Observer-based output-feedback control to eliminate torsional drill-string vibrations

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Abstract-Torsional stick-slip vibrations decrease the performance and reliability of drilling systems used for the exploration of energy and mineral resources. In this work, we present the design of a nonlinear observer-based outputfeedback control strategy to eliminate these vibrations. We apply the controller to a drill-string model based on a real-life rig. To facilitate the design and implementation of the controller, we employ model reduction to obtain a low-order approximation of this model. Conditions, guaranteeing asymptotic stability of the desired equilibrium, corresponding to nominal drilling operation, are presented. The proposed control strategy has a significant advantage over existing vibration control systems in current drilling rigs as it only requires surface measurements instead of expensive down-hole measurements and can handle multiple modes of torsional vibration. Case study results using the proposed control strategy show that stick-slip oscillations can indeed be eliminated in realistic drilling scenarios.

I. INTRODUCTION

Drilling systems, as schematically shown in Fig. 1, are used to drill deep wells for the exploration and production of oil and gas, mineral resources and geo-thermal energy. Surface and down-hole measurements [1]–[3] show that these systems experience different types of oscillations, which decrease the drilling rate of penetration due to damage to the drill bit (e.g. bit tooth wear), drill pipes and bottom hole assembly (e.g. twisted pipe). The focus of the current paper is on the aspect of mitigation of torsional stick-slip oscillations by means of control as these vibrations are known to be highly detrimental to drilling efficiency, reliability and safety.

For the design of controllers to eliminate torsional vibrations most studies rely on one- or two degree-of-freedom (DOF) models for the torsional drill-string dynamics only, see e.g. [4]–[6]. In these models, it is generally assumed that the resisting torque at the bit-rock interface can be modeled as a frictional contact with a velocity weakening effect as reported in [7], [8]. In fact, modelling of the coupled axial and torsional dynamics, as for example in [9], shows that the velocity weakening effect in the torque-on-bit (TOB) is a consequence of the drilling dynamics. The fact that such coupling effectively leads to a velocity weakening effect of the TOB (see e.g. [10], [11]) motivates to adopt a modellingfor-control approach for drill-string dynamics involving the torsional dynamics only, as we will pursue in this paper.



Fig. 1. Schematic drilling system (adapted from [12]).

Different control strategies to suppress torsional vibrations can be found in literature. In [13], the use of torque feedback in addition to a speed controller is investigated. The underlying idea is making the top rotary system behave in a "soft" manner, hence the name Soft Torque Rotary System, see also [5]. In these research contributions, it is assumed that the drilling system behaves like a 2-DOF torsional pendulum of which the first torsional mode can be damped using a PI-controller based on the surface angular velocity. In [8], [14], the above soft torque approach is compared with a control method based on torsional rectification, which outperforms the soft torque approach in simulation studies by using improved torque feedback based on the twist of the drill-string near the rotary. A linear \mathcal{H}_{∞} controller synthesis approach is presented in [6]. Herein, the bit-rock interaction, key in causing stick-slip, is not taken into account in the controller design and stability analysis of the closed-loop dynamics. A control design approach, where information of the nonlinear bit-rock interaction model is explicitly taken into account in the controller synthesis, is proposed in [4], [15], [16]. Drawbacks of the approaches in [4], [6] are, firstly, the necessity of down-hole measurements reflecting the twist of the drill-string between surface and bit, which can not be measured in practice, and, secondly, the fact that only one torsional mode of the drill-string is taken into account.

Increasing demands on the operating envelope and a tendency towards drilling deeper and inclined wells impose higher demands on the controllers used in drilling systems. Industrial controllers are not always able to eliminate stickslip vibrations under the imposed operating conditions. Two main reasons for this deficiency are the influence of multiple dynamical modes of the drill-string on torsional vibrations

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and uncertainty in the bit-rock interaction. The main contribution of this paper is an output-feedback control strategy mitigating torsional stick-slip vibrations while 1) only using surface measurements, 2) taking into account a multi-modal drill-string model and 3) including severe velocity weakening and uncertainty in the bit-rock interaction.

Preliminaries: In support of the controller design result in Section III-A we present the following definitions on input-to-state-stability and the strict passivity property. The concept of local input-to-state stability has been introduced in [17].

