Decentralized observer-based control via networked communication☆

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A B S T R A C T

This paper provides one of the first approaches to the design of decentralized observer-based output-feedback controllers for linear plants where the controllers, sensors and actuators are connected via a shared communication network subject to time-varying transmission intervals and delays. Due to the communication medium being shared, it is impossible to transmit all control commands and measurement data simultaneously. As a consequence, a protocol is needed to orchestrate what data is sent over the network at each transmission instant. To effectively deal with the shared communication medium using observer-based controllers, we adopt a switched observer structure that switches based on the available measured outputs and a switched controller structure that switches based on available control inputs at each transmission time. By taking a discrete-time switched linear system perspective, we are able to derive a general model that captures all these networked and decentralized control aspects. The proposed synthesis method is based on decomposing the closed-loop model into a multi-gain switched static output-feedback form. This decomposition allows for the formulation of linear matrix inequality based synthesis conditions which, if satisfied, provide stabilizing observer-based controllers, which are both decentralized and robust to network effects. A numerical example illustrates the strengths as well as the limitations of the developed theory.

1. Introduction

Recently, there has been an enormous interest in the control of large-scale networked systems that are physically distributed over a wide area (Murray, Åström, Boyd, Brockett, & Stein, 2003). Examples of such distributed systems are electrical power distribution networks (Blaabjerg, Teodorescu, Liserre, & Timbus, 2006), water transportation networks (Cembrano, Wells, Quevedo, Pérez, & Argelaguet, 2000), industrial factories (Moyne & Tilbury, 2007) and energy collection networks (such as wind farms Johnson & Thomas, 2009). The purpose of developing control theory in this large-scale setting is to work towards the goal of a streamlined design process which consistently results in efficient operation of these vital systems. Our contribution towards this goal is in the area of stabilizing controller design. This problem setting has many features that seriously challenge controller design.

The first feature which challenges controller design is that the controller is decentralized, in the sense that it consists of a number of local controllers that do not share information. Although a centralized controller could alternatively be considered, the achievable bandwidth associated with using a centralized control structure would be limited by long delays induced by the communication between the centralized controller and distant sensors and actuators over a (wireless) communication network (Al-Hammouri, Branicky, Liberatore, & Phillips, 2006). The difficulty of decentralized control synthesis lies in the fact that each local controller has only local information to utilize for control, which implies that the other local control actions are unknown and can be perceived as disturbances. This fundamental problem has received ample attention (Anderson & Moore, 1981; Sandell, Varaiya, Athans, & Safonov, 1978; Šiljak, 1991), but still many issues are actively researched today. A recent survey (Bakule, 2008) highlights newly developed techniques to solve this problem in different settings and recommends that research should consider interconnected systems which are controlled over realistic communication channels. This forms the exact topic of the presented paper.

The problem of synthesizing decentralized linear controllers is often referred to as the 'information-constrained' synthesis problem or the 'structured' synthesis problem due to the presence of zeros in the controller matrices corresponding to the decentralized structure. This synthesis problem is, in general, non-convex. It was shown in Rotkowitz and Lall (2005) that linear time-invariant systems which satisfy a property called 'quadratic invariance', with respect to the controller information structure, allow for convex synthesis of optimal static feedback controllers. For the specific case of block diagonal *static state* feedback

control design, (Geromel, Bernussou, & Peres, 1994) discovered that through a change of variable, linear matrix inequality (LMI) synthesis conditions could be formulated which guarantee robust stability. However, in the decentralized (block diagonal) dynamic output-feedback setting, the (robust) controller synthesis problem is far more complex (Stanković, Stipanović, & Šiljak, 2007).

The second feature which challenges controller design comes from the fact that when considering control of a large-scale system, it would be unreasonable to assume that all states are measured. Therefore an output-based controller is needed. This paper will, in fact, consider an observer-based control setup, which offers the additional advantage of reducing the number of sensors needed. The latter aspect alleviates the demands on the communication network design. However, it has been shown that, in general, it is hard to obtain decentralized observers providing state estimates converging to the 'true' states (Šiljak, 1991). In Stanković et al. (2007) and Zhu and Pagilla (2007), synthesis conditions for robust decentralized observer-based control with respect to unknown nonlinear subsystem coupling, which is sector bounded and state-dependent, were presented. In both papers, a decoupled quadratic Lyapunov function candidate was used to derive stabilizing gains that could be synthesized by transforming a linear minimization problem subject to a bilinear matrix inequality (BMI) into a two-step linear minimization problem subject to LMIs. It was also mentioned in Stanković et al. (2007) that in the simpler setting of the subsystem coupling matrices being linear and known, as is the setting in the current paper, the robust synthesis conditions are still obtained by convexifying the overlying problem of linear minimization subject to a BMI. Finally, we point out that all the aforementioned decentralized results, excluding the notable exception of Rotkowitz and Lall (2005) which includes communication delays, consider the communication channels between sensors, actuators and controllers to be *ideal*.

The third feature which challenges controller design arises from the fact that the implementation of a decentralized control strategy may not be economically feasible without a way to inexpensively connect the sensors, actuators and controllers. Indeed, the advantages of using a wired/wireless network compared to dedicated point-to-point (wired) connections between all sensors, controllers and actuators are inexpensive and easily modifiable communication links. However, the drawback is that the control system is susceptible to undesirable (possibly destabilizing) side-effects such as time-varying transmission intervals, time-varying delays, packet dropouts, quantization and a shared communication medium (the latter implying that not all information can be sent over the network at once). Clearly, the decentralized observer-based controller needs to have certain robustness properties with respect to these effects. For modeling simplicity, we only consider time-varying transmission intervals and the communication medium to be shared in this work, although extensions including the other side effects can be envisioned within the presented framework. In fact, the extension to including time-varying delays will be discussed explicitly in Remark 3.7.

In the Networked Control System (NCS) literature, there are many existing results on stability analysis which consider linear static controllers (Cloosterman, van de Wouw, Heemels, & Nijmeijer, 2009; Fujioka, 2008; Garcia-Rivera & Barreiro, 2007; Naghshtabrizi, Hespanha, & Teel, 2008; van de Wouw, Naghshtabrizi, Cloosterman, & Hespanha, 2009), linear dynamic controllers (Donkers, Heemels, van de Wouw, & Hetel, 2011; Walsh, Ye, & Bushnell, 2002), nonlinear dynamic controllers (Bauer, Maas, & Heemels, 2012; Heemels, Teel, van de Wouw, & Nešić, 2010; Nešić & Teel, 2004) and observer-based controllers (Montestruque & Antsaklis, 2004). However, results on controller synthesis for NCSs are rare (Hespanha, Naghshtabrizi, & Xu, 2007). LMI conditions for synthesis of state feedback (Cloosterman et al., 2010) and static output-feedback (Hao & Zhao, 2010) only became

available recently. For general linear dynamic controller synthesis, (Dačić & Nešić, 2007) considered the simultaneous design of the protocol, without considering time-varying transmission intervals or delays, and resulted in a linearized BMI algorithm. General linear dynamic controller synthesis conditions were also formulated in Gao, Meng, Chen, and Lam (2010), where the NCS included quantization, delay and packet dropout but without a shared communication medium, which resulted in LMI conditions only when a specific design variable (ϵ in Gao et al. (2010)) is fixed. Synthesis conditions for observer gains that stabilize the state estimation error (but not the state of the plant itself) in the presence of a shared communication medium were given in Dačić and Nešić (2008). The inclusion of varying transmission intervals were recently presented in Postoyan and Nešić (2010). In Zhang and Hristu-Varsakelis (2006), Gramian-based tools were used to synthesize observer-based gains that stabilize the closed-loop in the presence of a shared communication medium but they did not consider time-varying transmission intervals nor delays. Conditions for observer-based controller synthesis in the presence of time-varying delay, time-varying transmission intervals, and dropouts were given in Naghshtabrizi and Hespanha (2005). The synthesis conditions were derived by changing a non-convex feasibility problem into a linear minimization problem via a linear cone complementarity algorithm. It is worth mentioning that all the aforementioned NCS results consider the *centralized* controller problem setting.

To summarize, we note that although a decentralized observer-based control structure is reasonable to use in practice, its design is extremely complex due to the fact that we simultaneously face the issues of (i) a decentralized control structure, (ii) limited measurement information and (iii) communication side-effects. In this context, the contribution of this paper is twofold: firstly, a model describing the controller decentralization and the communication side-effects is derived, and, secondly, the most significant contribution is LMI-based synthesis conditions for decentralized switched observer-based controllers and decentralized switched static feedback controllers, which are robust to communication imperfections. For the simpler case of static output feedback, we refer the reader to Bauer, Donkers, van de Wouw, and Heemels (2012).

1.1. Nomenclature

The following notation will be used. $\mathrm{diag}(A_1, \ldots, A_N)$ denotes a block-diagonal matrix with the matrices $A_1, \ldots, A_N$ on the diagonal and $A^\top \in \mathbb{R}^{m \times n}$ denotes the transpose of the matrix $A \in \mathbb{R}^{n \times m}$. For a vector $x \in \mathbb{R}^n$, $\|x\| := \sqrt{x^\top x}$ denotes its Euclidean norm. We denote by $\|A\| := \sqrt{\lambda_{\max}(A^\top A)}$ the spectral norm of a matrix A, which is the square-root of the maximum eigenvalue of the matrix $A^\top A$. For brevity, we sometimes write symmetric matrices of the form $\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$ as $\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$. For a matrix $A \in \mathbb{R}^{n \times m}$ and two subsets $\mathbf{I} \subseteq \{1, \ldots, n\}$ and $\mathbf{J} \subseteq \{1, \ldots, m\}$, the $(\mathbf{I}, \mathbf{J})$-submatrix of A is defined as $(A)_{\mathbf{I},\mathbf{J}} := (a_{ij})_{i \in \mathbf{I}, j \in \mathbf{J}}$. In case $\mathbf{I} = \{1, \ldots, n\}$, we also write $(A)_{\bullet,\mathbf{J}}$.

2. The model and problem definition

We consider a collection of coupled continuous-time linear subsystems $\mathcal{P}_1, \ldots, \mathcal{P}_N$ given by

$$\mathcal{P}_i : \begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i \hat{u}_i(t) \\ \qquad + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left(A_{i,j} x_j(t) + B_{i,j} \hat{u}_j(t) \right), \\ y_i(t) = C_i x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_{i,j} x_j(t), \end{cases} \tag{1}$$
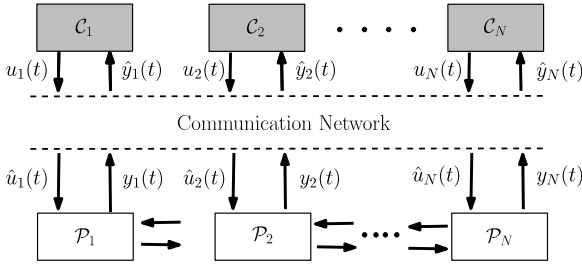
Fig. 1. Decentralized NCS.

for $i \in \{1, \ldots, N\}$, where $x_i \in \mathbb{R}^{n_{x_i}}$, $\hat{u}_i \in \mathbb{R}^{n_{u_i}}$, and $y_i \in \mathbb{R}^{n_{y_i}}$ are the subsystem state, input and output vectors, respectively. The subsystem interaction matrices, $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, $i \neq j$, represent how subsystem j affects subsystem i via state, input and output coupling, respectively. We consider this collection of subsystems to be disjoint, i.e. in the sense of Šiljak (1991), that is the entire collection can be compactly written as

$$\mathcal{P} : \begin{cases} \dot{x}(t) = Ax(t) + B\hat{u}(t), \\ y(t) = Cx(t), \end{cases} \tag{2}$$

with state $x = [x_1^\top, x_2^\top, \ldots, x_N^\top]^\top \in \mathbb{R}^{n_x}$, control input $\hat{u} = [\hat{u}_1^\top, \hat{u}_2^\top, \ldots, \hat{u}_N^\top]^\top \in \mathbb{R}^{n_u}$ and measured output $y = [y_1^\top, y_2^\top, \ldots, y_N^\top]^\top \in \mathbb{R}^{n_y}$. The matrix A is defined as

$$A := \begin{bmatrix} A_1 & A_{1,2} & \cdots & A_{1,N} \\ A_{2,1} & A_2 & & \vdots \\ \vdots & & \ddots & \\ A_{N,1} & \cdots & & A_N \end{bmatrix}$$

and the matrices B and C in (2) are defined similarly. The objective of this paper is to present an approach for synthesis of a controller for system (2) that has the following features: (i) discrete-time (desirable for networked sampled-data implementation); (ii) decentralized; (iii) output-based; (iv) robustly stabilizes the origin of system (2) despite the uncertain time-varying transmission intervals $h_k \in [\underline{h}, \bar{h}]$; (v) operates in the presence of a shared communication medium: not all measured outputs and control inputs can be communicated simultaneously and a protocol schedules which information is sent at the transmission instants.

