Linear repetitive control for a nonlinear mechanical ventilation system using feedback linearization Joey Reinders, Bram Hunnekens, Tom Oomen, and Nathan van de Wouw In 2021 5th IEEE Conference on Control Technology and Applications, San Diego, California, 2021

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Abstract—Mechanical ventilators sustain life of patients that are unable to breathe on their own. The aim of this paper is to improve pressure tracking performance of a nonlinear mechanical ventilation system using linear repetitive control, while guaranteeing stability. This is achieved by using feedback linearization and subsequently applying linear repetitive control to the linearized plant. The design procedure of this control strategy is developed in this paper. Thereafter, the controller is implemented in simulations and experiments showing superior pressure tracking performance of this control strategy compared to standard feedback control.

I. INTRODUCTION

Mechanical ventilators are essential equipment in Intensive Care Units (ICUs) to assist patients who cannot breathe on their own or need support to breathe sufficiently. The goal of mechanical ventilation is to ensure adequate oxygenation and carbon dioxide elimination [1], and thereby sustaining the patient's life. In 2005 over 790,000 patients required ventilation in the United States alone [2]. Therefore, improving mechanical ventilation improves treatment for a large population worldwide, especially during the flu-season or a world wide pandemic, such as the COVID-19 pandemic.

In pressure-controlled ventilation modes, the mechanical ventilator aims to track a pressure profile at the patient's airway set by a clinician [3]. An example of such profile is depicted in Fig. 1. The Inspiratory Positive Airway Pressure (IPAP) and Positive End-Expiratory Pressure (PEEP) induce airflow in and out of the lungs, respectively. This alternating flow of air allows the lungs to exchange CO_2 for O_2 in the blood.

Accurate tracking of the preset pressure profile ensures sufficient patient support and enhances patient comfort. According to [4], improved pressure tracking can prevent patient-ventilator asynchrony. In [5], patient-ventilator asynchrony is even associated with increased mortality rates. Furthermore, accurate tracking for a wide variety of patients

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improves consistency of treatment over these different patients.

The challenging problem of pressure tracking in presence of widely varying and uncertain patients parameters has spurred the development of a wide range of pressure control methodologies. Some examples of such control strategies applied to mechanical ventilation are variable-gain control in [4], adaptive feedback control in [6], model predictive control in [7], and adaptive hose compensation control in [8] and [9]. All those methods improve pressure tracking performance in mechanical ventilation.

Pressure tracking performance has been improved further using learning control strategies by exploiting the repetitive nature of breathing. Learning control strategies, such as Iterative Learning Control (ILC) ([10], [11], and [12]) and Repetitive Control (RC) ([13], [14], [15], and [16]), can achieve superior tracking performance utilizing the repetitive nature of breathing, i.e., the target pressure. In these learning control strategies, the controller learns an input signal using errors of previous tasks. In other application fields, with repetitive tasks, ILC and RC are successfully implemented, e.g., industrial robotics [10], wafer stages [17], printer systems [18] and [19], and in medical applications for a device to help with stroke rehabilitation [20].

Since an exact plant model is typically unavailable and breathing is a repeating process, such learning control strategies are particularly suitable for mechanical ventilation. In [21] and [22], ILC has been applied to mechanical ventilation. They show a significant performance improvement in experiments and simulations. However, only causal filters are used in the ILC design. In sharp contrast, non-causal filters can potentially improve performance significantly because of the delays that are present in ventilation systems, as mentioned in [23] and [9]. Furthermore, RC may be more suitable than ILC for ventilation systems because there is no system reset in between tasks, i.e., breaths. Therefore, in [24], RC with non-causal filters is applied to mechanical ventilation. Through an experimental case-study in [24], it is shown that RC improves performance significantly, up to a factor 10 in terms of the pressure error 2-norm, for a wide variety of patients.

