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On the convergence of linear passive complementarity systems

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Abstract— **The notion of convergent systems is a powerful tool both in the analysis and synthesis of nonlinear systems. Sufficient conditions for convergence have been under investigation for smooth systems and for classes of non-smooth switching systems in the literature. In this paper, we look at a very particular class of non-smooth systems, namely complementarity systems. These systems have the capability of capturing the non-smooth dynamics of various interesting applications from different fields of engineering. The main contribution of this paper is to show that a linear complementarity system is convergent if the underlying linear dynamics possesses a certain positive realness property.**

I. INTRODUCTION

In this paper, we will provide conditions under which a complementarity system is convergent. A system, which is excited by an input, is called (uniformly) convergent if it has a unique solution that is bounded on the whole time axis and this solution is globally asymptotically stable. Obviously, if such a solution does exist, all other solutions, regardless of their initial conditions, converge to this solution, which can be considered as a steady-state solution [12], [24]. The property of convergence was formalized in the notion of *convergent systems* and studied first for periodically excited systems in [29] and then for systems with arbitrary excitations in [12], see also [24]. Also, these kind of properties were considered in [41], [20]. For systems in Lur'e form convergence was investigated in [40]. For piecewise affine systems, sufficient conditions for convergence have been proposed in [28]. Other notions describing the property of solutions converging to each other are studied in literature. The notion of contraction has been introduced in [21] (see also references therein). An operator-based approach towards studying the property that all solutions of a system converge to each other is pursued in [13], [14]. In [1], a Lyapunov approach has been developed to study the global uniform asymptotic stability of all solutions of a system (in [1], this property is called incremental stability) and the so-called incremental input-to-state stability property, which is compatible with the input-to-state stability approach (see e.g. [35]).

The property of convergence can be beneficial from several points of view. Firstly, in many control problems it is required that controllers are designed in such a way that all solutions of the corresponding closed-loop system "forget" their initial conditions. Actually, one of the main tasks of feedback is to eliminate the dependency of solutions on initial conditions. In this case, all solutions converge to some steady-state solution that is determined only by the input of the closed-loop system. This input can be, for example, a command signal or a signal generated by a feedforward part of the controller or, as in the observer design problem, it can be the measured signal from the observed system. Such a convergence property of a system plays an important role in many nonlinear control problems including tracking, synchronization, observer design, and the output regulation problem, see e.g. [27], [26], [30], [37] and references therein. Secondly, from a dynamics point of view, convergence is an interesting property because it excludes the possibility of coexisting, different steady-state solutions: namely, a uniformly convergent system excited by a periodic input has a *unique* globally asymptotically stable periodic solution with the same period. Moreover, the notion of convergence is a powerful tool for analysis of time-varying systems. This tool can be used, for example, for performance analysis of nonlinear control systems, see e.g. [18].

The organization of the paper is as follows. In the remaining of this section, we summarize the notational conventions that are in force. In Section II, we introduce complementarity systems, convergent dynamics, and the notion of passivity. This will be followed by the main results in Section III. The paper is closed by conclusions in Section IV.

The following notational conventions will be in force. The set of real numbers is denoted by $\mathbb{R}$, nonnegative real numbers by $\mathbb{R}_+$, rational functions with real

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coefficients by $\mathbb{R}(s)$, complex numbers by $\mathbb{C}$, complex numbers with nonnegative real parts by $\mathbb{C}_+$. To denote k-tuples and $k \times \ell$ matrices having elements that belong these sets we write $\mathcal{X}^k$ and $\mathcal{X}^{k\times\ell}$, respectively, where $\mathcal{X}$ is any of the mentioned sets. The conjugate of a complex number z is denoted by $\bar{z}$. For complex vectors and matrices the superscripts T and H denote, respectively, the transpose and Hermitian. Inequalities for real vectors must be understood componentwise. For two matrices M and N with the same number of columns, $\operatorname{col}(M, N)$ will denote the matrix obtained by stacking M over N. For a nonempty set $\mathcal{Q}$ (not necessarily a cone), the dual cone of $\mathcal{Q}$ is the set $\{v \mid u^T v \geqslant 0 \text{ for all } u \in \mathcal{Q}\}$. It is denoted by $\mathcal{Q}^*$. We say that a triple of matrices (A, B, C) is *minimal* if (A, B) is controllable and (C, A) is observable. The notation $\|\xi\|$ is used for the Euclidean norm, i.e. $\|\xi\| = \sqrt{\xi^T\xi}$. We say that a square matrix M is *nonnegative definite* if $x^T M x \geqslant 0$ for all real vectors x. It is said to be *positive definite* if it is nonnegative definite and the implication $x^T M x = 0 \Rightarrow x = 0$. We use the notation $M \geqslant 0$ and $M > 0$ for nonnegative and positive definite matrices. In the obvious manner, we define nonpositive and negative definiteness.