Due to these design features, we consider a decentralized control structure consisting of N local controllers $\mathcal{C}_i$, $i \in \{1, \ldots, N\}$, which communicate with the sensors and actuators of the plant via a shared network. The decentralized control structure we consider 'parallels' the chosen plant decomposition as in (1). This is depicted in Fig. 1, where the ith controller receives measurements from and sends control commands to the ith subsystem only.

In the next sections, we will provide additional information regarding the setup in Fig. 1 by discussing the consequences of the design features of the controller in more detail. In particular, in Section 2.1, a description of the network imperfections is provided for which the controller has to be robust. In Section 2.2, a switching observer-based control structure will be presented that will switch based on the received measurement information and, finally, in Sections 2.3 and 2.4, a closed-loop model suitable for controller synthesis is derived incorporating the aforementioned aspects.

2.1. Network description

Communication between sensors, actuators and controllers will take place via a shared network, see Fig. 1. Here, we will consider two network effects: namely, time-varying transmission intervals and a shared communication medium, where the latter imposes the need for a scheduling protocol to determine what

measurement and control command data is transmitted at each transmission instant. In Remark 3.7, we will also explain how time-varying delays can be incorporated in a straightforward manner.

Assuming that the transmission intervals $h_k = t_{k+1} - t_k$ are contained in $[\underline{h}, \bar{h}]$ for some $0 < \underline{h} \leq \bar{h}$, i.e. $h_k \in [\underline{h}, \bar{h}]$ for all $k \in \mathbb{N}$ and a zero-order-hold assumption on the inputs $\hat{u}$, meaning that

$$\hat{u}(t) = \hat{u}_k \quad \text{for all } t \in [t_k, t_{k+1}), \ k \in \mathbb{N}, \tag{3}$$

the exact discrete-time equivalent of (2) is

$$\mathcal{P}_{h_k} : \begin{cases} x_{k+1} = \bar{A}_{h_k} x_k + \bar{B}_{h_k} \hat{u}_k, \\ y_k = Cx_k, \end{cases} \tag{4}$$

where $\bar{A}_{h_k} := e^{Ah_k}$ and $\bar{B}_{h_k} := \int_0^{h_k} e^{As} ds B$. In (4), $x_k := x(t_k)$, $y_k := y(t_k)$, with t_k the transmission instants, and $\hat{u}_k$ is the discrete-time control action available at the plant at $t = t_k$.

Since the plant and controller are communicating through a network with a shared communication medium, the actual input of the plant $\hat{u}_k \in \mathbb{R}^{n_u}$ is not equal to the controller output u_k and the actual input of the controller $\hat{y}_k \in \mathbb{R}^{n_y}$ is not equal to the sampled plant output y_k. Instead, $\hat{u}_k$ and $\hat{y}_k$ are 'networked' versions of u_k and y_k, respectively. In Section 2.2, we will detail how the controller output u_k will be determined based on $\hat{y}_k$ (see Fig. 1).

To explain the effect of the shared communication medium and thus the difference between $\hat{y}_k$ and y_k, and $\hat{u}_k$ and u_k, $k \in \mathbb{N}$, realize that the plant has n_y sensors and n_u actuators. In fact, the actuators and sensors are grouped into $\bar{N}$ nodes, where, in principle, it is allowed that a node can contain both sensors and actuators. The set of actuator and sensor indices corresponding to node $l \in \{1, \ldots, \bar{N}\}$ are denoted by

$$\bar{\mathbf{J}}_l^u \subseteq \{1, \ldots, n_u\}, \qquad \bar{\mathbf{J}}_l^y \subseteq \{1, \ldots, n_y\},$$

respectively.

At each transmission instant, only one node obtains access to the network and transmits its corresponding u and/or y values. Only the transmitted values will be updated, while all other values remain unchanged. This constrained data exchange can be expressed as

$$\hat{u}_k = \Gamma_{\sigma_k}^u u_k + (I - \Gamma_{\sigma_k}^u)\hat{u}_{k-1}, \tag{5a}$$

$$\hat{y}_k = \Gamma_{\sigma_k}^y y_k + (I - \Gamma_{\sigma_k}^y)\hat{y}_{k-1}, \tag{5b}$$

where the value of $\sigma_k \in \{1, \ldots, \bar{N}\}$ indicates which node is given access to the network at the transmission instant $k \in \mathbb{N}$, and $\Gamma_l^u \in \mathbb{R}^{n_u \times n_u}$ and $\Gamma_l^y \in \mathbb{R}^{n_y \times n_y}$, for $l \in \{1, \ldots, \bar{N}\}$, are diagonal matrices where

$$(\Gamma_l^u)_{i,i} := \begin{cases} 1, & \text{if } i \in \bar{\mathbf{J}}_l^u, \\ 0, & \text{otherwise}, \end{cases}$$

$$(\Gamma_l^y)_{i,i} := \begin{cases} 1, & \text{if } i \in \bar{\mathbf{J}}_l^y, \\ 0, & \text{otherwise}. \end{cases}$$

The mechanisms determining σ_k at transmission instant t_k are known as protocols. In this paper, we focus on the general class of periodic protocols (Donkers et al., 2011; Hong, 1995), which are characterized by

$$\sigma_{k+\tilde{N}} = \sigma_k, \quad \text{for all } k \in \mathbb{N}, \tag{6a}$$

$$\{\sigma_k | 1 \leq k \leq \tilde{N}\} \supseteq \{1, \ldots, \bar{N}\}, \tag{6b}$$

where $\tilde{N} \geq \bar{N}$ and $\tilde{N} \in \mathbb{N}$ is the period of the protocol. Note that $\{\sigma_k | 1 \leq k \leq \tilde{N}\} \supseteq \{1, \ldots, \bar{N}\}$ means that every node is addressed at least once in every period of the protocol. This condition is very natural as nodes that are never used do not need to be defined. The well-known Round Robin protocol (Walsh et al., 2002) belongs to this class of periodic protocols, which is characterized by (6) and $\tilde{N} = \bar{N}$. Implementation of such a protocol can be accomplished by using the channel access method known as (multi-channel) time division multiple access (TDMA).

Remark 2.1. A commonly studied (dynamic) protocol in the NCS literature is the Try-Once-Discard (TOD) protocol, which was introduced in Walsh et al. (2002) and recently studied in Dačić and Nešić (2007), Donkers et al. (2011) and Heemels et al. (2010). Although analysis of this protocol has shown improvement in the level of robustness with respect to network-induced effects (compared with periodic protocols), designing an output-based controller using a dynamic protocol is extremely challenging, even in the centralized setting, see, e.g., Dačić and Nešić (2007) in which constant transmission intervals have been considered. Considering the additional challenges that decentralization introduces into the NCS setting, in this paper, we choose to focus on periodic protocols for which an LMI-based design procedure will be offered.

To characterize the decentralized NCS, we need to determine the sets of actuators and sensors that are associated with node $l \in \{1, \ldots, \bar{N}\}$ and belong to subsystem $i \in \{1, \ldots, N\}$. To achieve this we can use the structure present in the disjoint decomposition. Due to the fact that we consider the decomposition of (2) to be disjoint, as given in (1), we have that the input vector $\hat{u}_k$, output vector y_k and state vector x_k are ordered such that the sets of indices corresponding to actuators $\hat{u}_k$, sensors y_k, and states x_k belonging to subsystem i are defined as

$$\mathbf{J}_i^u := \left\{ \sum_{j=0}^{i-1} n_{u_j} + 1, \sum_{j=0}^{i-1} n_{u_j} + 2, \ldots, \sum_{j=0}^{i} n_{u_j} \right\},$$

$$\mathbf{J}_i^y := \left\{ \sum_{j=0}^{i-1} n_{y_j} + 1, \sum_{j=0}^{i-1} n_{y_j} + 2, \ldots, \sum_{j=0}^{i} n_{y_j} \right\},$$

$$\mathbf{J}_i^x := \left\{ \sum_{j=0}^{i-1} n_{x_j} + 1, \sum_{j=0}^{i-1} n_{x_j} + 2, \ldots, \sum_{j=0}^{i} n_{x_j} \right\},$$

respectively, for $i \in \{1, \ldots, N\}$, where $n_{u_0} = n_{y_0} = n_{x_0} := 0$ and n_{u_i}, n_{y_i} and n_{x_i} denote the number of actuators, sensors and states, respectively, belonging to subsystem $i \in \{1, \ldots, N\}$. With these sets defined, we have that the set $\bar{\mathbf{J}}_l^u \cap \mathbf{J}_i^u$ consists exactly of the indices of the actuators which are associated with node l and belong to subsystem i. A similar interpretation holds for $\bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y$ regarding the indices of the sensors. We say that subsystem i is associated with node l if $\bar{\mathbf{J}}_l^u \cap \mathbf{J}_i^u \neq \emptyset$ or $\bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y \neq \emptyset$, meaning, that at least one sensor or actuator in node l belongs to subsystem i.

2.2. Decentralized networked observer-based controllers

In this paper, we will use decentralized observer-based controllers in the sense that for each subsystem of the plant we have one observer-based controller which does not exchange information, see Fig. 1. Therefore, the individual observers have no information about externally coupled states, inputs, or outputs.

To obtain approximate discrete-time subsystem models for usage in the observer, we discretize (2) with a suitably chosen constant transmission interval $h_\star$ and then discard the subsystem coupling matrices (as the observers to be designed cannot use information about either the external coupling or the time-varying nature of the sampling interval). The resulting discrete-time plant model for the ith subsystem is then

$$\mathcal{P}_{h_\star, i} : \begin{cases} \check{x}_{k+1,i} = \bar{A}_{h_\star, i} \check{x}_{k,i} + \bar{B}_{h_\star, i} \hat{u}_{k,i}, \\ \check{y}_{k,i} = C_i \check{x}_{k,i}, \end{cases} \tag{7}$$

for $i \in \{1, \ldots, N\}$, where $h_\star$ is a constant transmission interval, $\check{x}_{k,i} \in \mathbb{R}^{n_{x_i}}$, $\hat{u}_{k,i} := \hat{u}_i(t_k) \in \mathbb{R}^{n_{u_i}}$ and $\check{y}_{k,i} \in \mathbb{R}^{n_{y_i}}$ are the state, input and output vectors, respectively, of the ith approximate discrete-time model at discrete time $k \in \mathbb{N}$, and $\bar{A}_{h_\star, i} := (\bar{A}_{h_\star})_{\mathbf{J}_i^x, \mathbf{J}_i^x}$ and $\bar{B}_{h_\star, i} := (\bar{B}_{h_\star})_{\mathbf{J}_i^x, \mathbf{J}_i^u}$ where $\bar{A}_{h_\star}$ and $\bar{B}_{h_\star}$ have been defined

below (4). The observer-based controllers will use the approximate discrete-time subsystem models (7) that are based on the constant transmission interval $h_\star$, while the exact discrete-time model (4) corresponds to an uncertain and time-varying transmission interval h_k, which in general is not equal to $h_\star$. Hence, the variation in the transmission interval will act as a disturbance on the state estimation error dynamics as the observer model does not coincide with the true model. Clearly, the coupling terms are neglected in (7) which contributes to a further difference between the models (7) and the true model (4). Given $h_\star$, the designed observer and controller gains have to be designed in order to counteract these differences. Ways on how to determine a suitable $h_\star$ will be discussed at the end of Section 4.1.