Although learning control has substantially improved pressure tracking performance of mechanical ventilation, stability is typically ensured for a set of linearized plants and not for the full nonlinear dynamics of the ventilation system caused by the nonlinearity of the hose. Therefore, the aim of this paper is to use repetitive control to improve tracking



Fig. 1. Typical airway pressure for two breathing cycles of pressure controlled ventilation, showing the set-point (-----) and the typical response (-----).

performance of the non-linear mechanical ventilation system while providing stability guarantees. This is achieved by applying feedback linearization to obtain a linearized plant. Thereafter, linear RC is applied to this linearized plant, rendering the standard RC stability proofs valid.

The main contribution of this paper is the design of a control strategy for a non-linear mechanical ventilation system that combines feedback linearization and linear repetitive control. The first sub-contribution is that this combination of feedback linearization and linear repetitive control renders the stability and convergence analysis for linear repetitive control valid when it is applied to a non-linear system. The second sub-contribution is a performance analysis of this control strategy in simulations and in experiments.

The outline of this paper is as follows. In Section II, the considered system, the control goal, and the envisioned solution are presented. Then, in Section III, the dynamic model of the nonlinear ventilation system is presented. Thereafter, in Section IV, the control concept and design procedure are explained. Then, in Section V, the feedback linearized RC strategy is applied in a simulation case study. Next, in Section VI, the feedback linearized RC strategy is applied in an experimental case study. Finally, in Section VII, the main conclusions and extensions for future work are presented.

II. CONTROL PROBLEM

The considered nonlinear ventilation system is described in Section II-A. Thereafter, the control problem and challenges are presented in Section II-B. Then, in Section II-C, a high-level description of the control framework is given.

A. High-level system description

A schematic of the considered blower-patient-hose system, with the relevant parameters, is shown in Fig. 2. The main components of this system are the blower, the hose-filter system, and the patient.

The blower compresses ambient air to achieve the desired blower outlet pressure p_{out} . The change in p_{out} is controlled to achieve the desired airway pressure p_{aw} near the patient's mouth. The airway pressure is measured using a pilot line attached to the module and the end of the hose. All pressures are defined relative to the ambient pressure, i.e., $p_{amb} = 0$.

The hose-filter system connects the blower to the patient. The difference between the outlet pressure and the airway pressure results in a flow through the hose Q_{out} , related by



Fig. 2. Schematic representation of the blower-hose-patient system, with the corresponding resistances, lung compliance, pressures, and flows.



Fig. 3. Pressure drop over the hose Δp versus the outlet flow Q_{out} . Showing a measured data points of a calibration and a fit of the considered nonlinear hose model.

a nonlinear hose resistance model f_{hose} . A visualization of the nonlinear hose characteristics is shown in Fig. 3. This figure shows a non-linear model fitting the calibration data accurately. The change in airway pressure p_{aw} results in two flows, namely, the leak flow Q_{leak} and the patient flow Q_{pat} . The leak flow is used to flush exhaled CO₂-rich air from the hose. The patient flow is required to ventilate the patient.

The patient is modeled as a resistance R_{lung} and a compliance C_{lung} . The patient flow Q_{pat} is a result of the lung resistance and the difference between the airway pressure p_{aw} and the lung pressure p_{lung} , i.e., the pressure inside the lungs. The patient flow results in a change in the lung pressure, the relation between patient volume and lung pressure is given by the lung compliance.

B. Control goal and open challenge

This paper considers Pressure Controlled Mandatory Ventilation (PCMV) of fully sedated patients. The goal in PCMV is to track a given airway pressure reference, i.e., preset by the clinician, repeatedly, see Fig. 1 for an example reference. This reference is exactly periodic with a period length of N samples. Besides this reference pressure, no other disturbances are considered to be present.

Because of the plant variations, the delays in the system, and the repetitive nature of the reference signal, repetitive control (RC) has been successfully applied to achieve superior tracking performance in PCMV for a variety of patient in [24]. However, in [24] the stability properties of the closed-loop system are guaranteed for Frequency Response Functions (FRFs) characterizing the linearized dynamics at different steady-state pressure levels, i.e., linearizations of the actual non-linear mechanical ventilation system around certain steady-state working points. Therefore, the stability analysis does not hold for time-varying pressure levels during operation. The challenge of ensuring stability is tackled in this paper.