II. LINEAR COMPLEMENTARITY SYSTEMS

Consider the linear system

$$\dot{x}(t) = Ax(t) + Bz(t) + Ev(t) \tag{1a}$$

$$w(t) = Cx(t) + Dz(t) \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state, $v \in \mathbb{R}^p$ is the input, and $(z, w) \in \mathbb{R}^{m+m}$ are external variables that are constrained through the so-called *complementarity relations*

$$0 \leqslant z(t) \perp w(t) \geqslant 0. \tag{1c}$$

We call these systems *complementarity systems* and denote (1) by LCS. A wealth of examples, from various areas of engineering as well as operations research, of these piecewise linear (hybrid) systems can be found in [9], [34], [33], [15]. We refer to [2] for related/equivalent classes of systems, to [8], [16], [5], [31], [32], [17] for well-posedness analysis, to [6], [4], [7], [3] for controllability studies, to [10] for observability analysis, and to [11] for stability analysis.

A few words on the function classes of interest is in order. Let $\mathcal{B}$ denote the Bohl functions, i.e. sines, cosines, polynomials, and all finite sum and products of these. More precisely, a function f is said to be a *Bohl* function if there exist real matrices (F, G, H) with appropriate sizes such that $f(t) = H \exp(Ft) G$ holds for all $t \in \mathbb{R}$. We denote the set of locally square-integrable functions defined over the closed set $\Omega \subseteq \mathbb{R}$ by $L_{2,\mathrm{loc}}(\Omega)$. The set of absolutely continuous functions defined over the closed set $\Omega \subseteq \mathbb{R}$ is denoted by $AC(\Omega)$. We write $\mathcal{X}^k$ to denote the k-tuples of these function spaces.

In this paper, we are interested in absolutely continuous state trajectories, Bohl type inputs, and L-type complementarity variables. The following definition formalizes the solution concept that we will work with.

Definition II.1 Let $\Omega \subseteq \mathbb{R}$ be a closed set that contains zero. We say that a triple $(x, z, w) \in AC^n(\Omega) \times L^{m+m}_{2,\mathrm{loc}}(\Omega)$ is a *solution* on Ω of (1) for the initial state x_0 and the input $v \in \mathcal{B}^p$ if

$$x(0) = x_0 \tag{2a}$$

$$\dot{x}(t) = Ax(t) + Bz(t) + Ev(t) \tag{2b}$$

$$w(t) = Cx(t) + Dz(t) \tag{2c}$$

$$0 \leqslant z(t) \perp w(t) \geqslant 0 \tag{2d}$$

holds for almost all $t \in \Omega$.

A. CONVERGENT DYNAMICS

Instead of a definition of convergent dynamics for a general class of systems (e.g. see [27]), we will work with the following definition that is adapted to complementarity systems.

Definition II.2 A complementarity system (1) is said to be

- *convergent* if, for every bounded input $v \in \mathcal{B}^p$, there exists a solution $(\bar{x}_v, \bar{z}_v, \bar{w}_v)$ satisfying the following conditions:
  (i) $\bar{x}_v(t)$ is defined and bounded for all $t \in \mathbb{R}$,
  (ii) $\bar{x}_v(t)$ is globally asymptotically stable.
- *uniformly convergent* if it is convergent and $\bar{x}_v(t)$ is globally uniformly asymptotically stable.
- *exponentially convergent* if it is convergent and $\bar{x}_v(t)$ is globally exponentially stable.

For the definition of Lyapunov stability of solutions of differential equations we refer to [39], [27]. The solution $\bar{x}_v$ is called a *steady-state solution*; the subscript v emphasizes its dependency on the input v. As follows from the definition of convergence, any solution of a convergent system "forgets" its initial condition and converges to some steady-state solution. In general, the steady-state solution $\bar{x}_v$ may be non-unique. But for any two steady-state solutions $\bar{x}_{v_1}$

and $\bar{x}_{v_2}$, related to the same input v, it holds that $\|\bar{x}_{v_1}(t) - \bar{x}_{v_2}(t)\| \to 0$ as $t \to +\infty$. At the same time, for *uniformly* convergent systems the steady-state solution is unique, as formulated below.

Property II.3 ([27]) If system (1) is uniformly convergent, then the steady-state solution $\bar{x}_v$ is the only solution defined and bounded for all $t \in \mathbb{R}$.