Using (7) as the ith (approximate) subsystem model, we propose the ith observer-based controller to be of the form

$$\mathcal{C}_{\sigma_k, i} : \begin{cases} \tilde{x}_{k+1,i} = \bar{A}_{h_\star, i} \tilde{x}_{k,i} + \bar{B}_{h_\star, i} \hat{u}_{k,i} \\ \quad + L_{\sigma_k, i} \Gamma^y_{\sigma_k, i} (\hat{y}_{k,i} - C_i \tilde{x}_{k,i}), \\ u_{k,i} = K_{\sigma_k, i} \tilde{x}_{k,i}, \end{cases} \tag{8}$$

for $i \in \{1, \ldots, N\}$, where $\tilde{x}_{k,i} \in \mathbb{R}^{n_{x_i}}$, $\hat{y}_{k,i} \in \mathbb{R}^{n_{y_i}}$ and $u_{k,i} \in \mathbb{R}^{n_{u_i}}$ are the state estimate, input, and output vectors of the ith observer-based controller at the discrete time $k \in \mathbb{N}$, respectively. The matrices $\Gamma^u_{l,i} := (\Gamma^u_l)_{\mathbf{J}_i^u, \mathbf{J}_i^u}$, $\Gamma^y_{l,i} := (\Gamma^y_l)_{\mathbf{J}_i^y, \mathbf{J}_i^y}$, $L_{l,i} \in \mathbb{R}^{n_{x_i} \times n_{y_i}}$ and $K_{l,i} \in \mathbb{R}^{n_{u_i} \times n_{x_i}}$ are defined for $i \in \{1, \ldots, N\}$, $l \in \{1, \ldots, \bar{N}\}$, where $L_{l,i}$ and $K_{l,i}$ are the observer and controller gains, respectively. Hence, we adopt a switched observer and controller structure (notice the σ_k-dependence of $L_{\sigma_k, i}$ and $K_{\sigma_k, i}$ in (8)) to deal with the communication medium being shared. The presence of $\Gamma^y_{\sigma_k, i}$ in (8) is used so that the standard output injection is only applied to the newly received measurements. If no measurements are received from subsystem i at transmission time t_k (i.e. $\Gamma^y_{\sigma_k, i} = 0$) then (8) reduces to a standard model-based prediction step (according to the model in (7)).

Similar to the plant, the dynamics of all the controllers (8) can be described by a single discrete-time system, which will consist of block diagonal matrices due to the decoupled nature of the controllers:

$$\mathcal{C}_{\sigma_k} : \begin{cases} \tilde{x}_{k+1} = \bar{A}_D \tilde{x}_k + \bar{B}_D \hat{u}_k + L_{\sigma_k} \Gamma^y_{\sigma_k} (\hat{y}_k - C_D \tilde{x}_k) \\ u_k = K_{\sigma_k} \tilde{x}_k, \end{cases} \tag{9}$$

where $\bar{A}_D := \text{diag}(\bar{A}_{h_\star, 1}, \bar{A}_{h_\star, 2}, \ldots, \bar{A}_{h_\star, N})$, $\bar{B}_D$ and C_D defined similarly, and the observer gains

$$L_l = \text{diag}(L_{l,1}, L_{l,2}, \ldots, L_{l,N}), \quad \text{for } l \in \{1, \ldots, \bar{N}\}, \tag{10a}$$

$$K_l = \text{diag}(K_{l,1}, K_{l,2}, \ldots, K_{l,N}), \quad \text{for } l \in \{1, \ldots, \bar{N}\}. \tag{10b}$$

2.3. Closed-loop model

To derive an expression for the closed-loop dynamics, we will adopt the state vector

$$\bar{x}_k = [e_k^\top \ x_k^\top \ \hat{u}_{k-1}^\top \ \hat{y}_{k-1}^\top]^\top \in \mathbb{R}^n,$$

where e_k denotes the state estimation error defined as $e_k := \tilde{x}_k - x_k$, $k \in \mathbb{N}$, and $n = 2n_x + n_u + n_y$. Combining (4), (5) and (9) results in the overall closed-loop system

$$\bar{x}_{k+1} = \tilde{A}_{\sigma_k, h_k} \bar{x}_k, \tag{11}$$

where $\tilde{A}_{l,h}$ is given by (12), $l \in \{1, \ldots, \bar{N}\}$, and $h \in [\underline{h}, \bar{h}]$. In deriving (12) note that $\Gamma^y_{\sigma_k} (I - \Gamma^y_{\sigma_k}) = 0$ was used. The closed-loop system (11) is a discrete-time switched linear parameter-varying (SLPV) system where the switching, as given by σ_k, is due to the communication medium being shared and the parameter uncertainty is caused by the uncertainty in the transmission interval $h_k \in [\underline{h}, \bar{h}]$. $\tilde{A}_{l,h}$ is given in (12) (Box I).

$$\tilde{A}_{l,h} := \begin{bmatrix} \bar{A}_D - L_l \Gamma_l^y C_D + (\bar{B}_D - \bar{B}_h)\Gamma_l^u K_l & (\bar{A}_D - \bar{A}_h) - L_l \Gamma_l^y (C_D - C) + (\bar{B}_D - \bar{B}_h)\Gamma_l^u K_l & (\bar{B}_D - \bar{B}_h)(I - \Gamma_l^u) & 0 \\ \bar{B}_h \Gamma_l^u K_l & \bar{A}_h + \bar{B}_h \Gamma_l^u K_l & \bar{B}_h(I - \Gamma_l^u) & 0 \\ \Gamma_l^u K_l & \Gamma_l^u K_l & (I - \Gamma_l^u) & 0 \\ 0 & \Gamma_l^y C & 0 & (I - \Gamma_l^y) \end{bmatrix} \quad (12)$$

Box I.

2.4. Polytopic overapproximation

In the previous section, we obtained a decentralized NCS model in the form of a switched uncertain system. However, the form as in (11) and (12) is not convenient to develop efficient techniques for analysis or synthesis due to the nonlinear dependence of $\tilde{A}_{\sigma_k,h_k}$ in (12) on the uncertain parameter h_k, as observed before, see e.g. Heemels et al. (2010). To make the system amenable for analysis or synthesis, a procedure is borrowed from Donkers et al. (2011) to overapproximate system (11) and (12) by a polytopic system with norm-bounded additive uncertainty, i.e.

$$\bar{x}_{k+1} = \sum_{m=1}^{M} \alpha_k^m \left(\mathcal{F}_{\sigma_k,m} + \mathcal{G}_m \Delta_k \mathcal{H}_{\sigma_k} \right) \bar{x}_k, \quad (13)$$

where $\mathcal{F}_{l,m} \in \mathbb{R}^{n \times n}$, $\mathcal{G}_m \in \mathbb{R}^{n \times 2n_x}$, $\mathcal{H}_l \in \mathbb{R}^{2n_x \times n}$, for $l \in \{1, \ldots, \bar{N}\}$ and $m \in \{1, \ldots, M\}$, with M the number of vertices of the polytope. The vector $\alpha_k = [\alpha_k^1 \ldots \alpha_k^M]^\top \in \boldsymbol{\Omega}$, for all $k \in \mathbb{N}$, is time-varying with

$$\boldsymbol{\Omega} = \left\{ \alpha \in \mathbb{R}^M \,\middle|\, \sum_{m=1}^{M} \alpha^m = 1 \text{ and } \alpha^m \geq 0, \right.$$

$$\left. \text{for } m \in \{1, \ldots, M\} \right\} \quad (14)$$

and $\Delta_k \in \boldsymbol{\Delta}$, for all $k \in \mathbb{N}$, with the additive uncertainty set $\boldsymbol{\Delta} \subseteq \mathbb{R}^{2n_x \times 2n_x}$ given by

$$\boldsymbol{\Delta} = \left\{ \text{diag}(\Delta^1, \ldots, \Delta^{2Q}) \mid \Delta^{q+jQ} \in \mathbb{R}^{n_{\lambda_q} \times n_{\lambda_q}} , \right.$$

$$\left. \|\Delta^{q+jQ}\| \leq 1, q \in \{1, \ldots, Q\}, j \in \{0, 1\} \right\}, \quad (15)$$

where $n_{\lambda_q} \times n_{\lambda_q}$, $q \in \{1, \ldots, Q\}$, are the dimensions of the qth real Jordan block (Horn & Johnson, 1985) of A and Q is the number of real Jordan blocks of A. System (13) is an overapproximation of system (11) in the sense that for all $l \in \{1, \ldots, \bar{N}\}$, it holds that

$$\left\{ \tilde{A}_{l,h} \mid h \in [\underline{h}, \bar{h}] \right\}$$

$$\subseteq \left\{ \sum_{m=1}^{M} \alpha^m \left(\mathcal{F}_{l,m} + \mathcal{G}_m \Delta \mathcal{H}_l \right) \mid \alpha \in \boldsymbol{\Omega}, \Delta \in \boldsymbol{\Delta} \right\}. \quad (16)$$

Due to this inclusion, stability of (13) for all $\alpha_k \in \boldsymbol{\Omega}$ and $\Delta_k \in \boldsymbol{\Delta}$, $k \in \mathbb{N}$, implies stability of (11) for all $h_k \in [\underline{h}, \bar{h}]$. Although many overapproximation techniques are available, see e.g. the survey (Heemels et al., 2010), here we employ a gridding-based procedure based on Donkers et al. (2011) to overapproximate system (11), such that (16) holds. This choice is motivated by the favorable properties that this method possesses, see Heemels et al. (2010). Below we briefly summarize the main ideas on how to construct such a polytopic overapproximation in an effort to concisely introduce the relevant notation required later to formulate the main synthesis results.

To construct an overapproximation of (11) in the form (13) using a gridding-based approach, a set of grid points $\{\tilde{h}_1, \ldots, \tilde{h}_M\}$, where $\tilde{h}_m \in [\underline{h}, \bar{h}]$, $m \in \{1, \ldots, M\}$, must be chosen. The choice of grid points directly influences the tightness of the overapproximation. There are procedures in the literature which determine the

set of grid points $\{\tilde{h}_1, \ldots, \tilde{h}_M\}$ by iteratively placing each grid point at the location of the worst-case approximation error, thus, iteratively tightening the overapproximation. For the sake of brevity we do not provide the procedure here but instead refer the reader to Donkers et al. (2011) for details. Following a procedure similar to that of Donkers et al. (2011) leads to an overapproximation (13) of (11) satisfying (16), with

$$\mathcal{F}_{l,m} := \tilde{A}_{l,\tilde{h}_m}$$

for $l \in \{1, \ldots, \bar{N}\}$ and $m \in \{1, \ldots, M\}$ and, with B given in (2), we define

$$\mathcal{H}_l := \begin{bmatrix} 0 & T^{-1} & 0 & 0 \\ T^{-1}B\Gamma_l^u K_l & T^{-1}B\Gamma_l^u K_l & T^{-1}B(I - \Gamma_l^u) & 0 \end{bmatrix}, \quad (17)$$

for $l \in \{1, \ldots, \bar{N}\}$ and

$$\mathcal{G}_m := \begin{bmatrix} -T & -T \\ T & T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} U_m, \quad (18)$$

for $m \in \{1, \ldots, M\}$. The matrix T is given by the real Jordan form decomposition (Horn & Johnson, 1985) of the matrix A, as in (2), i.e. $A := T\Lambda T^{-1}$, where T is an invertible matrix and $\Lambda = \text{diag}(\Lambda_1, \ldots, \Lambda_Q)$ with $\Lambda_q \in \mathbb{R}^{n_{\lambda_q} \times n_{\lambda_q}}$, $q \in \{1, \ldots, Q\}$, the qth real Jordan block of A. Additionally,

$$U_m := \text{diag}(\delta_{1,m}^A I_1, \ldots, \delta_{Q,m}^A I_Q, \delta_{1,m}^E I_1, \ldots, \delta_{Q,m}^E I_Q) \quad (19)$$

where I_q is the $n_{\lambda_q} \times n_{\lambda_q}$ identity matrix (i.e. the size of I_q is equal to the size of the qth real Jordan block of A, which is also equal to the size of Δ^q in (15)) and $\delta_{q,m}$ is the worst case approximation error for each real Jordan block, $\Lambda_q, q \in \{1, \ldots, Q\}$ and for each grid point $m \in \{1, \ldots, M\}$. See Donkers et al. (2011) for details on how to compute $\delta_{q,m}$.