C. Control approach

Here, a high-level explanation of the control approach in this paper is presented. The goal of this control approach is to achieve the superior performance of RC, while guaranteeing stability for time-varying target pressure levels. This is achieved by first using feedback linearization to retrieve a linearized plant. Thereafter, linear RC is used to achieve the desired tracking performance.

As seen in (2), the hose resistance contains a nonlinear component. In order to linearize the nonlinear plant, this nonlinearity has to be compensated for. To achieve this, an estimate of the quadratic hose resistance \hat{R}_{quad} is used in a positive feedback loop similar to the hose-resistance compensation strategy in [9]. If the estimate is correct, i.e., $\hat{R}_{quad} = R_{quad}$, this results in a linearized plant. Details and the results of this feedback linearization method are presented in Section IV-A.

Thereafter, a linear repetitive controller is designed for this linearized plant. This is done similar to the method presented in [24]. However, in this paper we consider only one adult patient instead of designing the repetitive controller for a variety of patients. This is done without loss of generality and merely for the sake of simplicity and clarity. The design procedure of this repetitive controller is described in Section IV-B.

III. NONLINEAR VENTILATION SYSTEM DYNAMICS

In this section, the most important equations of the nonlinear ventilation system are presented. These equations are used to model the separate components: the blower, the hose, and the patient. Finally, it is described how these separate components are connected.

For the blower model, it is assumed that an internal control loop of the blower results in a transfer function from the control input $p_{control}$ to p_{out} that is equal to one, i.e.,

$$p_{out}(t) = p_{control}(t). \tag{1}$$

According to [9], the pressure drop over the hose-filter system can be modeled accurately with a nonlinear algebraic equation defined as

$$\Delta p = f_{hose}(Q_{out}) := R_{lin}Q_{out} + R_{quad}Q_{out}|Q_{out}|$$
(2)

with $\Delta p := p_{aw} - p_{out}$ the pressure drop over the hose, R_{lin} the linear resistance component, and R_{quad} the quadratic resistance component. The inverse of this nonlinear hose model gives an expression for the outlet flow

$$Q_{out} = f_{hose}^{-1}(\Delta p)$$

= sign(\Delta p) $\frac{-R_{lin} + \sqrt{R_{lin}^2 + 4R_{quad}|\Delta p|}}{2R_{quad}}$. (3)



Fig. 4. Block diagram showing how the different components of the plant are interconnected. The feedback linearization method is visualized in red. With feedback linearization this gives the linearized plant P_{lin} .

The leak is modeled as a linear resistance, which gives the leak flow

$$Q_{leak} = \frac{p_{aw} - p_{amb}}{R_{leak}} = \frac{p_{aw}}{R_{leak}}.$$
(4)

The patient is modeled with a linear one-compartmental lung model, as described in [25], as follows:

$$\dot{p}_{lung} = \frac{p_{aw} - p_{lung}}{C_{lung}R_{lung}} = \frac{1}{C_{lung}}Q_{pat}.$$
(5)

These different components are connected using conservation of flow, i.e., $Q_{out} = Q_{leak} + Q_{pat}$. Combining conservation of flow with (3), (4), and (5) results in the full nonlinear dynamics. The full nonlinear state-space model is omitted for brevity. This nonlinear dynamics can be represented by a block diagram as shown by the black part of Fig. 4, where the red part should be neglected for now.

IV. FEEDBACK LINEARIZATION FOR REPETITIVE CONTROL WITH GUARANTEED CLOSED-LOOP STABILITY

In this section, the proposed control structure and design methodology are presented. Before going into details about the design methodology, the challenge and the high-level solution are described.

