for a fixed nominal sampling interval h^* and nominal delay τ^*) and condition (17) is satisfied for that fixed θ^*, then system (7), (11) is GES for $\theta_k \in \Theta(\underline{h}, \bar{h}, \underline{\tau}, \bar{\tau})$, $\forall k \in \mathbb{N}$ (i.e. for $h_k \in [\underline{h}, \bar{h}]$ and $\tau_k \in [\underline{\tau}, \bar{\tau}]$, $\forall k \in \mathbb{N}$). Note that the condition in (17) involves two distinct terms:

(i) $L_v(L_a)^{p-1}\hat{\rho}(h^*)(1 + L_u)$, which reflects the effect of approximately discretising the nonlinear plant using a nominal parameter vector θ^* (i.e. corresponding to a nominal sampling interval h^* and a nominal delay τ^*);
(ii) $\frac{L_v(L_a)^{p-1}}{h^*}\rho_\theta\left(h^*, M_h, M_{t_1}, \dots, M_{t_{\bar{d}-\underline{d}}}\right)$, which reflects the effect of the uncertainty in the sampling interval and delay.

Moreover, a_3 in the right-hand side of (17) can be interpreted as a margin of stability of the approximate nominal discrete-time closed-loop system, see Assumption 1, which should dominate the effects under points (i), (ii) above.

For the application of Theorem 2, only a single Lyapunov function $V_{\theta^*}(\xi)$ and a single controller $u_{\theta^*}(\xi)$ need to be found, which is a relatively simple task. Note, however, that for a priori fixed θ^* there is no guarantee that condition (17) will be satisfied, because the discretisation error (expressed by the term under point (i) above) may be too large. If condition (17) is not satisfied one has to resort to designing a Lyapunov function $V_{\theta^*}(\xi)$ and a controller $u_{\theta^*}(\xi)$ for a smaller nominal sampling interval h^* (and corresponding θ^*) and, subsequently, checking whether condition (17) is satisfied. Although this approach is beneficial in the sense that one only needs the existence of a Lyapunov function and controller for a fixed θ^*, it may lead to an iterative design procedure for Lyapunov functions and controllers. Therefore, in Theorem 3 we formulate conditions under which we can always choose the nominal sampling interval h^*, the uncertainty on the sampling interval $\overline{h}-\underline{h}$ and the uncertainty on the delay $\overline{\tau}-\underline{\tau}$ sufficiently small such that (17) is satisfied.

Theorem 3. *Consider the exact discrete-time plant model (7) with $\theta_k \in \mathbb{R}^{\overline{d}-\underline{d}+1}$, $\forall k \in \mathbb{N}$. Moreover, consider the discrete-time controller (11), parametrised by $\theta^* \in \Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$, and the set $\Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$ of nominal parameter vectors as in (12) for given $\overline{h}^*$, $\underline{d}$, $\overline{d}$ and $\eta(\cdot)$. Furthermore, consider lower and upper bounds on the sampling interval and delay such that $0 < \underline{h}(\theta^*) < h^* \le \overline{h}(\theta^*)$, $0 \le \underline{\tau}(\theta^*) \le \tau^* = \eta(h^*) \le \overline{\tau}(\theta^*)$, $\lfloor \underline{\tau}(\theta^*)/\overline{h}(\theta^*) \rfloor = \underline{d}$ and $\lceil \overline{\tau}(\theta^*)/\underline{h}(\theta^*) \rceil = \overline{d}$ for all $\theta^* \in \Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$.*

If Assumptions 1–5 are satisfied for $\Theta^* = \Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$, then there exists an $h^*_{\max} \le \overline{h}^*$ such that for all $h^* \in (0, h^*_{\max}]$, there exist $\underline{h}(\theta^*)$, $\overline{h}(\theta^*)$, $\underline{\tau}(\theta^*)$, $\overline{\tau}(\theta^*)$, with $\underline{h}(\theta^*) < \overline{h}(\theta^*)$, $\underline{\tau}(\theta^*) < \overline{\tau}(\theta^*)$, and $0 < \beta < 1$ satisfying (17). Consequently, the family of closed-loop uncertain exact discrete-time systems (7), (11) is globally exponentially stable for all $\theta^* \in \Theta_0^*(h^*_{\max}, \underline{d}, \overline{d}, \eta(\cdot))$ and for $\theta_k \in \Theta(\underline{h}(\theta^*), \overline{h}(\theta^*), \underline{\tau}(\theta^*), \overline{\tau}(\theta^*))$, $\forall k \in \mathbb{N}$, with $\Theta(\underline{h}, \overline{h}, \underline{\tau}, \overline{\tau})$ as in (5).