We care to stress that the most appealing aspect of this particular overapproximation technique is the fact that it introduces arbitrarily little conservatism when employed in (quadratic-type) Lyapunov-based stability analysis (see Donkers et al., 2011 Theorem V.1), while having direct control over the complexity of the overapproximation through the number of grid points.

Remark 2.2. The stability analysis problem, i.e. determining whether the system (11) and (12) with *given* controller gains $K_l, L_l, l \in \{1, \ldots, \bar{N}\}$, is uniformly globally exponentially stable (UGES) for a given scheduling protocol, as in (6), and given bounds on the transmission interval, i.e. $h_k \in [\underline{h}, \bar{h}]$ for all $k \in \mathbb{N}$, can be addressed by using the overapproximated model (13) combined with the proposed LMI conditions in Donkers et al. (2011). The focus of the current paper is on the more challenging problem of controller synthesis, see Section 3.

Remark 2.3. Using reasoning similar as in Nešić, Teel, and Sontag (1999), it can be shown that UGES of the discrete-time model (11) and (12) with a protocol satisfying (6) implies UGES of the sampled-data NCS (2), (3), (5) and (9), with the same protocol and including the intersample behavior.

3. Controller synthesis

In the previous sections, we derived a model describing an LTI plant interconnected with a decentralized switched observer-based output-feedback controller by a communication network. In this section, we will present the main contribution of this paper consisting of LMI-based conditions for *designing* the decentralized controller and observer gains K_l and L_l, respectively, in (9) by using the overapproximated model (13).

For reasons of transparency, we choose to divide the presentation of our solution into two sections. In Section 3.1, LMI conditions which can be used to synthesize stabilizing controllers are derived for the case when the subsystems are restricted to communicate in a serial fashion (i.e. only one subsystem is allowed to communicate at each transmission instant). Then, in Section 3.2, the foundation laid in Section 3.1 is built upon to derive LMI conditions which synthesize stabilizing controllers for the more general case when the subsystems can communicate (also) in parallel (i.e. multiple subsystems are allowed to communicate at each transmission instant).

3.1. Serial subsystem communication

In this section, we will adopt the network assumption presented below. Next to the transparency reason already given before, a second reason to treat the case corresponding to this assumption separately, is that it represents a relevant subclass of the synthesis problem. In Section 3.2, details are provided regarding how this assumption can be removed.

Assumption 3.1. All sensors or actuators associated with a node must be members of the same subsystem, i.e. for each node $l \in \{1, \ldots, \bar{N}\}$, there exists a subsystem $i \in \{1, \ldots, N\}$ such that $\bar{\mathbf{J}}_l^u \subseteq \mathbf{J}_i^u$ and $\bar{\mathbf{J}}_l^y \subseteq \mathbf{J}_i^y$.

Remark 3.2. Due to Assumption 3.1, only one subsystem can communicate (a part of) its corresponding signals at each transmission time. Indeed, when node $l \in \{1, \ldots, \bar{N}\}$ attains access to the network, only one (corresponding) subsystem $i \in \{1, \ldots, N\}$ can communicate over the network and, hence, one gain $K_{l,i}$ and one gain $L_{l,i}$, as in (10), influence the closed loop dynamics given by either (11) or (16) (due to the presence of $\Gamma_{\sigma_k}^y$ in (9) and the fact that $\hat{u}_k$, given in (5), is the input to the plant). As a consequence, some of the gains $K_{l,i}$ and $L_{l,i}$, which are defined for all $i \in \{1, \ldots, N\}$, have no influence when node $l \in \{1, \ldots, \bar{N}\}$ attains access (in fact all of them that do not correspond to node l will have no influence). We care to stress that we explicitly account for this fact in the synthesis theorems in Section 3. Moreover, we choose to keep the more general definitions, as in (10), since we provide explicit details on how to remove Assumption 3.1 in Section 3.2 (meaning that possibly all of the gains $K_{l,i}$ and $L_{l,i}$, $i \in \{1, \ldots, N\}$, influence the closed-loop dynamics when node l communicates).

Before we can use the overapproximated model (13) for synthesis, an essential step must be taken so that the model (13) can be rewritten in a form which is suitable for controller synthesis. The essential step in achieving LMI-based synthesis conditions is reformulating (12) such that the design variables are non-structured matrices, instead of the structured (block diagonal) matrices K_{σ_k} and L_{σ_k}, as in (10), respectively, that are currently present. To achieve this, we first introduce the set of state indices belonging to subsystems associated with node l as

$$\bar{\mathbf{J}}_l^x \subseteq \{1, \ldots, n_x\}.$$

Due to Assumption 3.1, only one subsystem i is associated with node l and, hence, $\bar{\mathbf{J}}_l^x$ is the set of the state indices corresponding to the subsystem i that is associated with node l (i.e. if i is the

subsystem associated with node l then $\bar{\mathbf{J}}_l^x = \mathbf{J}_i^x$). With these sets defined, we introduce

$$\Upsilon_l^u := \begin{cases} (\Gamma_l^u)_{\bullet, \bar{\mathbf{J}}_l^u}, & \text{if } l \in \mathbf{L}_u, \\ 0, & \text{otherwise}, \end{cases} \tag{20a}$$

$$\Upsilon_l^y := \begin{cases} (\Gamma_l^y)_{\bullet, \bar{\mathbf{J}}_l^y}, & \text{if } l \in \mathbf{L}_y, \\ 0, & \text{otherwise}, \end{cases} \tag{20b}$$

$$\Upsilon_l^x := (I^x)_{\bullet, \bar{\mathbf{J}}_l^x}, \tag{20c}$$

for $l \in \{1, \ldots, \bar{N}\}$, where $I^x \in \mathbb{R}^{n_x \times n_x}$ is the identity matrix and

$$\mathbf{L}_u := \{l \in \{1, \ldots, \bar{N}\} \mid \bar{\mathbf{J}}_l^u \neq \emptyset\}, \tag{21a}$$

$$\mathbf{L}_y := \{l \in \{1, \ldots, \bar{N}\} \mid \bar{\mathbf{J}}_l^y \neq \emptyset\}, \tag{21b}$$

are the sets of node indices that contain at least one actuator or sensor, respectively. Note that Υ_l^u and Υ_l^y are simply matrices consisting of the non-zero columns of Γ_l^u and Γ_l^y, respectively. Finally, we define

$$\bar{K}_l := \begin{cases} (K_l)_{\bar{\mathbf{J}}_l^u, \bar{\mathbf{J}}_l^x}, & \text{if } l \in \mathbf{L}_u, \\ 0, & \text{otherwise}, \end{cases} \tag{22a}$$

$$\bar{L}_l := \begin{cases} (L_l)_{\bar{\mathbf{J}}_l^x, \bar{\mathbf{J}}_l^y}, & \text{if } l \in \mathbf{L}_y, \\ 0, & \text{otherwise}, \end{cases} \tag{22b}$$

for $l \in \{1, \ldots, \bar{N}\}$. Notice that $\bar{K}_l$ and $\bar{L}_l$ consist of all the non-restricted elements of $\Gamma_l^u K_l$ and $L_l \Gamma_l^y$, respectively. With these matrices defined and Assumption 3.1 adopted, we have that the following equations hold

$$\Gamma_l^u K_l = \Upsilon_l^u \bar{K}_l \Upsilon_l^{x\top}, \qquad L_l \Gamma_l^y = \Upsilon_l^x \bar{L}_l \Upsilon_l^{y\top}. \tag{23}$$

Now, (23) allows the closed-loop matrix $\tilde{A}_{\sigma_k, h_k}$ in (12) to be expressed in terms of the non-structured matrices $\bar{K}_{\sigma_k}$ and $\bar{L}_{\sigma_k}$ instead of the structured (block diagonal) matrices K_{σ_k} and L_{σ_k}. The benefit of this is that the synthesis problem for decentralized control, which naturally imposes 'structural' constraints, can now be formulated as a 'non-structured synthesis' problem when Assumption 3.1 is adopted. To help convey (20), (22) and (23), we will explicitly define these matrices for the example given in Section 4.

Recall that after employment of the overapproximation technique described in Section 2.4, we now have a system of the form (13), where the matrices $\mathcal{F}_{l,m} = \tilde{A}_{l,\tilde{h}_m}$ are given by (12) with $h \in \{\tilde{h}_1, \ldots, \tilde{h}_m\}$ and the matrices $\mathcal{H}_l$ and $\mathcal{G}_m$ are given in (17) and (18), respectively. Using (23), we can decompose $\mathcal{F}_{l,m}$ and $\mathcal{H}_l$ in the following way

$$\mathcal{F}_{l,m} = \mathcal{A}_{l,m} + \mathcal{B}_{l,m}\bar{K}_l\mathcal{E}_l - \mathcal{D}_l\bar{L}_l\mathcal{C}_l, \tag{24a}$$

$$\mathcal{H}_l = \mathcal{I}_l + \mathcal{G}_l\bar{K}_l\mathcal{E}_l, \tag{24b}$$

where

$$\mathcal{A}_{l,m} := \begin{bmatrix} \bar{A}_D & \bar{A}_D - \bar{A}_{\tilde{h}_m} & (\bar{B}_D - \bar{B}_{\tilde{h}_m})(I - \Gamma_l^u) & 0 \\ 0 & \bar{A}_{\tilde{h}_m} & \bar{B}_{\tilde{h}_m}(I - \Gamma_l^u) & 0 \\ 0 & 0 & I - \Gamma_l^u & 0 \\ 0 & \Gamma_l^y C & 0 & I - \Gamma_l^y \end{bmatrix}, \tag{25a}$$

$$\mathcal{B}_{l,m} := \begin{bmatrix} (\bar{B}_D - \bar{B}_{\tilde{h}_m})\Upsilon_l^u \\ \bar{B}_{\tilde{h}_m}\Upsilon_l^u \\ \Upsilon_l^u \\ 0 \end{bmatrix}, \qquad \mathcal{E}_l := \begin{bmatrix} \Upsilon_l^{x\top} & \Upsilon_l^{x\top} & 0 & 0 \end{bmatrix}, \tag{25b}$$

$$\mathcal{D}_l := \begin{bmatrix} \Upsilon_l^x \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathcal{C}_l := \begin{bmatrix} \Upsilon_l^{y\top} C_D & \Upsilon_l^{y\top}(C_D - C) & 0 & 0 \end{bmatrix}, \tag{25c}$$

$$\mathcal{I}_l := \begin{bmatrix} 0 & T^{-1} & 0 & 0 \\ 0 & 0 & T^{-1}B(I - \Gamma_l^u) & 0 \end{bmatrix},$$

$$\mathcal{J}_l := \begin{bmatrix} 0 \\ T^{-1}B\Upsilon_l^u \end{bmatrix}. \tag{25d}$$

Now we are ready to state our main result. Notice that (13) with (24) describes a discrete-time switched linear parameter-varying (SLPV) system with norm-bounded uncertainty. No results are available in the literature to synthesize the controller gains $\bar{K}_l$ and $\bar{L}_l$, at present. However, LMI-based synthesis conditions can be obtained by generalizing the results in Daafouz, Riedinger, and Iung (2002) and De Souza and Trofino (2000) in three directions. In particular, the first extension is the accommodation of norm-bounded uncertainty $\mathcal{G}_m \Delta \mathcal{H}_{\sigma_k}$ in (13), where $\Delta \in \boldsymbol{\Delta}$, and the second extension is that the switching sequence (6) that we consider is ordered (periodic in this case), whereas (Daafouz et al., 2002) considered the case of arbitrary switching. Finally, the third extension is generalizing the set of LMI-based conditions in Daafouz et al. (2002) so that solutions for the *multi-gain* switched static output-feedback problem can be included. Although the required extensions of the ideas in Daafouz et al. (2002) contribute towards our main result, we would like to emphasize that using (23) for the formulation of (24) is the foundation upon which our main result is built. Stabilizing controller and observer gains K_l and L_l for the NCS given by (11) with $h_k \in [\underline{h}, \bar{h}]$ and a protocol satisfying (6) can be synthesized according to the following theorem. In the formulation of the theorem, the matrix set

$$\mathbf{R} := \Big\{ \mathrm{diag}(r_1 I_1, \ldots, r_Q I_Q, r_{Q+1} I_1, \ldots, r_{2Q} I_Q)$$

$$\in \mathbb{R}^{2n_x \times 2n_x} \mid r_{\tilde{q}} \in \mathbb{R}, \, r_{\tilde{q}} > 0, \, \tilde{q} \in \{1, 2, \ldots, 2Q\} \Big\} \tag{26}$$

is used, where, as in (19), I_q is the $n_{\lambda_q} \times n_{\lambda_q}$ identity matrix.