Definition 1: The system $\dot{x}(t) \in F(x(t), e(t))$ is locally input-to-state stable (LISS) if there exist constants $c_1, c_2 > 0$, a function ρ of class \mathcal{KL} and a function μ of class \mathcal{K} such that for each initial condition $x(0) = x_0$, such that $||x_0|| \le c_1$, and each piecewise continuous bounded input function e(t)defined on $[0, \infty)$ and satisfying $\sup_{\tau \in [0,\infty)} ||e(\tau)|| \le c_2$, it holds that

- all solutions x(t) exist on $[0,\infty)$ and,
- all solutions satisfy

$$\|x(t)\| \le \rho(\|x_0\|, t) + \mu\left(\sup_{\tau \in [0,t]} \|e(\tau)\|\right), \ \forall t \ge 0.$$
 (1)

Consider the linear time-invariant minimal realization

$$\begin{aligned} \dot{x} &= Ax + Gw \\ z &= Hx + Dw \end{aligned}$$
 (2)

with the state $x \in \mathbb{R}^n$, input and output $w, z \in \mathbb{R}$.

Definition 2: The system (2) or the quadruple (A, G, H, D) is said to be strictly passive if there exist an $\varepsilon > 0$ and a matrix $P = P^{\top} > 0$ such that

$$\begin{bmatrix} A^{\top}P + PA + \varepsilon I & PG - H^{\top} \\ G^{\top}P - H & -D - D^{\top} \end{bmatrix} \le 0.$$
(3)

II. DRILL-STRING MODEL

We consider a jack-up drilling rig and a corresponding finite element method (FEM) model representation, with 18 elements. The rig is equipped with an AC top drive and Soft Torque system [18], however, stick-slip vibrations have been observed in the field. The model has been validated with field data under different conditions (in terms of weight-onbit (WOB) and angular velocity) and can be written as

$$M\ddot{q} + D\dot{q} + K_t q_d = S_w T_w(\dot{q}) + S_b T_{bit}(\dot{q}_1) + S_m T_m \quad (4)$$

with the coordinates $q \in \mathbb{R}^{\bar{n}}$ with $\bar{n} = 18$, the top drive motor torque input $T_m \in \mathbb{R}$ being the control input, the bit-rock interaction torque $T_{bit} \in \mathbb{R}$ and the interaction torques $T_w \in \mathbb{R}^{\bar{n}-1}$ between the borehole and the drill-string acting on the nodes of the FEM model. The coordinates q represent the angular displacements of the nodes of the finite element representation, where the first element (q_1) describes the rotation of the bit and the last element (q_{18}) the rotation of the top drive at surface. Next, we define the difference in angular position between adjacent nodes as $q_d := [q_1 - q_2 \quad q_2 - q_3 \quad \cdots \quad q_{17} - q_{18}]^{\top}$. In (4), the mass, damping and stiffness matrices are, respectively, given by $M \in \mathbb{R}^{\bar{n} \times \bar{n}}$, $D \in \mathbb{R}^{\bar{n} \times \bar{n}}$ and $K_t \in \mathbb{R}^{\bar{n} \times \bar{n}-1}$, the matrices $S_w \in \mathbb{R}^{\bar{n} \times \bar{n}-1}$, $S_b \in \mathbb{R}^{\bar{n} \times 1}$ and $S_m \in \mathbb{R}^{\bar{n} \times 1}$ represent the generalized force directions of the interaction torques, the bit torque and the input torque, respectively. The interaction torques T_w are modeled as Coulomb friction, that is

$$T_{w,i} \in T_i \operatorname{Sign}(\dot{q}_i), \quad \text{for } i = 2, \dots, 18, \tag{5}$$

with T_i representing the amount of friction at each element and the set-valued sign function defined as

Sign
$$(y) \triangleq \begin{cases} -1, & y < 0\\ [-1,1], & y = 0\\ 1, & y > 0. \end{cases}$$
 (6)