Proof. The proof is given in Appendix A.2. □

In Theorem 3, we require that Assumptions 1–3 and 5 hold for all $\theta^* \in \Theta_0^*(\overline{h}^*, \underline{d}, \overline{d}, \eta(\cdot))$. Hereto, in turn, we need to design a parametrised family of controllers $u_{\theta^*}(\xi)$ and construct a parametrised family of Lyapunov functions $V_{\theta^*}(\xi)$. When exploiting Theorem 3, one typically computes $\underline{h}(\theta^*)$, $\overline{h}(\theta^*)$, $\underline{\tau}(\theta^*)$, $\overline{\tau}(\theta^*)$ using (17) for each fixed $\theta^* \in \Theta_0^*(h^*_{\max}, \underline{d}, \overline{d}, \eta(\cdot))$. Note that, even for each fixed θ^*, different combinations of $\underline{h}(\theta^*)$, $\overline{h}(\theta^*)$, $\underline{\tau}(\theta^*)$, $\overline{\tau}(\theta^*)$ may satisfy (17), which may be used to investigate trade-offs between time-varying delays and time-varying sampling intervals.

Remark 2. We foresee that the condition on the global exponential stability of the approximate discrete-time closed-loop system in Assumption 1 can be relaxed to a requirement of global uniform asymptotic stability and that the global conditions in Assumptions 2–5 may be relaxed to conditions on compact sets, thereby enlarging the class of system which can be studied. However, under such relaxed conditions we only expect to guarantee semi-global practical asymptotic stability (as opposed to GES) of the closed-loop NCS.

Finally, let us remark that, using the results in Nesić, Teel, and Sontag (1999), we can conclude that, under the conditions of Theorems 2 and 3, also the closed-loop sampled-data NCS (1)–(3), (11) is globally exponentially stable.

4. Illustrative example

Let us consider a NCS as depicted in Fig. 1 with a class of scalar nonlinear continuous-time plants of the form

$$\dot{x} = f(x) + u, \tag{19}$$

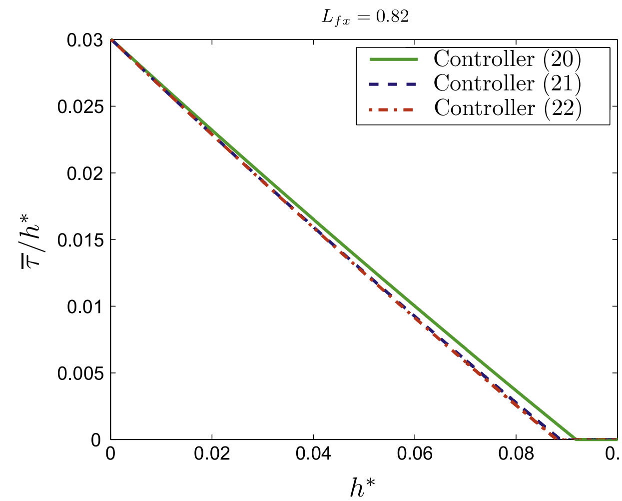

Fig. 4. Bounds $\overline{\tau}/h^*$ on the uncertainty of the delay for controllers (20)–(22) for $L_{fx} = 0.82$.

where $x \in \mathbb{R}$, $u \in \mathbb{R}$, and $f(x)$ is globally Lipschitz with Lipschitz constant L_{fx}. Consequently, the right-hand side of (19) satisfies Assumption 4 with $L_f = \max(1, L_{fx})$.

We consider the case in which the sampling interval h is constant and the uncertain time-varying network-induced delays satisfy $\tau_k \in [0, \overline{\tau}]$, for all $k \in \mathbb{N}$, with $\overline{\tau} \le h$. Here we choose $\tau^* = 0$ and $h^* = h$ and use an Euler-type discretisation scheme to construct the following family of approximate discrete-time plant models in terms of the extended state $\xi_k = \begin{bmatrix} \xi_k^1 & \xi_k^2 \end{bmatrix}^T = \begin{bmatrix} x_k & u_{k-1} \end{bmatrix}^T$, which yields $\xi_{k+1} = \begin{bmatrix} \xi_k^1 + h^*(f(\xi_k^1) + u_k), & u_k \end{bmatrix}^T =: \tilde{F}^a_{h^*}(\xi_k, u_k)$. We note that this family of approximate discrete-time models satisfies Assumption 3 with $h^*\hat{\rho}(h^*) = \frac{L_{fx}}{L_f}\left(e^{L_f h^*} - 1 - L_f h^*\right)$. Moreover, consider the following controllers

$$u_k = -f(x_k) - x_k \tag{20}$$

$$u_k = -f(x_k) - x_k - h^* x_k \tag{21}$$

$$u_k = -f(x_k) - \frac{\left(1 - \sqrt{1 - 4h^*}\right)}{2h^*} x_k, \quad \text{for } h^* \le \frac{1}{4}, \tag{22}$$

where the first controller is independent of h^* and can be regarded as an emulation-based controller, whereas the other two controllers are clearly parametrised by the nominal sampling interval h^*. Consider the following family of Lyapunov functions: $V(\xi) = |\xi^1| + \alpha|u(\xi^1) - \xi^2| + h^*\alpha|\xi^2|$, with $\alpha > 0$. This family of Lyapunov functions satisfies Assumption 2 with $L_v = \sqrt{2}\max(1 + \alpha L_u, \alpha(1 + h^*))$, which is bounded for bounded L_u, α and $\overline{h}^*$. The evolution of this family of Lyapunov functions along solutions of the family of closed-loop approximate discrete-time plant models, induced by the three controllers (20)–(22), can be shown to satisfy $\frac{V(\tilde{F}^a_{h^*}(\xi_k, u_k)) - V(\xi_k)}{h^*} \le -\alpha|\xi_k|$ with,