The main challenge is a result of the nonlinear system dynamics in combination with the repetitive target pressure for breathing. Because of the repetitive target pressure the Internal Model Principle (IMP), as described in [26], can be exploited. The IMP states that asymptotic disturbance rejection of an exogenous disturbance is achieved if a model of the disturbance generating system is included in a stable feedback loop. Therefore, we would like to include a disturbance generating system of our target pressure in our feedback loop to achieve asymptotic tracking of this target pressure for our nonlinear system.

RC is exploiting the IMP and can be used to achieve asymptotic rejection of a repetitive disturbance. However, stability analysis and design methods for RC are typically developed for linear systems. Therefore, in Section IV-A, we are using feedback linearization to reduce our plant to a linear plant. Thereafter, in Section IV-B, a repetitive controller design methodology for this linear plant is presented, such that high performance is achieved and stability is guaranteed.

A. Feedback linearization of the ventilation system

A linearized plant is retrieved by applying feedback linearization to the nonlinear ventilation system in Section III. The feedback linearization method uses an estimated value of the quadratic hose resistance \hat{R}_{quad} to compensate the nonlinear resistance term R_{quad} of the actual hose in (2). This is achieved by adding the estimated contribution of the nonlinear component of the hose, i.e., $p_{fbl} := \hat{R}_{quad} Q_{out} |Q_{out}|$, to the plant input, see the red part of Fig. 4. Intuitively, the pressure drop caused by the quadratic part of the hose resistance in (2) is added to the outlet pressure to compensate the nonlinear term in the hose model.

Mathematically, it is shown that this linearizes the system if $\hat{R}_{quad} = R_{quad}$. The proposed feedback linearization strategy results in the following pressure drop over the hose:

$$\Delta p = p_{control} + p_{fbl} - p_{aw}.$$
 (6)

Assuming that the estimate \hat{R}_{quad} is indeed the same as the true quadratic resistance parameter, i.e., $\hat{R}_{quad} = R_{quad}$, and substitution of (6) in $f_{hose}^{-1}(\Delta p)$ results in:

$$Q_{out} = \frac{p_{control} - p_{aw}}{R_{lin}}.$$
(7)

Therewith, the nonlinear resistance term R_{quad} is eliminated, if $\hat{R}_{quad} = R_{quad}$ holds. Hence, a linear system is retrieved. Concluding, feedback linearization results in the following linearized system:

$$\dot{p}_{lung} = \mathbf{A}_{l} p_{lung} + \mathbf{B}_{l} p_{control}$$

$$\begin{bmatrix} p_{aw} \\ Q_{pat} \end{bmatrix} = \mathbf{C}_{l} p_{lung} + \mathbf{D}_{l} p_{control}$$
(8)

with

$$\mathbf{A}_{h} = -\frac{R_{lin} + R_{leak}}{C_{lung}\bar{R}}, \ \mathbf{B}_{h} = \frac{R_{leak}}{C_{lung}\bar{R}},$$
$$\mathbf{C}_{h} = \begin{bmatrix} \frac{R_{lin}R_{leak}}{\bar{R}} & -\frac{R_{lin} + R_{leak}}{\bar{R}} \end{bmatrix}^{T},$$
$$\mathbf{D}_{h} = \begin{bmatrix} \frac{R_{leak}R_{lung}}{\bar{R}} & \frac{R_{leak}}{\bar{R}} \end{bmatrix}^{T},$$
(9)

and $\bar{R} := R_{lin}R_{leak} + R_{lin}R_{lung} + R_{leak}R_{lung}$. Note that due to the output delay of τ_d on p_{aw} , the measured airway pressure is defined as $\tilde{p}_{aw}(t) := p_{aw}(t - \tau_d)$. This gives the linearized plant transfer function with output delay:

$$P_{lin}(s) = \frac{\tilde{p}_{aw}(s)}{p_{control}(s)}$$
(10)
$$= \frac{(R_{leak} + C_{lung}R_{leak}R_{lung}s)e^{-\tau_d s}}{R_{leak} + C_{lung}R_{leak}R_{lung}s + R_{lin}(1 + C_{lung}(R_{leak} + R_{lung})s)}$$

with $s \in \mathbb{C}$ the Laplace variable. The linear plant P_{lin} is used for repetitive controller design in the next section.