The bit-rock interaction model is given by

$$T_{bit}(\dot{q}_1) \in \text{Sign}(\dot{q}_1) \left(T_d + (T_s - T_d) e^{-v_d |\dot{q}_1|} \right)$$
 (7)

with T_s the static torque, T_d the dynamic torque and $v_d = \frac{30}{N_d \pi}$ indicating the decrease from static to dynamic torque. The model (4), (5) and (7) together forms a differential inclusion that we can write in state-space Lur'e-type form as:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{G}\bar{w} + \bar{G}_2\bar{w}_2 + \bar{B}u$$

$$\bar{z} = \bar{H}\bar{x}$$

$$\bar{z}_2 = \bar{H}_2\bar{x}$$

$$\bar{y} = \bar{C}\bar{x}$$

$$\bar{w} \in -\varphi(\bar{z})$$

$$\bar{w}_2 \in -\phi(\bar{z}_2),$$
(8)

where $\bar{x} := \begin{bmatrix} q_d^\top & \dot{q}^\top \end{bmatrix}^\top \in \mathbb{R}^{2\bar{n}-1}$ is the state, $\bar{z} := \omega_{bit} \in \mathbb{R}$ and $\bar{z}_2 := \begin{bmatrix} \dot{q}_2, \dots, \dot{q}_{18} \end{bmatrix}^\top \in \mathbb{R}^{\bar{n}-1}$ are the angular velocity arguments of the set-valued nonlinearities φ and ϕ , respectively. The bit-rock interaction torque is given by $\bar{w} \in \mathbb{R}$ and the drill-string-borehole interaction torques are given by $\bar{w}_2 \in \mathbb{R}^{\bar{n}-1}$, $u := T_m \in \mathbb{R}$ is the control input and $\bar{y} := \begin{bmatrix} \omega_{td} & T_{pipe} \end{bmatrix}^\top \in \mathbb{R}^2$ is the measured output, which implies that only surface measurements will be employed. The angular velocities of the top drive and the bit are defined as $\omega_{td} := \dot{q}_{18}$ and $\omega_{bit} := \dot{q}_1$, respectively, and the pipe torque T_{pipe} is the torque in the drill-string directly below the top drive. The matrices \bar{A} , \bar{B} , \bar{G} , \bar{G}_2 , \bar{H} and \bar{H}_2 in (8), with appropriate dimensions are given by

$$\bar{A} = \begin{bmatrix} 0_{17\times17} & \bar{a} \\ -M^{-1}K_t & -M^{-1}D \end{bmatrix}, \ \bar{a} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}$$
$$\bar{B} = \begin{bmatrix} 0_{17\times1} \\ M^{-1}S_m \end{bmatrix}, \ \bar{G} = \begin{bmatrix} 0_{17\times1} \\ M^{-1}S_b \end{bmatrix}, \ \bar{G}_2 = \begin{bmatrix} 0_{17\times17} \\ M^{-1}S_w \end{bmatrix},$$
$$\bar{H} = \begin{bmatrix} 0_{1\times17} & 1 & 0_{1\times17} \end{bmatrix}, \ \bar{H}_2 = \begin{bmatrix} 0_{17\times18} & I_{17} \end{bmatrix},$$

and $\bar{C} \in \mathbb{R}^{2 \times 2\bar{n}-1}$ indicates the measured output. Note that $\varphi(\bar{z}) := T_{bit}(\bar{z})$ and $\phi(\bar{z}_2) := [T_{w,2}(\dot{q}_2), \ldots, T_{w,18}(\dot{q}_{18})]^{\top}$. The relevant frequency response functions for the linear part of the drill-string dynamics (8) are represented by the solid lines in Figs. 2, 3 and 4.

A. Reduced-order model

To facilitate the design and to decrease the implementation burden of observer-based output-feedback controllers (see Section III), we apply model reduction to obtain a loworder approximation of the drilling system dynamics (8), that





Fig. 2. Frequency response function of the fullorder and reduced-order model from input torque T_m to bit velocity ω_{bit} .