- for controller (20): $\alpha = 1/(1 + 2L_u)$ for $0 < h^* \le 1$ and $\alpha = (\frac{2}{h^*} - 1)/(1 + 2L_u)$ for $1 < h^* < 2$;
- for controller (21): $\alpha = (1 + h^*)/(1 + 2L_u + h^* L_u)$ for $0 < h^* \le \frac{1}{2}(\sqrt{5} - 1)$ and $\alpha = (\frac{2}{h^*} - 1 - h^*)/(1 + 2L_u + h^* L_u)$ for $\frac{1}{2}(\sqrt{5} - 1) \le h^* \le 1$;
- for controller (22): $\alpha = \frac{1 - \sqrt{1 - 4h^*}}{2h^* + L_u(2h^* + 1 - \sqrt{1 - 4h^*})}$ for $0 < h^* \le \frac{1}{4}$.

Consequently, we can conclude that, for all three controllers, Assumption 1 is satisfied with $a_1 = \alpha$, $a_2 = L_v$ and $a_3 = \alpha$. Now, Theorem 3 (and in particular condition (17)) can be used to show for which uncertainty level of the delay $\overline{\tau}(h^*)$, depending on h^*, the exact closed-loop discrete-time NCS is GES. For $L_{fx} = 0.82$, these results are depicted in Fig. 4. The estimated uncertainty bound on the delay depends on many factors, such as the (family of) controller(s) designed, the family of Lyapunov functions used, the particular choice for the nominal delay and nominal sampling interval, etc., all of which will influence the results presented in Fig. 4. The advantage of the framework for stability analysis proposed in this paper is exactly the fact that one may consider and compare a wide range of controllers (both emulation-based controllers and controllers designed for a (non-zero) nominal sampling interval and delay), in terms of both robustness for network-induced uncertainties and performance.

5. Conclusions

This paper presents results on the stability analysis of nonlinear Networked Control Systems (NCSs) with time-varying sampling intervals, time-varying delays and packet dropouts. As opposed to emulation-based approaches where the effects of sampling-and-hold and delays are ignored in the phase of controller design, we propose a prescriptive framework for controller design based on approximate discrete-time models constructed for a nominal (non-zero) sampling interval and a nominal delay. Subsequently, sufficient conditions for the global exponential stability of the closed-loop uncertain NCS are provided.

Appendix. Proofs

A.1. Proof of Theorem 2

Let us study the evolution of the candidate Lyapunov function $V_{\theta^*}(\xi)$ along solutions of the closed-loop uncertain exact discrete-time system (7), (11):

$$\Delta V_k := \frac{V_{\theta^*}(\bar{F}^e_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k))) - V_{\theta^*}(\xi_k)}{\bar{h}}. \tag{A.1}$$

Below, we exploit the mean value theorem to obtain $V_{\theta^*}(x) - V_{\theta^*}(y) \in \partial V^T_{\theta^*}(z)(x-y)$ for some $z = \sigma x + (1-\sigma)y$, $\sigma \in [0,1]$. Hence, $V_{\theta^*}(x) - V_{\theta^*}(y) \leq \sup_{\zeta \in \partial V_{\theta^*}(z)} |\zeta||x-y|$. Using Assumption 2, we obtain $V_{\theta^*}(x) - V_{\theta^*}(y) \leq L_v |z|^{p-1}|x-y|$, $z = \sigma x + (1-\sigma)y$, $\sigma \in [0,1]$. Exploiting the fact that $|z| = |\sigma x + (1-\sigma)y| \leq \sigma|x| + (1-\sigma)|y| \leq \max(|x|,|y|)$, we obtain that $V_{\theta^*}(x) - V_{\theta^*}(y) \leq L_v(\max(|x|,|y|))^{p-1}|x-y|$. Using Assumption 1 and the latter inequality in (A.1) gives

$$\begin{aligned}\Delta V_k \leq & -a_3\frac{h^*}{\bar{h}}|\xi_k|^p \\ & + \frac{L_v}{\bar{h}}\left(\max(|\bar{F}^e_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k))|, |\bar{F}^a_{\theta^*}(\xi_k, u_{\theta^*}(\xi_k))|)\right)^{p-1} \\ & \times |\bar{F}^e_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k)) - \bar{F}^a_{\theta^*}(\xi_k, u_{\theta^*}(\xi_k))|. \end{aligned} \tag{A.2}$$

For notational convenience we will drop the arguments of $\bar{F}^e_{\theta_k}$ and $\bar{F}^a_{\theta^*}$ from now on. Let us first investigate the term $\left(\max(|\bar{F}^e_{\theta_k}|, |\bar{F}^a_{\theta^*}|)\right)^{p-1}$ in (A.2). By the definitions of $\bar{F}^e_{\theta_k}$ and $\bar{F}^a_{\theta^*}$ in (7) and (10), respectively, and Assumption 5 we have that

$$\begin{aligned}|\bar{F}^e_{\theta_k}| &\leq |F^e_{\theta_k}| + |\xi_k| + |u_k| \leq |F^e_{\theta_k}| + (1+L_u)|\xi_k|, \\ |\bar{F}^a_{\theta^*}| &\leq |F^a_{\theta^*}| + |\xi_k| + |u_k| \leq |F^a_{\theta^*}| + (1+L_u)|\xi_k|. \end{aligned} \tag{A.3}$$