The latter property is very useful since it excludes the possibility of coexisting limit solutions (and of bifurcations leading to such coexistence). Convergent systems enjoy various properties which are encountered in asymptotically stable LTI systems, but which are not usually met in general asymptotically stable nonlinear systems. As an illustration, we provide a statement that summarizes some properties of uniformly convergent systems excited by periodic or constant inputs.

Property II.4 ([12], [27]) Suppose that system (1) with a given input v is uniformly convergent. If the input v is constant, the corresponding steady-state solution $\bar{x}_v$ is also constant; if the input v is periodic with period T, then the corresponding steady-state solution $\bar{x}_v$ is also periodic with the same period T.

This property is, for example, instrumental in the analysis of the steady-state behaviour of convergent systems to harmonic disturbances.

The next definition extends the uniform convergence property to the input-to-state stability (ISS) framework.

Definition II.5 System (1) is said to be *input-to-state convergent* if it is uniformly convergent and for every bounded input $v \in \mathcal{B}^p$ the system is input-to-state stable (ISS) along the steady-state solution $\bar{x}_v$, i.e. there exist a $\mathcal{KL}$-function $\beta(r,s)$ and a $\mathcal{K}$-function $\gamma(r)$ such that any solution x of the system corresponding to some input $\hat{v} := v + \Delta v$ satisfies

$$\begin{aligned}\|x(t) - \bar{x}_v(t)\| \leq & \beta(\|x(t_0) - \bar{x}_v(t_0)\|, t - t_0) \\ & + \gamma(\sup_{t_0 \leq \tau \leq t} \|\Delta v(\tau)\|).\end{aligned} \tag{3}$$

In general, the functions $\beta(r,s)$ and $\gamma(r)$ may depend on the particular input v.

Similar to the conventional ISS property [35], the property of input-to-state convergence is especially useful for studying convergence properties of interconnected systems. In [27], it is shown that a series connection of two input-to-state convergent systems is an input-to-state convergent system and that a feedback interconnection of an input-to-state convergent system and a uniformly asymptotically stable system is an input-to-state convergent system. The latter property can be used for establishing the separation principle for input-to-state convergent systems.

Sufficient conditions for convergence properties of smooth nonlinear systems were proposed in [12] (see also [24]). However, results for several classes of non-smooth systems only recently appeared. Sufficient conditions (in terms of LMIs) for exponential convergence and input-to-state convergence are stated in [25] for the class of continuous piecewise affine (PWA) systems and in [23] for a class of discontinuous PWA systems, see also [28].

B. LINEAR PASSIVE SYSTEMS

One of the main ingredients of the current paper is passivity. In what follows we review the notion of passivity and Kalman-Yakubovich-Popov lemma.

Definition II.6 ([38]) A linear system $\Sigma(A,B,C,D)$ given by

$$\dot{x}(t) = Ax(t) + Bz(t) \tag{4a}$$
$$w(t) = Cx(t) + Dz(t) \tag{4b}$$

is called *passive* if there exists a nonnegative function $V : \mathbb{R}^n \to \mathbb{R}_+$ such that for all $t_0 \leqslant t_1$ and all trajectories (z, x, w) of system (4) the following inequality holds:

$$V(x(t_0)) + \int_{t_0}^{t_1} z^T(t) w(t) dt \geqslant V(x(t_1)). \tag{5}$$

If exists the function V is called a *storage function*.

A closely related concept is positive realness.

Definition II.7 A rational matrix $H(s) \in \mathbb{R}^{m \times m}(s)$ is positive real if the following conditions are satisfied:

- H is analytic in $\mathbb{C}_+$;
- $H(\bar{s}) = \overline{H(s)}$ for all $s \in \mathbb{C}$;
- $H(s) + H^H(s) \geqslant 0$ for all $s \in \mathbb{C}_+$.

For the sake of completeness, we quote the well-known Kalman-Yakubovich-Popov lemma that links the notions of passivity and positive realness.

Theorem II.8 [38] *Assume that (A,B,C) is minimal. Let $G(s) := D + C(sI - A)^{-1}B$ be the transfer matrix of the system $\Sigma(A,B,C,D)$. Then the following statements are equivalent:*

1) *The system $\Sigma(A,B,C,D)$ is passive.*

2) *The matrix inequalities*

$$K = K^T \geqslant 0 \tag{6a}$$

and

$$\begin{bmatrix} A^T K + KA & KB - C^T \\ B^T K - C & -(D + D^T) \end{bmatrix} \leqslant 0 \tag{6b}$$

have a solution.