Fig. 5. Bit-rock interaction model.

approximates the input-output behavior from inputs u and \bar{w} to outputs \bar{y} and \bar{z} . The inputs and outputs related to the drillstring-borehole interaction $(T_{w,i})$ are not taken into account in the reduction process, but can be approximated using the transformation matrix obtained from the reduction procedure. Hence, system (8) can be represented as a Lur'e type system $\bar{\Sigma} = (\bar{\Sigma}_{lin}, \varphi)$, consisting of high-order linear dynamics $\bar{\Sigma}_{lin}$ with a single static output-dependent nonlinearity φ , related to the bit-rock interaction, in the feedback loop. We will use the model reduction approach for Lur'e-type systems as proposed in [19] to obtain the reduced-order linear system Σ_{lin} . Interconnecting this system with the nonlinearity φ yields the reduced-order drill-string system $\Sigma = (\Sigma_{lin}, \varphi)$. The reduced-order system model can then be written as (now again taking into account drill-string-borehole interactions):

$$\dot{x} = Ax + Gw + G_2w_2 + Bu$$

$$z = Hx$$

$$z_2 = H_2x$$

$$y = Cx$$

$$w \in -\varphi(z)$$

$$w_2 \in -\phi(z_2)$$
(9)

with state $x \in \mathbb{R}^n$, n = 7 and an experimentally validated bit-rock interaction as shown in Fig. 5. The relevant frequency response functions for the linear part of the dynamics in (9) are shown in Figs. 2, 3 and 4. Clearly, the first three resonance modes are dominant, while those dominant modes (and the rigid-body mode) are accurately captured, which motivates the choice to reduce to a model order of n = 7.



Fig. 3. Frequency response function of the fullorder and reduced-order model from bit torque T_{bit} to bit velocity ω_{bit} , i.e. bit mobility.

Fig. 4. Frequency response function of the fullorder and reduced-order model from input torque T_m to top drive velocity ω_{td} .

III. OUTPUT-FEEDBACK CONTROLLER DESIGN

We employ an observer-based controller synthesis strategy for Lur'e-type systems with discontinuities as in [4], [20]. The conditions for controller synthesis as in [4] achieving *global* asymptotic stability are infeasible for the realistic drill-string model presented here for three reasons: firstly, the incorporation of more realistic drill-string dynamics including multiple torsional flexibility modes, see Figs. 2, 3 and 4, secondly, the incorporation of a bit-rock interaction model based on field data, which shows a rather severe velocity weakening effect, see Fig. 5 and, thirdly, the restriction on the availability of only surface measurements. Therefore, we employ a controller synthesis strategy to design *locally* stabilizing controllers and we show that such local stability properties suffice in realistic drilling scenarios.

A. State-feedback controller

In this section, we discuss the design of a state-feedback controller for generic systems in the form

$$\dot{\xi} = A\xi + Bu_{fb} + G\tilde{w}
\tilde{z} = H\xi
\tilde{w} \in -\tilde{\varphi}(\tilde{z})$$
(10)

that stabilizes the origin $\xi = 0$ of the system state $\xi \in \mathbb{R}^n$. Stabilization of the origin of (10) corresponds to the desired operation of constant angular velocity of the drilling system. The control input is given by $u_{fb} \in \mathbb{R}^m$, the input and output of the set-valued nonlinearity $\tilde{\varphi}$ are given by $\tilde{z} \in \mathbb{R}$ and $\tilde{w} \in \mathbb{R}$, respectively, and the system matrices are $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $G \in \mathbb{R}^{n \times 1}$ and $H \in \mathbb{R}^{1 \times n}$. We introduce the linear static state-feedback law, $u_{fb} = K\hat{\xi} = K(\xi - e)$, where we take the "measurement" (or observer) error e := $\xi - \hat{\xi}$ into account, moreover, $K \in \mathbb{R}^{m \times n}$ is the control gain matrix and $\hat{\xi}$ the observer estimate of the state ξ ; the observer will be treated in more detail in Section III-B. The resulting closed-loop system is described by the differential inclusion:

$$\dot{\xi} = (A + BK)\xi + G\tilde{w} - BKe
\tilde{z} = H\xi$$

$$\tilde{w} \in -\tilde{\varphi}(\tilde{z}).$$
(11)

The transfer function $G_{cl}(s)$ from the input \tilde{w} to the output \tilde{z} of system (11) is given by $G_{cl} = H(sI - (A + BK))^{-1}G$, $s \in \mathbb{C}$. Now, let us state the following assumptions on the properties of the set-valued



Fig. 6. Schematic representation of system (11) after transformation using a dynamic multiplier.

nonlinearity $\tilde{\varphi}(\tilde{z})$. Hereto, we first define a set S_a for which a particular sector condition is satisfied $S_a := \{\tilde{z} \in \mathbb{R} | \tilde{z}_{a1} < \tilde{z} < \tilde{z}_{a2}\}$ with $\tilde{z}_{a1} < 0 < \tilde{z}_{a2}$.