Now, $|F^e_{\theta_k}|$ can be upperbounded as follows:

$$|F^e_{\theta_k}| = |x_{k+1}| \leq |x_k| + \sum_{j=0}^{\bar{d}-\underline{d}} \int_{s_k+t^k_j}^{s_k+t^k_{j+1}} |f(x(s), u_{k+j-\bar{d}})| ds. \tag{A.4}$$

Using Assumption 4 and the Gronwall–Bellman inequality we obtain:

$$\begin{aligned}|F^e_{\theta_k}| &\leq |x_k| + L_f \sum_{j=0}^{\bar{d}-\underline{d}} \int_{s_k+t^k_j}^{s_k+t^k_{j+1}} \left(|x(s)| + |u_{k+j-\bar{d}}|\right) ds \\ &\leq |x_k| + \sum_{j=0}^{\bar{d}-\underline{d}} \left(e^{L_f t^k_{j+1}} - e^{L_f t^k_j}\right) \\ &\quad \times \left(|x_k| + \max_{i\in\{0,\ldots,\bar{d}-\underline{d}\}} \left(|u_{k+i-\bar{d}}|\right)\right). \end{aligned} \tag{A.5}$$

Let us now use the fact that $|x_k| + \max_{i\in\{0,\ldots,\bar{d}-\underline{d}\}}(|u_{k+i-\bar{d}}|) \leq |x_k| + \max(|\bar{u}_k|, |u_k|)$ and the fact that $\sum_{j=0}^{\bar{d}-\underline{d}}(e^{L_f t^k_{j+1}} - e^{L_f t^k_j}) = e^{L_f h_k} - 1 \leq e^{L_f \bar{h}} - 1$ to obtain $|F^e_{\theta_k}| \leq |\xi_k| + \left(e^{L_f\bar{h}} - 1\right)(|\xi_k| + \max(|\xi_k|, |u_k|))$, where we also used that $|x_k| \leq |\xi_k|$ and $|\bar{u}_k| \leq |\xi_k|$. Using Assumption 5, we obtain that

$$|F^e_{\theta_k}| \leq \left(1 + (1+\max(1, L_u))\left(e^{L_f\bar{h}} - 1\right)\right)|\xi_k|. \tag{A.6}$$

Combining (A.3) and (A.6) and using the definition $L_e := \left(2 + L_u + (1+\max(1,L_u))(e^{L_f\bar{h}} - 1)\right)$ yields

$$|\bar{F}^e_{\theta_k}| \leq L_e|\xi_k|. \tag{A.7}$$

Next, $|\bar{F}^a_{\theta^*}|$ can be upperbounded using Assumptions 3 and 5:

$$\begin{aligned} & |\bar{F}^a_{\theta^*} - \bar{F}^e_{\theta^*}| \leq h^*\hat{\rho}(h^*)(1+L_u)|\xi_k| \\ \Rightarrow & |\bar{F}^a_{\theta^*}| \leq \left(L_e + h^*\hat{\rho}(h^*)(1+L_u)\right)|\xi_k| \\ & \qquad = L_a|\xi_k|, \end{aligned} \tag{A.8}$$

where we used (A.7), the fact that $h^* \leq \bar{h}$ and the definition of L_a in (16). Combining (A.7) and (A.7), the term $\left(\max(|\bar{F}^e_{\theta_k}|, |\bar{F}^a_{\theta^*}|)\right)^{p-1}$ in (A.2) can be upperbounded as follows:

$$\left(\max(|\bar{F}^e_{\theta_k}|, |\bar{F}^a_{\theta^*}|)\right)^{p-1} \leq (L_a)^{p-1}|\xi_k|^{p-1}, \tag{A.9}$$

where we used that $L_a \geq L_e$. Next, we investigate the term $|\bar{F}^e_{\theta_k} - \bar{F}^a_{\theta^*}|$ in (A.2) in more detail:

$$|\bar{F}^e_{\theta_k} - \bar{F}^a_{\theta^*}| \leq |\bar{F}^e_{\theta_k} - \bar{F}^e_{\theta^*}| + |\bar{F}^e_{\theta^*} - \bar{F}^a_{\theta^*}|. \tag{A.10}$$

Using Assumptions 3 and 5, the second term in the right-hand side of (A.10) can be upperbounded as follows:

$$|\bar{F}^e_{\theta^*} - \bar{F}^a_{\theta^*}| \leq h^*\hat{\rho}(h^*)(1+L_u)|\xi_k|, \tag{A.11}$$