B. Repetitive controller design

In this section, a linear repetitive controller is designed for the linearized plant P_{lin} in (10). To achieve this, a brief background, stability properties, and the design methodology of an add-on repetitive controller are explained.

A closed-loop control system with a feedback controller and an add-on RC is depicted in Fig. 5. For the design



Fig. 5. Block diagram of a classic feedback control system including an add-on repetitive controller.

and implementation of the repetitive controller discrete-time filters are developed. Therefore, the transfer function of the linearized plant P_{lin} , in (10), is discretized, this discrete transfer function is denoted by $P_{lin,d}$. Furthermore, in Fig. 5, *C* is a linear stabilizing feedback controller, *R* is the addon RC, the robustness filter is denoted by *Q*, the learning filter is denoted by *L*, and *N* denotes the length of a single breath in samples, the breath length is visualized in Fig. 1. The repetitive controller is designed in the *z*-domain, based on the discrete-time plant model $P_{lin,d}$.

For *N*-periodic disturbances, a model of the disturbance generating system can be obtained using a memory loop. Including this memory loop in the control loop, see Fig. 5 with Q = L = 1, results in a transfer function from the reference to the error with infinite disturbance rejection at the harmonics of *N*. Hence, a reference signal that is exactly periodic with period length *N* is perfectly tracked. In the remainder of this section, stability and filter design for RC are briefly addressed.

The stability conditions considered in this paper are a special case of the conditions in [15, Theorem 4]. Stability conditions independent of N are desired because the breath length can be changed by a clinician. Hence, conditions independent of N allow for filter design independent of the target signals length N. Therefore, the Single-Input Single-Output (SISO) stability condition in Theorem 1 is commonly used, which is a special case of the multi-variable case in [15, Theorem 4] and is independent of N.

Theorem 1: [15, Theorem 4] Assume that $S = (1 + P_{lin,d}C)^{-1}$ and T = 1 - S are asymptotically stable. Then, the closed-loop system with repetitive control of Fig. 5 is asymptotically stable for all N if

$$|Q(z)(1 - T(z)L(z))| < 1, \forall z = e^{i\omega}, \omega \in [0, 2\pi).$$
(11)

Using the stability condition in Theorem 1, the following two-step design procedure is followed for SISO RC systems, see [13], [27], [28], and [19].

Procedure 1: (Frequency-domain SISO RC design, from [19]).

1) Given a parameteric model of the complementary sensitivity T(z), construct a learning filter L(z) as an approximate stable inverse of T(z), i.e., $L(z) \approx T^{-1}(z)$.

2) Using a model $T(e^{i\omega})$, design a robustness filter Q(z) such that Theorem 1 is satisfied.

This procedure describes a systematic robust design method for RC. In step 1, the *L* filter is based on a parametric model of the system. This first step can be motivated by considering $L = T^{-1}$, which results in |Q(z)(1 - T(z)L(z))| = $0 < 1, \forall z = e^{i\omega}, \omega \in [0, 2\pi)$. Therefore, stability is guaranteed if $L = T^{-1}$. In case *T* is non-minimum phase or strictly proper, algorithms such as Zero Phase Error Tracking Control (ZPETC), see [29], can be used to obtain a stable *L* filter. Then, in step 2, a robustness filter is added to ensure stability and improve robustness to modeling errors. This is done by using a model of the complementary sensitivity *T* and checking the stability condition in Theorem 1.

Moreover, the memory loop allows for implementation of non-causal filters. This is possible because the signals can be shifted by N samples in time. This property is used to compensate for the error introduced by the output delays using pre-actuation, i.e., the plant is actuated before the reference is changing. Furthermore, the robustness filter Q is implemented as a non-causal filtering to avoid phase delay, this is achieved by shifting a symmetric FIR-filter.