Assumption 1: The set-valued nonlinearity $\tilde{\varphi}: \mathbb{R} \to \mathbb{R}$ satisfies

- $0 \in \tilde{\varphi}(0);$
- $\tilde{\varphi}$ is continuously differentiable and bounded $\forall \ \tilde{z} \in S_a$;
- $\tilde{\varphi}$ locally satisfies the [0,k] sector condition, with k > 0, in the sense that $\tilde{w} [\tilde{w} + k\tilde{z}] \leq 0 \quad \forall \ \tilde{w} \in \{\tilde{w} \in -\tilde{\varphi}(\tilde{z}) | \tilde{z} \in S_a\}.$

The intended control goal is to render the closed-loop system (11) *locally input-to-state stable* with respect to the input e, as formalized in Definition 1, by a proper design of the controller gain K. We use the concept of a dynamic multiplier to transform the original system into a feedback interconnection of two passive systems. In Fig. 6, a block diagram of the system including the dynamic multiplier with transfer function $M(s) = 1 + \gamma s$, $s \in \mathbb{C}$, is shown; furthermore the loop transformation gain $\frac{1}{k}$ is included given the fact that the nonlinearity $\tilde{\varphi}(\cdot)$ belongs to the sector [0, k]. The linear system Σ_1 in Fig. 6 can be written in state-space form as follows:

$$\Sigma_1 : \begin{cases} \dot{\xi} = (A + BK)\xi + G\tilde{w} - BKe \\ \breve{z} = \breve{H}\xi + \breve{D}\tilde{w} + \breve{Z}e \end{cases}$$
(12)

with $\check{H} := H + \gamma H (A + BK)$, $\check{D} := \frac{1}{k} + \gamma HG$ and $\check{Z} := -\gamma HBK$. For system Σ_2 in Fig. 6 we can write:

$$\Sigma_{2}: \begin{cases} \dot{\tilde{z}} = -\frac{1}{\gamma}\tilde{z} + \frac{1}{\gamma}\check{z} - \frac{1}{\gamma k}\tilde{w} \\ \tilde{w} \in -\tilde{\varphi}(\tilde{z}) \end{cases}$$
(13)

The following theorem states sufficient conditions under which system (11) is LISS with respect to input e.

Theorem 1: Consider system (11) and suppose there exists a constant $\gamma > 0$ such that $(A + BK, G, \breve{H}, \breve{D})$ is strictly passive. Then system (11) is LISS, with respect to input *e* for any $\tilde{\varphi}(\cdot)$ satisfying Assumption 1.

Proof: The proof can be found in [21].

B. Observer design

We will use an observer to find an estimate of the states of system (10), since we only rely on surface measurements. Following [20], we propose the following observer, with measured output $\tilde{y} = C\xi$ ($\tilde{y} \in \mathbb{R}^k$ and $C \in \mathbb{R}^{n \times k}$):

$$\hat{\xi} = (A - LC)\hat{\xi} + Bu_{fb} + G\hat{w} + L\tilde{y}
\hat{z} = (H - NC)\hat{\xi} + N\tilde{y}
\hat{w} \in -\tilde{\varphi}(\hat{z})
\hat{y} = C\hat{\xi}$$
(14)

and observer gain matrices $L \in \mathbb{R}^{n \times k}$ and $N \in \mathbb{R}^{1 \times k}$. Next, we state an additional assumption on the nonlinearity $\tilde{\varphi}(\cdot)$. Hereto, we first define the set S_b as $S_b := \{\tilde{z} \in \mathbb{R} | \tilde{z}_{b1} < \tilde{z} < \tilde{z}_{b2} \}$ with $\tilde{z}_{b1} < 0 < \tilde{z}_{b2}$, such that for all $\tilde{z} \in S_b$ the monotonicity property holds.