$\forall \xi_k \in \mathbb{R}^{n+\bar{d}m}$. The first term in the right-hand side of (A.10) reflects the difference in the exact discrete-time plant induced by the difference between θ^* and θ_k. Let $u(t) = u_{k+j-\bar{d}}$ for $t \in [s_k + t^k_j, s_k + t^k_{j+1})$, $u^*(t) = u_{k+j-\bar{d}}$ for $t \in [s_k + t^*_j, s_k + t^*_{j+1})$ and $x(t)$, $x^*(t)$ represent the solutions (with initial condition $x(s_k) = x^*(s_k) = x_k$) corresponding to the inputs $u(t)$, $u^*(t)$, respectively. Using these notational conventions and the definition in (7), it can be shown that the term $|\bar{F}^e_{\theta_k} - \bar{F}^e_{\theta^*}|$ satisfies

$$|\bar{F}^e_{\theta_k} - \bar{F}^e_{\theta^*}| = |F^e_{\theta_k} - F^e_{\theta^*}| = \left|x(s_k + h_k) - x^*(s_k + h^*)\right|. \tag{A.12}$$

Let us consider the case that $h^* \leq h_k$ (the case that $h^* > h_k$ can be treated in an analogous fashion). In this case, (A.12) can be written as

$$\begin{aligned}|\bar{F}^e_{\theta_k} - \bar{F}^e_{\theta^*}| &= \left|x(s_k + h_k) - x^*(s_k + h^*)\right| \\ &\leq \left|x(s_k + h^*) - x^*(s_k + h^*)\right| \\ &\quad + \left|x(s_k + h_k) - x(s_k + h^*)\right|. \end{aligned} \tag{A.13}$$

Using Assumption 4, it can be shown that

$$\begin{aligned}\left|x(s_k+h^*)-x^*(s_k+h^*)\right| &\le \int_{s_k}^{s_k+h^*}\left|f(x(s),u(s))-f(x^*(s),u^*(s))\right|ds\\ &\le \int_{s_k}^{s_k+h^*} L_f\left(\left|x(s)-x^*(s)\right|+\left|u(s)-u^*(s)\right|\right)ds,\end{aligned} \tag{A.14}$$

and

$$\begin{aligned}\left|x(s_k+h_k)-x(s_k+h^*)\right| &\le \left|\int_{s_k+h^*}^{s_k+h_k} f(x(s),u(s))ds\right|\\ &\le \int_{s_k+h^*}^{s_k+h_k} L_f\left(|x(s)|+|u(s)|\right)ds.\end{aligned} \tag{A.15}$$

Combining (A.13)–(A.15) and by exploiting the integral variant of the Gronwall–Bellman inequality to rewrite the inequality in (A.14) and the fact that $|u(t)| \le \max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)$ for $t \in [s_k, s_k+h_k)$ to rewrite the right-hand side of (A.15), we obtain:

$$\begin{aligned}|\bar F^e_{\theta_k}-\bar F^e_{\theta^*}| &\le L_f\int_{s_k}^{s_k+h^*}\left|u(s)-u^*(s)\right|ds\\ &+\int_{s_k}^{s_k+h^*} L_f\int_{s_k}^{\sigma}\left|u(s)-u^*(s)\right|ds\, L_f\left(e^{\int_\sigma^{s_k+h^*}L_f dr}\right)d\sigma\\ &+\int_{s_k+h^*}^{s_k+h_k} L_f\left(|x(s)|+\max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\right)ds.\end{aligned} \tag{A.16}$$

Exploiting the Gronwall–Bellman inequality again to rewrite the last term in (A.16) and the fact that $s_k \le \sigma \le s_k+h^*$ gives

$$\begin{aligned}|\bar F^e_{\theta_k}-\bar F^e_{\theta^*}| \le\ & L_f e^{L_f h^*}\int_{s_k}^{s_k+h^*}\left|u(s)-u^*(s)\right|ds\\ &+e^{L_f h^*}\left(e^{L_f(h_k-h^*)}-1\right)\left(|x_k|+\max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\right).\end{aligned} \tag{A.17}$$

Next, we investigate the term $\int_{s_k}^{s_k+h^*}|u(s)-u^*(s)|ds$ in (A.17). Since we consider the case that $h^* \le h_k$, (2) yields that, for $t \in [s_k, s_k+h^*)$, $u(t) = \sum_{j=0}^{\bar d-\underline{d}} u_{k+j-\bar d}\ \mathbf{1}_{[\tilde t^k_j,\tilde t^k_{j+1}]}(t-s_k)$ and $u^*(t) = \sum_{j=0}^{\bar d-\underline{d}} u_{k+j-\bar d}\mathbf{1}_{[t^*_j,t^*_{j+1}]}(t-s_k)$ with $\tilde t^k_j = \min(h^*, t^k_j)$ and $\mathbf{1}_{[a,b]}(t)$ the indicator function defined by $\mathbf{1}_{[a,b]}(t) := \begin{cases}1 & \text{for } t\in[a,b]\\ 0 & \text{otherwise}\end{cases}$. Consequently,

$$\begin{aligned}\int_{s_k}^{s_k+h^*}\left|u(s)-u^*(s)\right|ds \le\ & \max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\\ &\times\sum_{j=0}^{\bar d-\underline{d}}\int_0^{h^*}\left|\mathbf{1}_{[\tilde t^k_j,\tilde t^k_{j+1}]}(\sigma)-\mathbf{1}_{[t^*_j,t^*_{j+1}]}(\sigma)\right|d\sigma.\end{aligned} \tag{A.18}$$