Theorem 3.3. *Consider the system* (11) *and* (12) *with* $h_k \in [\underline{h}, \bar{h}]$, $k \in \mathbb{N}$, *and its overapproximation given by* (13), (18) *and* (24). *Assume that* Assumption 3.1 *holds, the protocol satisfies* (6) *and any node* $l \in \{1, \ldots, \bar{N}\}$ *containing at least one sensor, i.e.* $\bar{\mathbf{J}}_l^y \neq \emptyset$, *consists of linearly independent sensors, i.e.* $(C)_{\bar{\mathbf{J}}_l^y, \bullet}$ *has full row rank. Suppose there exist symmetric matrices* P_j, *matrices* $R_{j,m} \in \mathbf{R}$, *with* $\mathbf{R}$ *as in* (26), *and matrices* $G_l, Z_{1,l}, Z_{2,l}, X_{1,l}$ *and* $X_{2,l}$ *where* $j \in \{1, \ldots, \tilde{N}\}$, $m \in \{1, \ldots, M\}$, $l \in \{1, \ldots, \bar{N}\}$ *such that*

$$\begin{bmatrix} G_{\sigma_j} + G_{\sigma_j}^\top - P_j & \Xi_1(j,m)^\top & 0 & \Xi_2(j)^\top \\ \star & P_{j+1} & \mathcal{G}_m R_{j,m} & 0 \\ \star & \star & R_{j,m} & 0 \\ \star & \star & \star & R_{j,m} \end{bmatrix} \succ 0, \tag{27}$$

for $j \in \{1, \ldots, \tilde{N}\}$, $m \in \{1, \ldots, M\}$, *and*

$$X_{1,l}\mathcal{E}_l = \mathcal{E}_l G_l, \quad \text{for } l \in \mathbf{L}_u, \tag{28a}$$

$$X_{2,l}\mathcal{C}_l = \mathcal{C}_l G_l, \quad \text{for } l \in \mathbf{L}_y, \tag{28b}$$

for which we define

$$\Xi_1(j,m) := \mathcal{A}_{\sigma_j,m} G_{\sigma_j} + \mathcal{B}_{\sigma_j,m} Z_{1,\sigma_j} \mathcal{E}_{\sigma_j} - \mathcal{D}_{\sigma_j} Z_{2,\sigma_j} \mathcal{C}_{\sigma_j},$$

$$\Xi_2(j) := \mathcal{I}_{\sigma_j} G_{\sigma_j} + \mathcal{J}_{\sigma_j} Z_{1,\sigma_j} \mathcal{E}_{\sigma_j},$$

for $j \in \{1, \ldots, \tilde{N}\}$, $m \in \{1, \ldots, M\}$, *with* $P_{\tilde{N}+1} := P_1$ *and the sets* $\mathbf{L}_u$ *and* $\mathbf{L}_y$ *are defined in* (21), *respectively. Then the controller gains* K_l, *defined by* (22), (23) *and* $\bar{K}_l = Z_{1,l} X_{1,l}^{-1}$, $l \in \mathbf{L}_u$, *and the observer gains, defined by* (22), (23) *and* $\bar{L}_l = Z_{2,l} X_{2,l}^{-1}$, $l \in \mathbf{L}_y$, *render the system* (11) *and* (12), *with* $h_k \in [\underline{h}, \bar{h}]$, $k \in \mathbb{N}$, *and the mentioned periodic protocol, UGES.*

Proof. See the Appendix. $\quad\square$

Remark 3.4. The requirement that any node $l \in \{1, \ldots, \bar{N}\}$ containing at least one sensor consists of linearly independent sensors, i.e. $(C)_{\bar{\mathbf{J}}_l^y, \bullet}$ has full row rank, is a rather mild condition. Indeed, the natural situation of C having full row rank is a sufficient condition for this requirement.

Remark 3.5. Note that Theorem 3.3 provides only sufficient conditions for robustly stabilizing decentralized controller synthesis. Interestingly, these conditions also become necessary when using quadratic-type node-dependent Lyapunov functions provided a suitable node-dependent linear state-space transformation $\bar{z}_k = \tilde{T}_\ell \bar{x}_k$ is chosen. This fact can be seen by, first, noting that the GNB overapproximation technique introduces arbitrarily little conservatism (at an increasing computational cost) (Donkers et al., 2011, Theorem V.1) and, second, extending the result in De Souza and Trofino (2000, Lemma 1) that proves the necessity part for the simpler case of static feedback of linear discrete-time periodic systems. However, to the best of the authors' knowledge, determining this suitable state-space transformation remains a difficult and open problem.

3.2. Parallel subsystem communication

In this section, we will generalize the reasoning in Section 3.1 to allow the subsystems to communicate in parallel. Specifically, we will explain why Assumption 3.1 is included and how it is possible to remove Assumption 3.1 from Theorem 3.3. If Assumption 3.1 does not hold, sensors and/or actuators from two (or more) subsystems are grouped into one node and, hence, communicate at the same transmission instant. The consequence of two subsystems communicating is that $\bar{K}_l$ and $\bar{L}_l$ as defined in (22) will remain structured, as these gains will then contain elements that must be equal to zero. Due to these design variables containing structure, solutions to Theorem 3.3 using (23) will not be valid. In order to remove Assumption 3.1, (23) needs to be generalized to

$$\Gamma_l^u K_l = \sum_{i=1}^N \Upsilon_{l,i}^u \bar{K}_{l,i} \Upsilon_{l,i}^{x\top}, \qquad L_l \Gamma_l^y = \sum_{i=1}^N \Upsilon_{l,i}^x \bar{L}_{l,i} \Upsilon_{l,i}^{y\top}, \tag{29}$$

where (20) is then generalized to

$$\Upsilon_{l,i}^u := \begin{cases} (\Gamma_l^u)_{\bullet, \bar{\mathbf{J}}_l^u \cap \mathbf{J}_i^u}, & \text{if } l \in \mathbf{L}_{u,i}, \\ 0, & \text{otherwise}, \end{cases}$$

$$\Upsilon_{l,i}^y := \begin{cases} (\Gamma_l^y)_{\bullet, \bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y}, & \text{if } l \in \mathbf{L}_{y,i}, \\ 0, & \text{otherwise}, \end{cases}$$

$$\Upsilon_{l,i}^x := \begin{cases} (I^x)_{\bullet, \bar{\mathbf{J}}_l^x \cap \mathbf{J}_i^x}, & \text{if } \bar{\mathbf{J}}_l^x \cap \mathbf{J}_i^x \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

where (21) is generalized to

$$\mathbf{L}_{u,i} := \{l \in \{1, \ldots, \bar{N}\} \mid \bar{\mathbf{J}}_l^u \cap \mathbf{J}_i^u \neq \emptyset\}, \tag{30a}$$

$$\mathbf{L}_{y,i} := \{l \in \{1, \ldots, \bar{N}\} \mid \bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y \neq \emptyset\}, \tag{30b}$$

and, finally, (22) is generalized to

$$\bar{K}_{l,i} := \begin{cases} (K_l)_{\bar{\mathbf{J}}_l^u \cap \mathbf{J}_i^u, \bar{\mathbf{J}}_l^x \cap \mathbf{J}_i^x}, & \text{if } l \in \mathbf{L}_{u,i}, \\ 0, & \text{otherwise}, \end{cases} \tag{31a}$$

$$\bar{L}_{l,i} := \begin{cases} (L_l)_{\bar{\mathbf{J}}_l^x \cap \mathbf{J}_i^x, \bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y}, & \text{if } l \in \mathbf{L}_{y,i}, \\ 0, & \text{otherwise}. \end{cases} \tag{31b}$$

Notice that, using (29), if multiple subsystems communicate simultaneously then each non-zero gain $\bar{K}_{l,i}$ and $\bar{L}_{l,i}$ is non-structured. In the case that Assumption 3.1 is adopted, for each

$l \in \{1, \ldots, \bar{N}\}$, there exists only one $i \in \{1, \ldots, N\}$ where $\Upsilon_{l,i}^x \neq 0$, and thus (29) simplifies to (23). With (29), we have that (24) becomes

$$\mathcal{F}_{l,m} = \mathcal{A}_{l,m} + \sum_{i=1}^{N} (\mathcal{B}_{l,m,i} \bar{K}_{l,i} \mathcal{E}_{l,i} - \mathcal{D}_{l,i} \bar{L}_{l,i} \mathcal{C}_{l,i}), \tag{32a}$$

$$\mathcal{H}_l = \mathcal{I}_l + \sum_{i=1}^{N} \mathcal{J}_{l,i} \bar{K}_{l,i} \mathcal{E}_{l,i}, \tag{32b}$$

where $\mathcal{B}_{l,m,i}, \mathcal{E}_{l,i}, \mathcal{D}_{l,i}, \mathcal{C}_{l,i}$ and $\mathcal{J}_{l,i}$ are of the form $\mathcal{B}_{l,m}, \mathcal{E}_l, \mathcal{D}_l, \mathcal{C}_l$ and $\mathcal{J}_l$ in (25) with $\Upsilon_{l,i}^u, \Upsilon_{l,i}^y$ and $\Upsilon_{l,i}^x$ substituted for $\Upsilon_l^u, \Upsilon_l^y$ and Υ_l^x, respectively. These extensions lead to the following theorem, which is a generalization of Theorem 3.3.

Theorem 3.6. *Consider the system (11) and (12) with $h_k \in [\underline{h}, \bar{h}]$, $k \in \mathbb{N}$, and its overapproximation given by (13), (18), and (32). Assume that the protocol satisfies (6) and any node $l \in \{1, \ldots, \bar{N}\}$ containing at least one sensor from subsystem i, i.e. $\bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y \neq \emptyset$, consists of linearly independent subsystem sensors, i.e. $(C)_{\bar{\mathbf{J}}_l^y \cap \mathbf{J}_i^y, \bullet}$ has full row rank. Suppose there exist symmetric matrices P_j, matrices $R_{j,m} \in \mathbf{R}$, with $\mathbf{R}$ as in (26), and matrices $G_l, Z_{1,l,i}, Z_{2,l,i}, X_{1,l,i}$ and $X_{2,l,i}$ where $i \in \{1, \ldots, N\}, j \in \{1, \ldots, \tilde{N}\}, m \in \{1, \ldots, M\}, l \in \{1, \ldots, \bar{N}\}$ such that (27) holds for $j \in \{1, \ldots, \tilde{N}\}, m \in \{1, \ldots, M\}$, and*

$$X_{1,l,i} \mathcal{E}_{l,i} = \mathcal{E}_{l,i} G_l, \quad \text{for } l \in \mathbf{L}_{u,i}, \ i \in \{1, \ldots, N\}$$
$$X_{2,l,i} \mathcal{C}_{l,i} = \mathcal{C}_{l,i} G_l, \quad \text{for } l \in \mathbf{L}_{y,i}, \ i \in \{1, \ldots, N\}$$

for which we define

$$\Xi_1(j,m) := \mathcal{A}_{\sigma_j,m} G_{\sigma_j} + \sum_{i=1}^{N} (\mathcal{B}_{\sigma_j,m,i} Z_{1,\sigma_j,i} \mathcal{E}_{\sigma_j,i} - \mathcal{D}_{\sigma_j,i} Z_{2,\sigma_j,i} \mathcal{C}_{\sigma_j,i}),$$

$$\Xi_2(j) := \mathcal{I}_{\sigma_j} G_{\sigma_j} + \sum_{i=1}^{N} \mathcal{J}_{\sigma_j,i} Z_{1,\sigma_j,i} \mathcal{E}_{\sigma_j,i},$$

for $j \in \{1, \ldots, \tilde{N}\}, m \in \{1, \ldots, M\}$, with $P_{\tilde{N}+1} := P_1$ and the sets $\mathbf{L}_{u,i}$ and $\mathbf{L}_{y,i}, i \in \{1, \ldots, N\}$, are defined in (30), respectively. Then the controller gains K_l, defined by (29), (31) and $\bar{K}_{l,i} = Z_{1,l,i} X_{1,l,i}^{-1}, l \in \mathbf{L}_{u,i}, i \in \{1, \ldots, N\}$, and the observer gains, defined by (29), (31) and $\bar{L}_{l,i} = Z_{2,l,i} X_{2,l,i}^{-1}, l \in \mathbf{L}_{y,i}, i \in \{1, \ldots, N\}$, render the system (11), with $h_k \in [\underline{h}, \bar{h}], k \in \mathbb{N}$, and the mentioned periodic protocol, UGES.