Concluding, this control strategy allows design methods for linear repetitive control, with its stability guarantees, to be applied to the nonlinear ventilation system. This is achieved by first linearizing the nonlinear system by feedback linearization. Thereafter, a linear repetitive controller is designed for this linearized plant.

V. SIMULATION RESULTS

In this section, the proposed control strategy of Section IV is applied in a simulation case study. First, the considered use-case is described in Section V-A. Thereafter, the designed controller is presented in Section V-B. Finally, the simulation results are presented in Section V-C.

A. Simulation case description

In the simulation and experimental case study an adult patient scenario from the ISO standard for PCMV obtained from Table 201.104 in NEN-EN-ISO 80601-2-12:2011 (NEN, Delft, The Netherlands) is considered. For this standardized scenario, the patient parameters, the ventilator settings, and the considered hose parameters are given in Table I. In the simulations and experimental case study, three different control strategies are compared. More specifically, a pure PID control strategy, the proposed linear repetitive control strategy with feedback linearization, and linear repetitive control without feedback linearization are compared.

B. Controller design

In this section, the final controller designs for the simulation case study are presented. First, the benchmark PID controller is presented. Then, the feedback linearization method is explained. Finally, the filter design for the RC strategies is presented.

The benchmark PID controller, which is also used in the RC strategies, is a pure integral controller that is implemented as shown in Fig. 5. This controller is robustly

TABLE I

PATIENT PARAMETERS AND VENTILATION SETTINGS USED FOR FILTER DESIGN AND IN THE SIMULATIONS AND EXPERIMENTS.

Parameter	Adult	Unit
R _{lung}	5	mbar s / L
C_{lung}	50	L/mbar $\cdot 10^{-3}$
Respiratory rate	15	breaths / min
PEEP	5	mbar
IPAP	15	mbar
Inspiratory time	1.5	8
Expiratory time	2.5	S
R _{lin}	1.7	mbar s/L
R _{quad}	1.6	mbar s ² /L ²
R _{leak}	43.1	mbar s/L

designed to ensure stability for a large variation of plants. The transfer function of this controller is $C(z) = \frac{0.01257}{z-1}$, with sampling time 2×10^{-3} s.

The feedback linearization method is implemented as described in Section IV-A, where \hat{R}_{quad} is assumed to be retrieved using a calibration such that $\hat{R}_{quad} = R_{quad}$. This feedback linearization results in the linearized plant P_{lin} given by (8).

Before designing the RC filters, it is argued that for both RC strategies the same filters should be used. Linearizing the nonlinear plant at a blower outlet flow zero, i.e., $Q_{out} = 0$, gives the same linear plant as P_{lin} . Furthermore, it is known in practice that the outlet flow through the hose will be both positive and negative. Therefore, designing the filters with the plant that is linearized around zero outlet flow, i.e., P_{lin} , makes sense for the system without feedback linearization as well.

Using the discretization of the linearized plant P_{lin} , denoted by Plin.d, and the proposed PID controller C, interconnected as shown in Fig. 5, the complementary sensitivity T is computed and Procedure 1 is followed to compute the RC filters. First, a stable inverse of T is computed using ZPETC which gives the learning filter L. In simulations, we exactly know what T is and can implement non-causal filters in the memory loop. Therefore, an exact inverse of T can be implemented as a learning filter L. Hence, the stability criterion in Theorem 1 is always satisfied for a robustness filter Q = 1. To resemble the experiments more closely, the same Q filter as in the experiments is used in the simulations. As a Q filter, a 20th-order symmetric FIR filter with a cutoff frequency of 23 Hz is used. Using this Q filter, stability of the system with feedback linearization is ensured using Theorem 1. Note that we cannot check stability for the nonlinear system without feedback linearization.