We consider four cases in evaluating the integral $I := \int_0^{h^*}|\mathbf{1}_{[\tilde t^k_j,\tilde t^k_{j+1}]}(\sigma)-\mathbf{1}_{[t^*_j,t^*_{j+1}]}(\sigma)|d\sigma$ in (A.18):

- If $\tilde t^k_j \le \tilde t^k_{j+1} \le t^*_j \le t^*_{j+1}$, then $I = (\tilde t^k_{j+1}-\tilde t^k_j)+(t^*_{j+1}-t^*_j) \le (t^*_j-\tilde t^k_j)+(t^*_{j+1}-\tilde t^k_{j+1})$, since $\tilde t^k_{j+1} \le t^*_j$ and $-t^*_j \le -\tilde t^k_{j+1}$;
- If $\tilde t^k_j \le t^*_j < \tilde t^k_{j+1} \le t^*_{j+1}$, then $I = (t^*_j-\tilde t^k_j)+(t^*_{j+1}-\tilde t^k_{j+1})$;
- If $t^*_j < \tilde t^k_j \le t^*_{j+1} \le \tilde t^k_{j+1}$, then $I = (\tilde t^k_j-t^*_j)+(\tilde t^k_{j+1}-t^*_{j+1})$;
- If $t^*_j \le t^*_{j+1} \le \tilde t^k_j \le \tilde t^k_{j+1}$, then $I = (t^*_{j+1}-t^*_j)+(\tilde t^k_{j+1}-\tilde t^k_j) \le (\tilde t^k_j-t^*_j)+(\tilde t^k_{j+1}-t^*_{j+1})$, since $\tilde t^k_j \ge t^*_{j+1}$ and $-\tilde t^k_j \le -t^*_{j+1}$.

From the above four cases we can conclude that

$$\begin{aligned}&\int_0^{h^*}\left|\mathbf{1}_{[\tilde t^k_j,\tilde t^k_{j+1}]}(\sigma)-\mathbf{1}_{[t^*_j,t^*_{j+1}]}(\sigma)\right|d\sigma\\ &\quad\le |\tilde t^k_j-t^*_j|+|\tilde t^k_{j+1}-t^*_{j+1}|.\end{aligned} \tag{A.19}$$

Moreover, it holds that $|\tilde t^k_j - t^*_j| = |\min(h^*, t^k_j)-t^*_j| = \begin{cases}|h^*-t^*_j| & \text{if } t^k_j \ge h^*\\ |t^k_j-t^*_j| & \text{if } t^k_j < h^*\end{cases} \le |t^k_j-t^*_j|$ for all $j \in \{0,\dots,\bar d-\underline{d}\}$, since $t^*_j \le h^*$, and that $|\tilde t^k_{\bar d-\underline{d}+1}-t^*_{\bar d-\underline{d}+1}| = |\min(h^*,h_k)-h^*| = 0$. Using this fact in (A.19) gives

$$\begin{aligned}&\int_0^{h^*}\left|\mathbf{1}_{[\tilde t^k_j,\tilde t^k_{j+1}]}(\sigma)-\mathbf{1}_{[t^*_j,t^*_{j+1}]}(\sigma)\right|d\sigma\\ &\quad\le\begin{cases}|t^k_j-t^*_j|+|t^k_{j+1}-t^*_{j+1}| & \text{if } j\in\{0,\dots,\bar d-\underline{d}-1\}\\ |t^k_j-t^*_j| & \text{if } j=\bar d-\underline{d}.\end{cases}\end{aligned} \tag{A.20}$$

Let us now define $\Delta t^k_j := t^k_j - t^*_j,\ j \in \{0,1,2,\dots,\bar d-\underline{d}\}$. Since $t^*_j \in [\underline{t}_j, \bar t_j]\ \forall j \in \{1,2,\dots,\bar d-\underline{d}\}$ and $t^k_j \in [\underline{t}_j,\bar t_j]\ \forall k$ and $j \in \{1,2,\dots,\bar d-\underline{d}\}$, we have that $\Delta t^k_j \in [-\overline{\Delta t}_j, \overline{\Delta t}_j]$, with $\overline{\Delta t}_j = \bar t_j - \underline{t}_j,\ j \in \{1,2,\dots,\bar d-\underline{d}\}$. Substituting (A.20) in (A.18) and using the definition of Δt^k_j above, we obtain

$$\begin{aligned}\int_{s_k}^{s_k+h^*}\left|u(s)-u^*(s)\right|ds &\le \max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\\ &\quad\times\left(\sum_{j=0}^{\bar d-\underline{d}-1}\left(|\Delta t^k_j|+|\Delta t^k_{j+1}|\right)+|\Delta t^k_{\bar d-\underline{d}}|\right)\\ &= 2\max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\sum_{j=1}^{\bar d-\underline{d}}|\Delta t^k_j|,\end{aligned} \tag{A.21}$$

since $|\Delta t^k_0| = 0$. Using (A.21) in (A.17), we obtain

$$\begin{aligned}|\bar F^e_{\theta_k}-\bar F^e_{\theta^*}| &\le 2L_f e^{L_f h^*}\max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\sum_{j=1}^{\bar d-\underline{d}}|\Delta t^k_j|\\ &+e^{L_f h^*}\left(e^{L_f(h_k-h^*)}-1\right)\left(|x_k|+\max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|)\right).\end{aligned} \tag{A.22}$$