Proof. The proof follows directly from Theorem 3.3. □

Remark 3.7. The NCS model presented here can be extended to include time-varying communication delays $\tau_k \in [\underline{\tau}, \bar{\tau}]$, where $\tau_k < h_k$ for all $k \in \mathbb{N}$, using the results in Donkers et al. (2011), in a straightforward manner. Such an extension only requires re-defining $\bar{B}_{h_k}$ to be $\bar{B}_{h_k, \tau_k} = \int_{\tau_k}^{h_k} e^{A(h_k - s)} ds B$ and adding an additional term $W_{h_k, \tau_k} \hat{u}_{k-1} = \int_0^{\tau_k} e^{A(h_k - s)} ds B \hat{u}_{k-1}$ to x_{k+1} in (4). As a direct consequence, the closed-loop system matrix (12) will depend on τ_k. This delay-induced uncertainty can be incorporated into an overapproximated system of the form (13), where the additive uncertainty set Δ then becomes part of $\mathbb{R}^{3n_x \times 3n_x}$ instead of $\mathbb{R}^{2n_x \times 2n_x}$. The decomposition of this overapproximated system into the form (13) with (24) can still be achieved and, hence, Theorem 3.6 can still be applied.

4. Example

In this section, we illustrate the presented theory using a well-known benchmark example Bauer, Maas et al. (2012), Dačić and Nešić (2007), Donkers et al. (2011), Heemels et al. (2010), Nešić and Teel (2004) and Walsh et al. (2002) in the NCS literature consisting of a linearized model of an unstable batch reactor. This benchmark example has been used primarily to compare conservatism in stability analysis techniques, where the dynamic output-based stabilizing controller is assumed to be given. In Dačić and Nešić (2007), dynamic output-feedback controllers were synthesized for this problem in the presence of a shared communication medium, but with a constant transmission interval. This is the first paper which synthesizes dynamic output-based stabilizing controllers for this problem with both a shared communication medium and time-varying transmission intervals. Moreover, we synthesize controllers while imposing constraints regarding controller decentralization. First we will synthesize stabilizing decentralized controllers for a single batch reactor in Section 4.1 and then, in Section 4.2, synthesize stabilizing controllers for multiple batch reactors. The single batch reactor will be considered primarily for reasons of comparison to previous work, and the multiple batch reactor case will be considered to explore the (computational) limitations of the presented synthesis technique.

4.1. Single batch reactor

The system matrices for the linearized batch reactor are given in Walsh et al. (2002). This system is not in an ideal form to be expressed as a collection of disjoint subsystems as in (1). So we use a linear state transformation $z = \bar{S}x$, where

$$\bar{S} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

and we reverse the order of the output vector y to arrive at the following system matrices for the system in the form (2):

$$\left[\begin{array}{c|c} A & B \\ \hline C & \end{array} \right]$$

$$= \left[\begin{array}{cc|cc|cc} -4.290 & 0.675 & -0.581 & -0.581 & 5.679 & 0 \\ 4.273 & -0.761 & 0.048 & -1.295 & 1.136 & 0 \\ \hline -0.208 & 1.039 & 2.399 & 3.681 & 0 & -3.146 \\ 0 & 0 & -1.019 & -9.016 & 0 & 3.146 \\ \hline 1 & 0 & 0 & 0 & & \\ 0 & 0 & 1 & 0 & & \end{array} \right]. \tag{33}$$

The two disjoint subsystems of (33) are denoted by the dashed lines. We will use the system matrices in (33) as the plant model (2) for the remainder of this example. We will compare four different controller structures, denoted C1–C4.

C1—The first controller (C1) is a centralized controller ($N = 1$) of the form (9) where the communication medium is not shared, meaning all sensors and actuators are in one node ($\bar{N} = 1$) and $\Gamma_1^u = \Gamma_1^y = I$. This is the simplest setting for which Theorem 3.3 applies.

C2—The second controller (C2) is a decentralized controller ($N = 2$) of the form (9) where the decentralized structure is indicated in (33) by the dashed lines. The communication medium is not shared, meaning all sensors and actuators are in one node ($\bar{N} = 1$) and $\Gamma_1^u = \Gamma_1^y = I$. Since the communication medium is not shared and the controller is decentralized, the subsystems will communicate in parallel and Theorem 3.6 must be used.

C3—The third controller (C3) is a decentralized controller ($N = 2$) of the form (9) where the decentralized structure is indicated in (33) with the dashed lines. In addition, the communication medium is shared. We specify that each sensor and actuator is placed into a separate node. Hence, there are $\bar{N} = 4$ nodes, where $\Gamma_1^u = \text{diag}(1, 0), \Gamma_2^u = \text{diag}(0, 1), \Gamma_3^u = \Gamma_4^u = \Gamma_1^y = \Gamma_2^y = \text{diag}(0, 0), \Gamma_3^y = \text{diag}(1, 0)$ and $\Gamma_4^y = \text{diag}(0, 1)$. We

specify the protocol to be the well-known Round Robin protocol given by (6) with $\sigma_l = l, l \in \{1, \ldots, 4\}$ and $\tilde{N} = 4$. With this decentralized structure and communication protocol, the subsystems communicate in a serial fashion and Theorem 3.3 applies.

To help clarify (23) we now explicitly provide the matrices $\Upsilon_l^u, \Upsilon_l^y, \Upsilon_l^x, \bar{K}_l$ and $\bar{L}_l, l \in \{1, \ldots, 4\}$ associated with this controller. If we define the elements of K_l and L_l as

$$K_l := \left[\begin{array}{cc|cc} K_{l,1} & K_{l,2} & 0 & 0 \\ \hline 0 & 0 & K_{l,3} & K_{l,4} \end{array}\right],$$

$$L_l^\top := \left[\begin{array}{cc|cc} L_{l,1} & L_{l,2} & 0 & 0 \\ \hline 0 & 0 & L_{l,3} & L_{l,4} \end{array}\right],$$

then we have that (23) translates to

$$\Gamma_1^u K_1 = \Upsilon_1^u \bar{K}_1 \Upsilon_1^{x\top} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} K_{1,1} & K_{1,2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\Gamma_2^u K_2 = \Upsilon_2^u \bar{K}_2 \Upsilon_2^{x\top} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} K_{2,3} & K_{2,4} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Gamma_3^u K_3 = \Gamma_4^u K_4 = 0, \qquad L_1 \Gamma_1^y = L_2 \Gamma_2^y = 0,$$

$$L_3 \Gamma_3^y = \Upsilon_3^x \bar{L}_3 \Upsilon_3^{y\top} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} L_{3,1} \\ L_{3,2} \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$L_4 \Gamma_4^y = \Upsilon_4^x \bar{L}_4 \Upsilon_4^{y\top} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_{4,3} \\ L_{4,4} \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

C4—The fourth controller (C4) is an exact discretization of the dynamic controller considered in Bauer, Maas et al. (2012), Donkers et al. (2011), Heemels et al. (2010), Nešić and Teel (2004) and Walsh et al. (2002) which, when discretized, becomes of the form $\tilde{x}_{k+1} = A_c \tilde{x}_k + B_c \hat{y}_k$, $u_k = C_c \tilde{x}_k + D_c \hat{y}_{k-1}$, where

$$\left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & h_\star & 0 \\ 0 & 1 & 0 & h_\star \\ \hline -2 & 0 & -2 & 0 \\ 0 & 8 & 0 & 5 \end{array}\right],$$

$h_\star$ is the nominal sampling interval used for controller discretization and the decentralized structure is indicated with dashed lines. This discrete-time controller was also studied in Dačić and Nešić (2007). We consider the communication medium to be shared and impose the same nodes and Round Robin protocol as specified for controller C3.

For each of the controllers C1–C3 we took different values of $h_\star$ and used the YALMIP interface (Löfberg, 2004) with the SeDuMi solver (Sturm, 1999) to verify the conditions of Theorem 3.3 or Theorem 3.6 in order to find stabilizing gains K_l and L_l which maximize $\bar{h}$ such that the NCS (11) is stable for $[\underline{h}, \bar{h}] = [10^{-3}, \bar{h}]$, $k \in \mathbb{N}$, i.e. for a fixed lower bound on the transmission interval. In the NCS literature, this problem setting is also known as finding the maximum allowable transmission interval (MATI) that still guarantees stability (Bauer, Maas et al., 2012; Donkers et al., 2011; Heemels et al., 2010; Nešić & Teel, 2004; Walsh et al., 2002). Unlike the aforementioned references which consider the controller as given, we now have the advantage of using controller synthesis to push the MATI to an even higher value. For C4, the closed-loop model and stability analysis technique given in Donkers et al. (2011) (see Remark 2.2) were used to verify stability in order to maximize the uncertainty range $[\underline{h}, \bar{h}] = [10^{-3}, \bar{h}]$, $k \in \mathbb{N}$. For C1–C4 we considered an overapproximation of the closed-loop dynamics using $M = 10$ grid points.
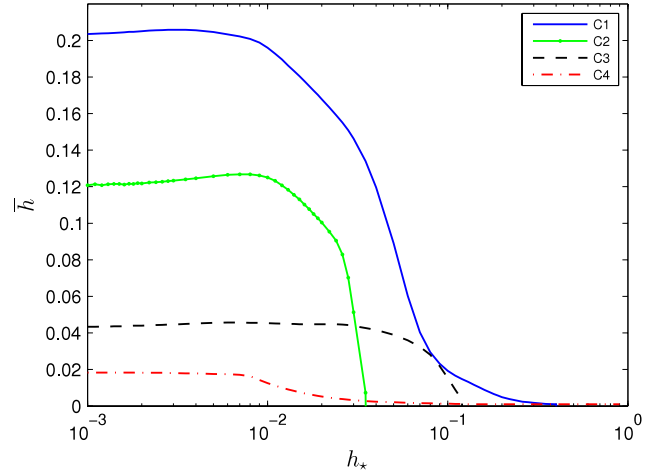
Fig. 2. Maximum $\bar{h}$ (with $\underline{h} = 10^{-3}$) for which (i) stabilizing controller gains for the batch reactor system could be synthesized for C1 and C3 using Theorem 3.3 and C2 using Theorem 3.6 and (ii) stability could be guaranteed for C4.

The result of applying Theorem 3.3 to C1 and C3 and applying Theorem 3.6 to C2 is plotted in Fig. 2. The regions for which closed-loop stability can be guaranteed for controller structures C1, C2, C3 and C4 lie below the lines corresponding to C1, C2, C3 and C4, respectively. Furthermore, the regions lying below the lines corresponding to C1–C3 represent the set of stabilizing controllers that can be found by iteratively applying Theorem 3.3 or Theorem 3.6. One can see that, as expected, the case of a centralized controller and a communication medium which is not shared (C1) achieves the largest robustness margins and yields the largest set of stabilizing controllers. Imposing decentralized structural constraints (C2) and both decentralized structural constraints and a shared communication medium (C3) results in lower robustness margins and smaller sets of controllers. Lastly, analyzing stability of the 'conventional' batch reactor controller (C4) yields the smallest region. The stability analysis technique used to analyze C4 was shown in Donkers et al. (2011) to greatly reduce conservatism compared to robustness margins proven in previous work. However, every point $(h_\star, \bar{h})$, which lies between the lines corresponding to C3 and C4 in Fig. 2, represents a decentralized observer-based controller (9) that has improved closed-loop robustness compared to the existing controller C4. Hence, the presented technique, which synthesizes decentralized dynamic controllers for C3, results in finding an entire set of controllers that have significantly improved robustness margins compared to the given decentralized controller C4.