3) $G(s)$ *is positive real.*

Moreover, the following holds:

1) $V(x) = \frac{1}{2}x^T Kx$ *defines a quadratic storage function if and only if* K *satisfies the above system of linear matrix inequalities.*
2) *All solutions* K *of* (6) *are positive definite.*

In this paper, we will work with a somewhat stronger positive realness notion. We say that a rational transfer matrix $H(s)$ is *strictly positive real* if $H(s - \epsilon)$ is positive real for some $\epsilon > 0$. One can modify the above theorem to capture a state-space characterization of strict positive realness as follows.

Theorem II.9 ([22], [36]) *Assume that* (A, B, C) *is minimal. Let* $G(s) := D + C(sI - A)^{-1}B$ *be the transfer matrix of the* $\Sigma(A, B, C, D)$*. Suppose that the real parts of the poles of* $G(s)$ *less than* $-\mu$ *for some positive constant* μ*. Then the following statements are equivalent:*

1) *The matrix inequalities*

$$K = K^T > 0$$

and

$$\begin{bmatrix} (A^T + \mu I)K + K(A + \mu I) & KB - C^T \\ B^T K - C & -(D + D^T) \end{bmatrix} \leqslant 0$$

have a solution for $\mu > 0$.

2) $G(s - \mu)$ *is positive real for* $\mu > 0$.

III. FROM PASSIVITY TO CONVERGENCE

The main aim of this paper is to show that a linear complementarity system has convergent dynamics if the underlying linear system has a strictly positive real transfer matrix.

We first quote the following auxiliary results that will be used later on. The first guarantees existence of a solution that is defined on the whole time axis.

Lemma III.1 *Consider the LCS* (1)*. Let* $v \in \mathcal{B}^p$ *be bounded. Suppose that*

1) *it admits a unique solution on* $\mathbb{R}_+$ *for initial states* $x_0 \in \mathcal{X} \subseteq \mathbb{R}^n$ *and for the input* v,
2) *these solutions depend on the initial state* x_0 *in a continuously manner,*
3) *there exists a positively invariant compact set* $\bar{\mathcal{X}} \subseteq \mathcal{X}$*, i.e. whenever* $x(\bar{t}) \in \bar{\mathcal{X}}$ *for some* $\bar{t} \geqslant 0$ *it holds that* $x(t) \in \bar{\mathcal{X}}$ *for all* $t \geqslant \bar{t}$.

Then, there exists a solution on $\mathbb{R}$ *that is bounded.*

The proof is analogue to that of [40, Lemma 2] and will be omitted for the sake of shortness.

The second auxiliary result concerns existence/uniqueness of solutions and their continuous dependence on the initial states.

Theorem III.2 ([5], [16]) *Consider the LCS* (1)*. Suppose that the system* $\Sigma(A, B, C, D)$ *is passive,* (A, B, C) *is minimal, and* $\mathrm{col}(B, D + D^T)$ *is of full column rank. Let*

$$\mathcal{Q}_D = \{v \mid v \geqslant 0,\ Dv \geqslant 0 \text{ and } v^T Dv = 0\}.$$

Then, the following statements are equivalent:

1) *There exists a solution of* (1) *on* $\mathbb{R}_+$ *for the initial state* $x_0 \in \mathbb{R}^n$ *and the input* $v \in \mathcal{B}^p$.
2) *The initial state* x_0 *satisfies* $Cx_0 \in \mathcal{Q}_D^*$.

Moreover, when the input is fixed continuous dependence on initial states is guaranteed.

For later use, we define $\mathcal{X} = \{x \mid Cx \in \mathcal{Q}_D^*\}$. Note that the set $\mathcal{X}$ is positively invariant according to the above theorem.

These auxiliary results will be employed in the proof of our main result that is stated in the following theorem.

Theorem III.3 *Consider the LCS* (1)*. Suppose that* (A, B, C) *is minimal, and* $\mathrm{col}(B, D + D^T)$ *is of full column rank. Let* $G(s) := D + C(sI - A)^{-1}B$ *be a strictly positive real transfer matrix. Then, the LCS* (1) *is*

1) *exponentially convergent.*
2) *input-to-state convergent.*

Proof. *1*: Proof of exponential convergence consists of two steps. First, we prove existence of a positively invariant compact set for any given input. In view of Lemma III.1, this would prove existence of a solution that is defined and bounded on $\mathbb{R}$. The second step is to prove the global exponential stability of this solution.