Finally, a learning gain α is added to the learning filter. This means that the learning filter is multiplied by a gain $\alpha \in (0, 1]$. Low values for this learning gain reduce the convergence rate but do avoid that non-periodic disturbances, such as noise, are fed back into the loop. Therewith, the pressure tracking performance upon convergence can be improved. For the controllers in this study, it is found that $\alpha = 0.5$ gives the desired trade-off between convergence speed and reduction of the effect of noise.



Fig. 6. Simulation results of the 20th breath in time domain upon convergence of the repetitive controllers. Showing the target pressure (---), the PID control strategy (--), repetitive control without feedback linearization (--), and repetitive control with feedback linearization (--).

C. Results

The control strategies designed in the previous section are implemented in simulations with the use case as presented in Section V-A. The results of these simulations are presented in Fig. 6 and 7.

The time-domain results for the three control strategies are shown in Fig. 6. This figure shows the airway pressure of all control strategies upon convergence. The figure shows that the PID controller achieves a sub-optimal rise-time and has significant overshoot, which can damage the patient's lungs. Furthermore, the figure shows that both RC strategies achieve near perfect tracking upon convergence, the airway pressure is almost exactly the same as the target pressure. This improves the patient's comfort and avoids harmful peak pressures.

The error 2-norm per breath for every control strategy is shown in Fig. 7. It shows that the error two norm of the PID controller is approximately 35 mbar. Furthermore, it shows that both RC strategies converge to a significantly smaller error 2-norm just above 10^{-1} mbar. The error 2-norm also shows that the feedback linearization results in a smaller initial error and faster convergence of the RC controller. The reduced initial error is a result of the feedback linearization. The faster convergence is a result of the fact that *L* resembles the linearized T^{-1} better over the entire flow region. More precisely, in the feedback linearized case $L = T^{-1}$ holds exactly for every pressure level. In the case without feedback linearization a linear T^{-1} that holds for every pressure level does not exist.

Concluding, RC with feedback linearization results in faster convergence and a slightly lower error upon convergence. Furthermore, stability is guaranteed. Next, these control strategies are compared in experiments.

VI. EXPERIMENTAL RESULTS

In this section, the proposed control strategy of Section IV is applied in an experimental case study, the particular case study, i.e., patient type and pressure levels, is the same as in Section V. First, the experimental setup is presented



Fig. 7. Error 2-norm per breath for every control strategy in simulations. This figures contains the PID control strategy (\mathbf{x}) , the RC without feedback linearization (\mathbf{x}) , and the RC with feedback linearization (\mathbf{x}) .



Fig. 8. Experimental setup with the most important parts, i.e., blower driven ventilator, ASL 5000 breathing simulation, dSpace module, and the hose.

in Section VI-A. Thereafter, the final designed controller is presented in Section VI-B. Finally, the experimental results are presented in Section VI-C.

A. Experimental setup

The main components of the experimental setup used in this case study are depicted in Fig. 8. The figure shows a Macawi blower-driven mechanical ventilation module (DEMCON macawi respiratory systems, Best, The Netherlands). Furthermore, the ASL 5000TMBreathing Simulator (IngMar Medical, Pittsburgh, PA) is shown in the figure. This breathing simulator is used to emulate a linear onecompartmental patient model. Furthermore, a typical hosefilter system for ventilation of a patient in a hospital setting is shown. The developed control algorithms are implemented in a dSPACE system (dSPACE GmbH, Paderborn, Germany).

B. Controller design

Again, three different controllers are considered, the benchmark PID controller, the linear RC without feedback linearization, and the linear RC with feedback linearization. In this section, the final designs for these controllers for the experimental case study are presented.



Fig. 9. Showing an FRF measurement of the complementary sensitivity T (____) and a fourth order fit of the complementary sensitivity (____).

The benchmark PID controller, which is also used in the RC strategies, is the same controller as in the simulations. Also the feedback linearization strategy is the same as in the simulation case study. The estimate \hat{R}_{quad} is retrieved through a calibration procedure prior to ventilation, as shown in Fig. 3. Furthermore, the same argumentation as in the simulations is used to design the same Q and L filter for both RC strategies.