An additional useful aspect of Fig. 2 is that the $h_\star$ which provides the most robustness to network-induced uncertainties (largest $\bar{h}$ in this case) can be determined for the different controller/network configurations C1–C4. Thus by performing a parametric sweep of $h_\star$ over a certain range (typically between $\underline{h}$ and $\bar{h}$), we have a way of determining suitable values of $h_\star$ that provide a certain desirable level of robustness.

As a final remark, the amount of time taken to solve the LMI feasibility problem given in Theorem 3.3 or Theorem 3.6 was, on average, 5 s for C1 and C2 and 20 s for C3 using a laptop containing a 2.5 GHz Core2 Duo CPU and 4 GB of RAM, which illustrates the computational feasibility of the presented approach for small-scale problems.

4.2. Multiple batch reactors

In this section, we will apply the synthesis techniques to the case where $\nu \in \mathbb{N}$ batch reactors are considered. Thus, the system matrices $\hat{A}$, $\hat{B}$ and $\hat{C}$ considered in this section are block diagonal,
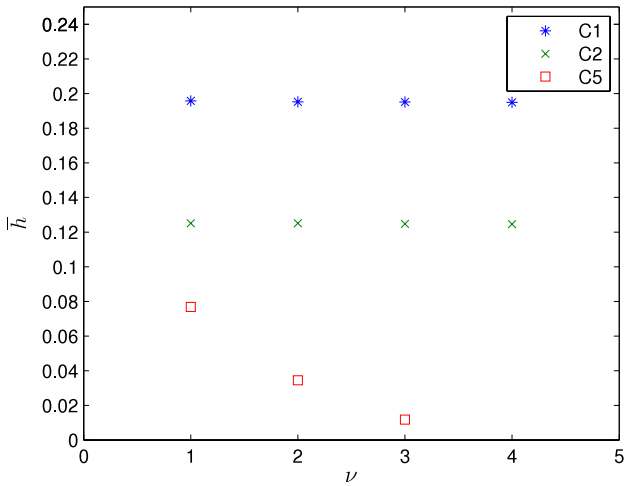
Fig. 3. Maximum $\bar{h}$ (with $\underline{h} = 10^{-3}$ and $h_\star = 0.010$) for which (i) stabilizing controller gains for the batch reactor system could be synthesized for C1 using Theorem 3.3 and C2 and C5 using Theorem 3.6.
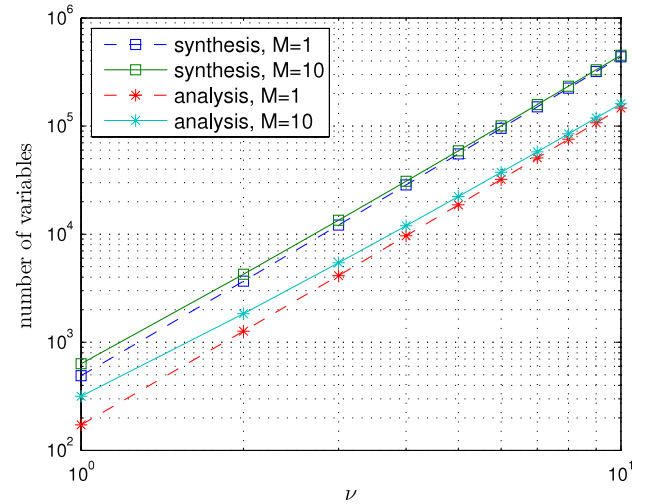

Fig. 4. The number of variables which must be solved as a function of the number of batch reactors ν. Both the cases of controller synthesis and stability analysis are plotted with different numbers of grid points M.

where the number of blocks is equal to ν and the blocks themselves are equal to the A, B and C matrices in (33), respectively. Hence, we have that $\hat{A} = \text{diag}(A, A, \ldots, A)$, $\hat{B} = \text{diag}(B, B, \ldots, B)$ and $\hat{C} = \text{diag}(C, C, \ldots, C)$. The scenario we aim to study is a factory setting where multiple batch reactors are using wireless communication to transmit their sensor values. We can perform a similar analysis as in the previous section, which compares the resulting robustness for different controller configurations. We will again consider an overapproximation of the closed-loop dynamics using $M = 10$ grid points, as in the previous section.

For this multi-batch-reactor situation, we will again consider the controller structures C1 and C2 introduced in the previous section. In this section, C2 is a decentralized controller ($N = 2\nu$) of the form (9) where the decentralized structure *for each batch reactor* is indicated in (33) with the dashed lines. Although synthesizing robustly stabilizing controllers C1 or C2 for a single batch reactor *does* guarantee robust stability when applied to multiple batch reactors (due to a lack of network and subsystem coupling), the main reason for considering these two controller structures is to explore the computational limitations of the developed synthesis technique when the closed-loop dimension is increased. In addition to C1 and C2, we also want to investigate the resulting robustness for a decentralized controller structure that does introduce network coupling, denoted C5, described below.

C5—The fifth controller (C5) is a decentralized controller ($N = 2\nu$) of the form (9) where the decentralized structure *for each batch reactor* is indicated in (33) with the dashed lines. In addition, the communication medium is shared. We specify that each sensor is placed into a separate node and all actuators are updated at each transmission instant. Hence, there are $\bar{N} = 2\nu$ nodes, where $\Gamma_l^u = I$ for all $l \in \{1, \ldots, 2\nu\}$ and $(\Gamma_l^y)_{(r,r)} = 1$ when $l = r$ and is zero otherwise for all $l \in \{1, \ldots, 2\nu\}$. We specify the protocol to be the well-known Round Robin protocol given by (6) with $\sigma_l = l, l \in \{1, \ldots, 2\nu\}$ and $\tilde{N} = 2\nu$. With this decentralized structure and communication protocol, the subsystems' actuators communicate in a parallel fashion and Theorem 3.6 applies.

The controller structure C5 models the practical situation where each (decentralized) controller is co-located at each actuator. Although the batch reactors themselves are not coupled, the presence of a shared communication network couples the individual batch reactor's dynamics. Unlike C1 and C2, a controller C5 that robustly stabilizes a single batch reactor is *not* guaranteed to be stabilizing when applied to multiple batch reactors. Hence, stabilizing multiple batch reactors by using this practical (wireless)

controller structure requires a synthesis technique that includes both decentralized and shared networked aspects, such as the one provided in Theorem 3.6.

The result of applying the synthesis theorems to this setting is shown in Fig. 3. Due to the lack of a shared communication medium, the amount of robustness that can be guaranteed employing controllers corresponding to C1 and C2 is equal to that in Fig. 2 for any number of batch reactors. This is, of course, expected since the controller structures C1 and C2 do not couple the batch reactors in any way. However, for C5, which considers a shared communication medium, we can see that the amount of robustness that can be guaranteed decreases with an increasing number of batch reactors as more sensors are required to share the medium. For the case of $\nu = 3$ batch reactors, the proposed synthesis technique can be used to synthesize a decentralized controller that robustly stabilizes the closed-loop NCS for $\bar{h} = 0.011$. Due to the memory limitations of the computer used for computation (mentioned before), stabilizing controllers could be synthesized for a maximum of $\nu = 4$ batch reactors for C1 and C2, whereas stabilizing controllers could be synthesized for a maximum number of $\nu = 3$ batch reactors for C5. This indicates the limitations of this technique when implemented in current commercially available computer hardware.

To provide a better (solver/hardware independent) indication of how the computational complexity (i.e. memory/computational time required) scales with the state dimension, we will provide an analytical expression which specifies the number of free variables which must be determined to synthesize controller C5 as a function of the number of batch reactors (and grid points). This expression is

$$n_{vars} = n_P + n_R + n_G + n_X + n_Z \tag{34}$$

where n_P, n_R, n_G, n_X and n_Z indicate the number of free variables in the P, R, G, X and Z matrices of Theorem 3.6, respectively. For the controller structure C5, it can be determined that

$$n_P = 144\nu^3 + 12\nu^2, \qquad n_R = 16M\nu^2, \qquad n_G = 288\nu^3, \\ n_X = 8\nu^2 + 4\nu, \qquad n_Z = 16\nu^2 + 2\nu, \tag{35}$$

where $M \in \mathbb{N}$ is the number of grid points (and ν is the number of batch reactors). This representation illustrates the computational penalty incurred from each component, and gives insight into how the computational complexity would be reduced if certain elements were removed or modified. For example, performing robust stability analysis (as mentioned in Remark 2.2) only

requires the computation of the P and R matrices and, therefore, $n_{vars} = n_P + n_R$. Therefore, we can conclude that for this switched system framework, the computational complexity has polynomial growth (of third order in this case) in terms of the number of batch reactors considered. Interestingly, the decentralized synthesis technique presented in this paper has the same order of complexity as the simpler stability analysis problem.

In Fig. 4, the number of free variables is plotted as a function of the number of batch reactors, as specified in (34) with (35). This plot considers the number of variables required to both synthesize controllers C5 and determining robust stability analysis of a given controller C5 (based on Donkers et al., 2011). We can observe how many more additional variables the synthesis technique presented in this paper requires than the stability analysis technique. Recalling that the maximum number of batch reactors able to be synthesized for C5 was $\nu = 3$ (which corresponds to a closed-loop dimension of size 36), we can see that $\approx 10^4$ variables need to be determined by the LMI solver, which, from Fig. 4, implies that robust stability analysis of C5 can be assessed for at least $\nu = 4$ batch reactors (which corresponds to a closed-loop dimension of size 48). We also observe that the addition of more grid points does not introduce a large computational penalty in terms of additional variables as M enters linearly in (34) and (35). Finally, Fig. 4 provides an indication of the amount of memory (and computational time) required to solve larger problems, and suggests where the limitations of one-shot LMI-based techniques currently are.

5. Conclusion

In this paper, we have presented LMI-based synthesis conditions for designing decentralized observer-based control laws in the presence of a shared communication medium, which are robust with respect to time-varying transmission intervals and time-varying delays. This result was obtained by expressing the observer-based controller design problem as a multi-gain switched static output-feedback problem (with additive uncertainty), for which the gains can be efficiently solved by LMI-based feasibility conditions. These LMI-based synthesis conditions, if satisfied, provide stabilizing gains for both the decentralized problem setting and the NCS problem setting in isolation, as well as the unification of these two problem settings. Using a benchmark example in the NCS literature, it was shown that this synthesis technique was able to find an entire set of controllers that significantly improved the closed-loop robustness compared to that of a dynamical controller, extensively studied in the literature. However, the computational complexity of the proposed approach limits this one-shot technique to synthesizing (decentralized) controllers for small and mid-size state-space dimensions. This limitation is primarily due to the fact that, although offering low levels of conservatism and efficient verification for small-scale problems, the number of variables that must be solved using a (switched) quadratic Lyapunov function candidate grows polynomially with respect to the state dimension. Therefore, this advocates that future techniques should not only focus on providing low levels of conservatism but also focus on having low levels of computational complexity. Accomplishing lower levels of computational complexity, improved solvers, and distributed solving of LMIs would enhance the possible application of the proposed methodology for large-scale systems. In any event, the framework laid down in this paper forms one of the first systematic methodologies for the synthesis of stabilizing controllers that incorporate both decentralized and (shared) networked features.

Appendix. Proof of Theorem 3.3

Before going to the construction of a Lyapunov function to prove the theorem, we first establish some technical facts that we need in the following:

- Due to (27), $P_j \succ 0$ for $j \in \{1, \ldots, \tilde{N}\}$.
- Feasibility of (27) implies that $G_{\sigma_j} + G_{\sigma_j}^\top - P_j \succ 0$, and thus G_l is invertible for all $l \in \{1, \ldots, \bar{N}\}$. Indeed, suppose that $G_{\sigma_j}\bar{x} = 0$ for some $\bar{x}$, then $0 = \bar{x}^\top (G_{\sigma_j} + G_{\sigma_j}^\top)\bar{x} \succeq \bar{x}^\top P_j \bar{x}$. Since $P_j \succ 0$, this implies that $\bar{x} = 0$ and thus G_l, $l \in \{1, \ldots, \bar{N}\}$, is invertible.
- Using the fact that $\mathcal{C}_l$ has full row rank when $(C)_{\overline{y}_l^y, \bullet} = \Upsilon_l^{y\top} C$ has full row rank and $\mathcal{E}_l$ is full row rank by definition, then it follows from (28) and invertibility of G_l, $l \in \{1, \ldots, \bar{N}\}$, that $X_{1,l}$, $l \in \mathbf{L}_u$, must have full rank and $X_{2,l}$, $l \in \mathbf{L}_y$, must have full rank and thus be invertible. Hence, the controller gains $\bar{K}_l = Z_{1,l}X_{1,l}^{-1}$, $l \in \mathbf{L}_u$, and observer gains $\bar{L}_l = Z_{2,l}X_{2,l}^{-1}$, $l \in \mathbf{L}_y$, are well defined.