Next, it is exploited that $|x_k| \le |\xi_k|$, $|\bar u_k| \le |\xi_k|$, Assumption 5 implies that $|u_k| \le L_u|\xi_k|$ and $\max_{i\in\{0,\dots,\bar d-\underline{d}\}}(|u_{k+i-\bar d}|) = \max(|u_{k-\bar d}|,\dots,|u_{k-\underline{d}}|) \le \max(|\bar u_k|,|u_k|)$ to rewrite (A.22) as follows:

$$|\bar F^e_{\theta_k}-\bar F^e_{\theta^*}| \le \rho_\theta\left(h^*, M_h, M_{t_1},\dots,M_{t_{\bar d-\underline{d}}}\right)|\xi_k| \tag{A.23}$$

with $\rho_\theta\left(h^*, M_h, M_{t_1},\dots,M_{t_{\bar d-\underline{d}}}\right)$ defined in (18) and where we note that (A.23) holds for the case that $h^* \le h_k$ (treated here in detail) and the case that $h^* > h_k$ (which can be treated in an analogous fashion).

Next, we return to the evaluation of the increment ΔV_k of the candidate Lyapunov function given in (A.2) by using (A.9)–(A.11) and (A.23):

$$\begin{aligned}\Delta V_k \le |\xi_k|^p\Bigg(&-a_3\frac{h^*}{\bar h}+\frac{L_v\,(L_a)^{p-1}}{\bar h}\times\Big(h^*\hat\rho(h^*)\,(1+L_u)\\ &+\ \rho_\theta\left(h^*, M_h, M_{t_1},\dots,M_{t_{\bar d-\underline{d}}}\right)\Big)\Bigg).\end{aligned} \tag{A.24}$$

The satisfaction of condition (17) in the theorem for some $0 < \beta < 1$ implies that

$$\frac{L_v\,(L_a)^{p-1}}{\bar{h}}\Big(h^*\hat{\rho}(h^*)\,(1+L_u) + \rho_\theta\left(h^*, M_h, M_{t_1}, \ldots, M_{t_{\bar{d}-\underline{d}}}\right)\Big) \leq (1-\beta)a_3\frac{h^*}{\bar{h}} \tag{A.25}$$

since $\bar{h} \geq \underline{h} > 0$. Substitution of (A.25) in (A.24) gives

$$\Delta V_k \leq -a_3\beta\frac{h^*}{\bar{h}}|\xi_k|^p. \tag{A.26}$$

Since $\underline{h} < h^* \leq \bar{h}$, there exists an $0 \leq \varepsilon < 1$ such that $h^* = \varepsilon\underline{h} + (1-\varepsilon)\bar{h}$. Consequently, $\frac{h^*}{\bar{h}} = \varepsilon\underline{h}/\bar{h} + (1-\varepsilon) \geq (1-\varepsilon)$ and (A.26) gives

$$\Delta V_k \leq -a_3\beta(1-\varepsilon)|\xi_k|^p. \tag{A.27}$$

Note that (A.27) with the definition of ΔV_k in (A.1) implies that

$$\frac{V_{\theta^*}(\bar{F}^e_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k))) - V_{\theta^*}(\xi_k)}{h_k} \leq -a_3\beta(1-\varepsilon)|\xi_k|^p, \tag{A.28}$$

$\forall\theta_k \in \Theta$, since $h_k \in \left[\underline{h}, \bar{h}\right]$, $\forall k \in \mathbb{N}$. Given the fact that the function V_{θ^*} satisfies the conditions in (14) of Theorem 1 (see Assumption 1 and (A.28)) we can conclude that the closed-loop uncertain exact discrete-time system (7), (11) is globally exponentially stable.

A.2. Proof of Theorem 3

Note that the term $\frac{L_v(L_a)^{p-1}}{h^*}\rho_\theta\left(h^*, M_h, M_{t_1}, \ldots, M_{t_{\bar{d}-\underline{d}}}\right)$ in (17) can always be made arbitrarily small by an appropriate choice of $\bar{h} - \underline{h}$ and $\bar{\tau} - \underline{\tau}$ (i.e. by making the uncertainty intervals $[\underline{h}, \bar{h}]$ and $[\underline{\tau}, \bar{\tau}]$ sufficiently small). Moreover, using the fact that $\hat{\rho}$ is a $\mathcal{K}_\infty$ function and the fact that Assumptions 1–3 and 5 hold for all $\theta^* \in \Theta_0^*$, where the definition of Θ_0^* in (12) allows h^* to be taken arbitrarily close to zero, the term $L_v(L_a)^{p-1}\hat{\rho}(h^*)\,(1+L_u)$ in (17) can always be made arbitrarily small by making the nominal sampling interval h^* small enough. Consequently, there exists an $h^*_{\max} \leq \bar{h}^*$ such that for all $h^* \in \left(0, h^*_{\max}\right]$, there exist $\underline{h}, \bar{h}, \underline{\tau}, \bar{\tau}$, with $\underline{h} < \bar{h}$ and $\underline{\tau} < \bar{\tau}$, and $0 < \beta < 1 - \varepsilon$ satisfying (17). In turn, this implies, using (A.24)–(A.27) and the definition of ΔV_k in (A.1), that there exists $0 < \beta < 1 - \varepsilon$ such that