The filters for the repetitive controller are retrieved by first taking an FRF measurements of the open-loop plant. This open-loop FRF measurement and the considered feedback controller are combined to retrieve a non-parametric model of the complementary sensitivity T_{FRF} as depicted in Fig. 9. Next, a fourth-order fit of this process sensitivity is used to retrieve a parameteric model of the complementary sensitivity T_{fit} , also shown in Fig. 9. A stable inverse of this parametric model T_{fit} is computed with ZPETC to retrieve the learning filter L. Next, stability is checked with Theorem 1 and this result is visualized in Fig. 10. This figure shows that stability is guaranteed with Q = 1. However, to improve robustness against plant variations a Q filter is added as depicted in Fig. 10. This Q filter is a 20th-order symmetric FIR filter with a cutoff frequency of 23 Hz. This results in more robustness against plant variations. Finally, to reduce the effect of non-periodic disturbances a learning gain α of 0.5 is added to both repetitive control strategies.

C. Results

The control strategies designed in the previous section are implemented in the experimental setup of Section VI-A with the use case as presented in Section V-A. The results of these experiments are presented in Fig. 11 and 12.

The time-domain results of the 20th breath for the three control strategies are shown in Fig. 11. This figure shows the airway pressure of all control strategies upon convergence. The results are similar to the simulation results. The figure shows that the PID controller achieves a sub-optimal rise-time and has significant overshoot, which is non-optimal for the patient's lungs. Furthermore, the figure shows that the RC strategies achieve near perfect tracking upon convergence, the airway pressure is almost exactly the same as the target



Fig. 10. Stability condition for the experimental setup. Showing Q as a low-pass filter with cut-off frequency at 23 Hz (----), the stability condition (11) with Q = 1 (-----), and the stability condition (11) with Q the low-pass filter (----).



Fig. 11. Experimental results of the 20th breath in time domain upon convergence of the repetitive controllers. Showing the target pressure (---), the PID control strategy (----), repetitive control without feedback linearization (-----), and repetitive control with feedback linearization (-----).

pressure. This improves the patient's comfort and avoids harmful peak pressures.

The error 2-norm per breath for every control strategy is shown in Fig. 12. It shows that the error two norm with the PID controller remains around 35 mbar. Furthermore, it shows that both RC strategies converge to a significantly smaller error 2-norm of approximately 3 mbar. Also it shows that the feedback linearization results in a smaller initial error.

Besides the different initial error norm these experiments do not show a large difference between the RC strategies. Therefore, it would be interesting in future work to test this control strategy on a system with a more dominant nonlinearity. This might increase the effect of feedback linearization, resulting in faster convergence. However, stability is guaranteed for the closed-loop system with feedback linearization and repetitive control. In contrast to the other control strategies, for which no stability guarantees are provided.



Fig. 12. Error 2-norm per breath for every control strategy in experiments. This figures contains the PID control strategy (\mathbf{x}), the RC without feedback linearization (\mathbf{x}), and the RC with feedback linearization (\mathbf{x}). This figure shows that both RC strategies achieve very accurate tracking performance.

VII. CONCLUSIONS

In this paper, linear repetitive control in combination with feedback linearization is applied to a nonlinear mechanical ventilation such that the closed-loop stability properties of linear repetitive control are valid for this nonlinear plant.

The main contribution of this paper is the combination of feedback linearization and linear repetitive control, such that the closed-loop nonlinear ventilation system is stable. In addition, the performance of the control strategy is analyzed by means of simulations and experiments. In simulations, it is shown that convergence speed of the repetitive controller is improved by applying feedback linearization. The performance in terms of error 2-norm upon convergence are similar for the method with and without feedback linearization. Thereafter, in experiments it is shown that the repetitive controller achieves accurate pressure tracking. The experiments show no significant effect on the performance by adding feedback linearization. However, the closed-loop system with feedback linearization and repetitive control is guaranteed to be stable. Whereas, no stability guarantees for the other presented control strategies are provided.

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