Now we are ready to prove that the controller gains $\bar{K}_l = Z_{1,l}V_{1,l}^{-1}$, $l \in \mathbf{L}_u$, and observer gains $\bar{L}_l = Z_{2,l}V_{2,l}^{-1}$, $l \in \mathbf{L}_y$, with (22) and (23) stabilize (11) and (12) with $h_k \in [\underline{h}, \bar{h}]$ and a given protocol satisfying (6) by proving that (27) and (28) guarantee the existence of a Lyapunov function proving uniform global exponential stability (UGES) of (13), (18) and (24) with $\alpha_k \in \Omega$, $\Delta_k \in \Delta$ and the same protocol satisfying (6). This is a direct consequence as (13) is an overapproximation of (11) and (12) in the sense that (16) holds.

Let us consider the following Lyapunov function candidate

$$V_k(\bar{x}_k) = \bar{x}_k^\top P_j^{-1} \bar{x}_k, \tag{36}$$

where $j = k - r\tilde{N}$ for some $r \in \mathbb{N}$ such that $j \in \{1, \ldots, \tilde{N}\}$. Clearly, there exist $c_1, c_2 > 0$ such that $c_1\|\bar{x}\|^2 \leq V_k(\bar{x}) \leq c_2\|\bar{x}\|^2$ for all $k \in \mathbb{N}$ and $\bar{x} \in \mathbb{R}^n$ due to the positive definiteness of $P_1^{-1}, \ldots, P_{\tilde{N}}^{-1}$. Due to the fact that σ_k is periodic, see (6), we only have to show that the Lyapunov function candidate decreases along solutions of (13), (18) and (24) for $k = \{1, \ldots, \tilde{N}\}$. UGES of (13), (18) and (24) is established if this Lyapunov function candidate satisfies

$$P_j^{-1} - \sum_{m_1=1}^{M} \alpha^{m_1} (\mathcal{F}_{\sigma_j, m_1} + \mathcal{G}_{m_1} \Delta \mathcal{H}_{\sigma_j})^\top$$

$$\times P_{j+1}^{-1} \sum_{m_2=1}^{M} \alpha^{m_2} (\mathcal{F}_{\sigma_j, m_2} + \mathcal{G}_{m_2} \Delta \mathcal{H}_{\sigma_j}) \succ 0 \tag{37}$$

for all $j \in \{1, \ldots, \tilde{N}\}$, $\Delta \in \Delta$ and $\alpha \in \Omega$ where $P_{\tilde{N}+1} = P_1$ as this would guarantee the existence of an $\epsilon > 0$ such that $\Delta V_k(\bar{x}_k) := V_{k+1}(\bar{x}_{k+1}) - V_k(\bar{x}_k) \leq -\epsilon\|\bar{x}_k\|^2$ for all $\bar{x}_k \in \mathbb{R}^n$ and all $k \in \mathbb{N}$.

Now we will prove that satisfaction of (37) for all $\Delta \in \Delta$ and $\alpha \in \Omega$ is implied by satisfaction of (27) and (28) with

$\bar{K}_l = Z_{1,l} X_{1,l}^{-1}$, $l \in \mathbf{L}_u$ and $\bar{L}_l = Z_{2,l} X_{2,l}^{-1}$, $l \in \mathbf{L}_y$. By a Schur complement we can observe that the condition in (37) is equivalent to satisfying $\sum_{m=1}^{M} \alpha^m Q_{j,m} \succ 0$, where

$$Q_{j,m} := \begin{bmatrix} P_j^{-1} & (\mathcal{F}_{\sigma_j,m} + \mathcal{G}_m \Delta \mathcal{H}_{\sigma_j})^\top \\ (\mathcal{F}_{\sigma_j,m} + \mathcal{G}_m \Delta \mathcal{H}_{\sigma_j}) & P_{j+1} \end{bmatrix},$$

$\alpha \in \mathbf{\Omega}$ and $\Delta \in \mathbf{\Delta}$ for all $j = \{1, \ldots, \tilde{N}\}$. A necessary and sufficient condition for the satisfaction of $\sum_{m=1}^{M} \alpha^m Q_{j,m} \succ 0$ for all $\alpha \in \mathbf{\Omega}$ and for all $\Delta \in \mathbf{\Delta}$ is to require that $Q_{j,m} \succ 0$ for all $\Delta \in \mathbf{\Delta}$ and for all $j \in \{1, \ldots, \tilde{N}\}$, $m \in \{1, \ldots, M\}$. Now observe that for all $\Delta_j \in \mathbf{\Delta}$, it holds that $\mathcal{H}_{\sigma_j}^\top (R_{j,m}^{-1} - \Delta^\top R_{j,m}^{-1} \Delta) \mathcal{H}_{\sigma_j} \succeq 0$, for all $R_{j,m}^{-1} \in \mathbf{R}$, $j \in \{1, \ldots, \tilde{N}\}$ and $m \in \{1, \ldots, M\}$ by the definitions of $\mathbf{\Delta}$ in (15) and $\mathbf{R}$ in (26). Hence, $Q_{j,m} \succ 0$ is satisfied if

$$\begin{bmatrix} P_j^{-1} & (\mathcal{F}_{\sigma_j,m} + \mathcal{G}_m \Delta \mathcal{H}_{\sigma_j})^\top \\ (\mathcal{F}_{\sigma_j,m} + \mathcal{G}_m \Delta \mathcal{H}_{\sigma_j}) & P_{j+1} \end{bmatrix} \succ \begin{bmatrix} \mathcal{H}_{\sigma_j}^\top (R_{j,m}^{-1} - \Delta^\top R_{j,m}^{-1} \Delta) \mathcal{H}_{\sigma_j} & 0 \\ 0 & 0 \end{bmatrix},$$

or equivalently that $S_{j,m}^\top \bar{Q}_{j,m} S_{j,m} \succ 0$, where

$$\bar{Q}_{j,m} := \begin{bmatrix} G_{\sigma_j}^\top P_j^{-1} G_{\sigma_j} & (\mathcal{F}_{\sigma_j,m} G_{\sigma_j})^\top & 0 & (\mathcal{H}_{\sigma_j} G_{\sigma_j})^\top \\ \star & P_{j+1} & \mathcal{G}_m R_{j,m} & 0 \\ \star & \star & R_{j,m} & 0 \\ \star & \star & \star & R_{j,m} \end{bmatrix},$$

and

$$S_{j,m} := \begin{bmatrix} G_{\sigma_j}^{-1} & 0 \\ 0 & I \\ R_{j,m}^{-1} \Delta \mathcal{H}_{\sigma_j} & 0 \\ -R_{j,m}^{-1} \mathcal{H}_{\sigma_j} & 0 \end{bmatrix}.$$

The matrix inequality $S_{j,m}^\top \bar{Q}_{j,m} S_{j,m} \succ 0$ is satisfied if $\bar{Q}_{j,m} \succ 0$ since $S_{j,m}$ is full column-rank. Using the fact that it holds that $G_{\sigma_j}^\top P_j^{-1} G_{\sigma_j} \succeq G_{\sigma_j} + G_{\sigma_j}^\top - P_j$, the satisfaction of $\bar{Q}_{j,m} \succ 0$ is implied by the satisfaction of

$$\left[\begin{array}{cc|cc} G_{\sigma_j} + G_{\sigma_j}^\top - P_j & (\mathcal{F}_{\sigma_j,m} G_{\sigma_j})^\top & 0 & (\mathcal{H}_{\sigma_j} G_{\sigma_j})^\top \\ \star & P_{j+1} & \mathcal{G}_m R_{j,m} & 0 \\ \hline \star & \star & R_{j,m} & 0 \\ \star & \star & \star & R_{j,m} \end{array} \right] \succ 0. \quad (38)$$

Note that $G_{\sigma_j}^\top P_j^{-1} G_{\sigma_j} \succeq G_{\sigma_j} + G_{\sigma_j}^\top - P_j$ follows from the fact that if $P_j^{-1} \succ 0$ then $(G_{\sigma_j} - P_j)^\top P_j^{-1} (G_{\sigma_j} - P_j) \succeq 0$.

Finally, combining $Z_{2,\sigma_j} \mathcal{C}_{\sigma_j} = \bar{L}_{\sigma_j} \mathcal{C}_{\sigma_j} G_{\sigma_j}$, which is derived from (28b) and $\bar{L}_{\sigma_j} = Z_{2,\sigma_j} X_{2,\sigma_j}^{-1}$, and $Z_{1,\sigma_j} \mathcal{E}_{\sigma_j} = \bar{K}_{\sigma_j} \mathcal{E}_{\sigma_j} G_{\sigma_j}$, which is derived from (28a) and $\bar{K}_{\sigma_j} = Z_{1,\sigma_j} X_{1,\sigma_j}^{-1}$, with (24), we can substitute

$$\begin{aligned} \mathcal{F}_{\sigma_j,m} G_{\sigma_j} &= \mathcal{A}_{\sigma_j,m} G_{\sigma_j} + \mathcal{B}_{\sigma_j,m} \bar{K}_{\sigma_j} \mathcal{E}_{\sigma_j} G_{\sigma_j} - \mathcal{D}_{\sigma_j} \bar{L}_{\sigma_j} \mathcal{C}_{\sigma_j} G_{\sigma_j} \\ &= \mathcal{A}_{\sigma_j,m} G_{\sigma_j} + \mathcal{B}_{\sigma_j,m} Z_{1,\sigma_j} \mathcal{E}_{\sigma_j} - \mathcal{D}_{\sigma_j} Z_{2,\sigma_j} \mathcal{C}_{\sigma_j}, \end{aligned}$$

$$\begin{aligned} \mathcal{H}_{\sigma_j} G_{\sigma_j} &= \mathcal{I}_{\sigma_j} G_{\sigma_j} + \mathcal{J}_{\sigma_j} \bar{K}_{\sigma_j} \mathcal{E}_{\sigma_j} G_{\sigma_j} \\ &= \mathcal{I}_{\sigma_j} G_{\sigma_j} + \mathcal{J}_{\sigma_j} Z_{1,\sigma_j} \mathcal{E}_{\sigma_j}, \end{aligned}$$

into (38), which yields (27). The above substitution of $Z_{2,l} \mathcal{C}_l = \bar{L}_l \mathcal{C}_l G_l$, thus using (28b), is only needed when $l \in \mathbf{L}_y$ since, by definition, $\Upsilon_l^y = 0$ when $l \notin \mathbf{L}_y$ (and thus $\mathcal{C}_l = 0$). Similarly, substitution of $Z_{1,l} \mathcal{E}_l = \bar{K}_l \mathcal{E}_l G_l$, thus using (28a), is only needed when $l \in \mathbf{L}_u$ since $\Upsilon_l^u = 0$ when $l \notin \mathbf{L}_u$ (and thus $\mathcal{J}_l = 0$ and $\mathcal{B}_{l,m} = 0$ for all $m \in \{1, \ldots, M\}$).

We have shown that satisfaction of (27) and (28) yield $\bar{K}_l$ and $\bar{L}_l$ which satisfy (37) for all $j \in \{1, \ldots, \tilde{N}\}$, $\alpha \in \mathbf{\Omega}$ and $\Delta \in \mathbf{\Delta}$. Hence, using standard Lyapunov arguments, UGES of (13), (18) and (24) with $\alpha_k \in \mathbf{\Omega}$, $\Delta_k \in \mathbf{\Delta}$ and the given protocol satisfying (6) is guaranteed and also yields UGES of (11) and (12) with $h_k \in [\underline{h}, \bar{h}]$ and the same periodic protocol.

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