$$\frac{V_{\theta^*}(\bar{F}^e_{\theta_k}(\xi_k, u_{\theta^*}(\xi_k))) - V_{\theta^*}(\xi_k)}{h_k} \leq -a_3\beta|\xi_k|^p, \tag{A.29}$$

for all $\theta^* \in \Theta_0^*(h^*_{\max}, \underline{d}, \bar{d}, \eta(\cdot))$ and for all $\theta_k \in \Theta(\underline{h}(\theta^*), \bar{h}(\theta^*), \underline{\tau}(\theta^*), \bar{\tau}(\theta^*))\ \forall k \in \mathbb{N}$, where Θ typically depends on θ^* since $\underline{h}, \bar{h}, \underline{\tau}$ and $\bar{\tau}$ typically depend on h^* when guaranteeing the satisfaction of condition (17). Using Theorem 1, we can now conclude that the closed-loop exact uncertain discrete-time model is GES for all $\theta^* \in \Theta_0^*(h^*_{\max}, \underline{d}, \bar{d}, \eta(\cdot))$ and $\theta_k \in \Theta(\theta^*)\ \forall k \in \mathbb{N}$.

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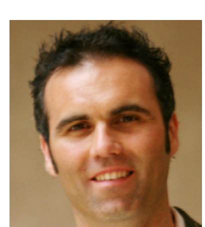

N. van de Wouw (born, 1970) obtained his M.Sc.-degree (with honours) and Ph.D.-degree in Mechanical Engineering from the Eindhoven University of Technology, Eindhoven, the Netherlands, in 1994 and 1999, respectively. From 1999 until 2011 he has been affiliated with the Department of Mechanical Engineering of the Eindhoven University of Technology in the group of Dynamics and Control as an Assistant/Associate Professor. In 2000, Nathan van de Wouw has been working at Philips Applied Technologies, Eindhoven, The Netherlands, and, in 2001, he has been working at the Netherlands Organisation for Applied Scientific Research (TNO), Delft, The Netherlands. He has held positions as a

visiting professor at the University of California at Santa Barbara, USA, in 2006/2007 and at the University of Melbourne, Australia, in 2009/2010. Nathan van de Wouw has published a large number of journal and conference papers and the books 'Uniform Output Regulation of Nonlinear Systems: A convergent Dynamics Approach' with A.V. Pavlov and H. Nijmeijer Birkhauser 2005 and 'Stability and Convergence of Mechanical Systems with Unilateral Constraints' with R.I. Leine (Springer-Verlag, 2008). His current research interests are the analysis and control of nonlinear/nonsmooth systems and networked control systems.

D. Nešić is a Professor in the Department of Electrical and Electronic Engineering (DEEE) at The University of Melbourne, Australia. He received his BE degree in Mechanical Engineering from The University of Belgrade, Yugoslavia in 1990, and his Ph.D. degree from Systems Engineering, RSISE, Australian National University, Canberra, Australia in 1997. Since February 1999 he has been with The University of Melbourne. His research interests include networked control systems, discrete-time, sampled-data and continuous-time nonlinear control systems, input-to-state stability, extremum seeking control, applications of symbolic computation in control theory, hybrid control systems, and so on. He was awarded a Humboldt Research Fellowship (2003) by the Alexander von Humboldt Foundation, an Australian Professorial Fellowship (2004–2009) and Future Fellowship (2010–2014) by the Australian Research Council. He is a Fellow of IEEE and a Fellow of IEAust. He is currently a Distinguished Lecturer of CSS, IEEE (2008–2010). He served as an Associate Editor for the journals Automatica, IEEE Transactions on Automatic Control, Systems and Control Letters and European Journal of Control.

W.P.M.H. Heemels received the M.Sc. in Mathematics (with the highest distinction) and the Ph.D. degrees (cum laude) from the Eindhoven University of Technology (TU/e), Eindhoven, the Netherlands, in 1995 and 1999, respectively.

From 2000 to 2004, he was with the Electrical Engineering Department, TU/e, as an Assistant Professor, and from 2004 to 2006 with the Embedded Systems Institute (ESI) as a Research Fellow. Since 2006, he has been with the Department of Mechanical Engineering, TU/e, where he is currently a Full Professor in the Hybrid and Networked Systems Group. He held visiting research positions at the Swiss Federal Institute of Technology (ETH), Zurich, Switzerland (2001) and at the University of California at Santa Barbara (2008). In 2004, he was also at the Research & Development laboratory, Océ, Venlo, the Netherlands. His current research interests include hybrid and nonsmooth dynamical systems, networked control systems and constrained systems including model predictive control.

Dr. Heemels is an Associate Editor for the journals Automatica and *Nonlinear Analysis: Hybrid Systems* and will serve as the General Chair of the 4th IFAC Conference on Analysis and Design of Hybrid Systems 2012 in Eindhoven, The Netherlands.