In both steps, we will employ an inequality that is derived in what follows. Let ξ, ρ, η, and ζ be such that

$$\dot{\xi} = A\xi + B\rho + E\zeta \tag{7a}$$
$$\eta = C\xi + D\rho. \tag{7b}$$

Take $W(\xi) = \xi^T K\xi$ where K is as in Theorem II.9. Note that

$$\dot{W}(\xi) = \dot{\xi}^T K\xi + \xi^T K\dot{\xi}$$
$$= \begin{bmatrix}\xi\\ \rho\end{bmatrix}^T \begin{bmatrix}A^TK+KA & KB\\ B^TK & 0\end{bmatrix}\begin{bmatrix}\xi\\ \rho\end{bmatrix} + 2\xi^T KE\zeta.$$

Further, strict positive realness implies that

$$\dot{W}(\xi) \leqslant -2\mu\xi^T K\xi + 2\rho^T\eta + 2\xi^T KE\zeta. \tag{8}$$

To prove the existence of the above-mentioned positively invariant compact set, take any bounded input $v \in \mathcal{B}^p$. Consider a single trajectory (z, x, w) of the LCS (1) for the initial state x_0 and for the input v . Take $\xi = x$, $\rho = z$, $\zeta = v$, and $\eta = w$. The equations (7) are obviously satisfied by these choices. Note that $\rho^T\eta = 0$ due to the complementarity relations. In this case, inequality (8) results in

$$\dot{W}(x) \leqslant -2\mu W(x) + 2x^T KEv \tag{9}$$
$$\leqslant -2\|x\|_K(\mu\|x\|_K - 2\|Ev\|_K) \tag{10}$$

where $\|\xi\|_K = \sqrt{\xi^T K\xi}$. This means that $\dot{W}(x) \leqslant 0$ whenever $\mu\|x\|_K \geqslant 2\|Ev\|_K$. So,

$$\{x \in \mathcal{X} \mid \|x\|_K \leqslant 2\mu^{-1}\sup_{t\in\mathbb{R}}\|Ev(t)\|_K\}$$

characterizes a positively invariant compact set. Then, Lemma III.1 can be used to show that there exists a solution that is defined and bounded on $\mathbb{R}$. We denote this solution by $(\bar{z}_v, \bar{x}_v, \bar{w}_v)$.

To prove the global exponential stability of this solution, consider the trajectories (z^i, x^i, w^i) for $i = 1, 2$ of the LCS (1) for some initial state x_0^i and the same function v. Take $\xi = x^1 - x^2$, $\rho = z^1 - z^2$, $\zeta = 0$, and $\eta = w^1 - w^2$. Clearly, these functions satisfy (7). Note that $\rho^T\eta \leqslant 0$ due to the complementarity relations (1c). Therefore, it follows from (8) that

$$\dot{V}(x^1 - x^2) \leqslant -2\mu V(x^1 - x^2). \tag{11}$$

From Belmann-Gronwall lemma, one can conclude that

$$V(x^1(t) - x^2(t)) \leqslant e^{-2\mu(t-t')}V(x^1(t') - x^2(t')) \tag{12}$$

for all $t \geqslant t'$. By taking one of the x^i in (12) as $\bar{x}_v$, we can conclude its global exponential stability.

2: To prove input-to-state convergence, take any bounded $v \in \mathcal{B}^p$. Let (z, x, w) be a solution for the initial state x_0 and the input $v + \Delta v$. Take $\xi = x - \bar{x}_v$, $\rho = z - \bar{z}_v$, $\zeta = \Delta v$, and $\eta = w - \bar{w}_v$. Clearly, these choices satisfy (7). Note that $\rho^T\eta \leqslant 0$ due to the complementarity relations (1c). Then, inequality (8) results in

$$\dot{W}(x - \bar{x}_v) \leqslant -2\mu W(x - \bar{x}_v) + 2(x - \bar{x}_v)^T KE\Delta v. \tag{13}$$

Then, one gets

$$\dot{W}(x - \bar{x}_v) \leqslant -\mu W(x - \bar{x}_v)$$

whenever $\|x - \bar{x}_v\| \geqslant \beta\|\Delta v\|$ for some positive constant β that depends on K and E. Then, it follows from [19, Thm. 4.19] that the system (7) with $\xi = x - \bar{x}_v$, $\rho = z - \bar{z}_v$, $\zeta = \Delta v$ and $\eta = w - \bar{w}_v$ is input-to-state stable. Since exponential convergence has already been proven, the system (1) is input-to-state convergent. ■

IV. CONCLUSIONS AND FUTURE WORKS

In this paper, we discussed the convergence property in the context of complementarity systems. After adapting a suitable definition of convergence for these systems, we proved that a complementarity system is exponentially and input-to-state convergent if the underlying linear system has a strictly positive real transfer matrix.

A line of further research is to investigate convergence of the so-called switched complementarity systems that capture dynamics of various non-smooth systems including the power converters.

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