Assumption 2: The set-valued nonlinearity $\tilde{\varphi} : \mathbb{R} \to \mathbb{R}$ is such that $\tilde{\varphi}$ is monotone for all $\tilde{z} \in S_b$, i.e. for all $z_1 \in S_b$ and $z_2 \in S_b$ with $w_1 \in \tilde{\varphi}(z_1)$ and $w_2 \in \tilde{\varphi}(z_2)$, it holds that $(w_1 - w_2)(z_1 - z_2) \ge 0$.

The observer error has been defined as $e = \xi - \hat{\xi}$ before. Consequently, the observer error dynamics can be written as

$$\dot{e} = (A - LC) e + G(\tilde{w} - \hat{w})
\tilde{w} \in -\tilde{\varphi}(\tilde{z})
\hat{w} \in -\tilde{\varphi} \left(H\hat{\xi} + N\left(\tilde{y} - C\hat{\xi}\right)\right).$$
(15)

The following theorem provides sufficient conditions for the design of the observer gains L and N such that the origin e = 0 is a locally exponentially stable (LES) equilibrium point of the observer error dynamics (15).

Theorem 2: Consider system (10) and the observer (14) with (A - LC, G, H - NC, 0) strictly passive and the matrix G being of full column rank. If it holds that

$$\|\xi(t)\| \le \epsilon \frac{\tilde{z}_{b,min}}{\|H\|}, \quad \forall t \ge 0,$$

for some $\epsilon \in (0, 1)$ and $\tilde{z}_{b,min} := \min(|\tilde{z}_{b1}|, |\tilde{z}_{b2}|)$, then e = 0 is a locally exponentially stable equilibrium point of the observer error dynamics (15) for any $\tilde{\varphi}$ satisfying Assumptions 1 and 2 with the region of attraction containing the set

$$\left\{ e \in \mathbb{R}^n | \|e_0\| \le (1-\epsilon) \, \frac{\tilde{z}_{b,min}}{\|H - NC\|} \left(\frac{\lambda_{max}(P_o)}{\lambda_{min}(P_o)} \right)^{-\frac{1}{2}} \right\}$$

with the initial observer error $e(0) = e_0$. The matrix P_o results from the existence of $P_o = P_o^\top > 0$ and $Q_o = Q_o^\top > 0$ such that $P_o(A - LC) + (A - LC) P_o = -Q_o$ and $G^\top P_o = H - NC$, which is equivalent to the strict passivity of (A - LC, G, H - NC, 0).

Proof: The proof can be found in [21].

C. Output-feedback control

The state-feedback controller and the observer from the previous sections together form an observer-based output-feedback controller. We use the estimated state $\hat{\xi}$ of the observer (14) in the closed-loop system (11) and prove local asymptotic stability of the equilibrium (ξ , e) = (0, 0) of the interconnected system (11), (15).

Theorem 3: Consider system (11) and observer (14). Suppose the conditions in Theorem 1 are satisfied for system (11) and that the observer error dynamics in (15) satisfies the conditions in Theorem 2. Then, $(\xi, e) = (0, 0)$ is a locally asymptotically stable equilibrium point of the interconnected system (11), (15) for any $\tilde{\varphi}$ satisfying Assumptions 1 and 2. *Proof:* The proof can be found in [21].



IV. SIMULATION RESULTS

In this section, we will show the application of the observer-based output-feedback controller (see Section III) to the reduced-order drill-string model presented in Section II. To stabilize the desired equilibrium x_{eq} of system (9) we have to design the controller gain K and the observer gains L and N. The control torque $u = u_{ff} + K\hat{\xi}$, consists of the constant feedforward torque u_{ff} and the feedback torque $K\hat{\xi}$ based on the observer estimate $\hat{\xi}$. We assume that the resistive torques along the drill-string are constant and can be compensated by u_{ff} . The equilibrium x_{eq} and feedforward torque u_{ff} can be obtained from the equilibrium condition of system (9) that has to satisfy $Ax_{eq} - G\varphi(Hx_{eq}) - G_2\phi(H_2x_{eq}) + Bu_{ff} = 0$ and the requirement that $y_1 = \omega_{td}$ matches the desired equilibrium velocity ω_{eq} .

Now, we have to write the system (9) in the form (10) and such that the set-valued nonlinearity satisfies the conditions in Assumptions 1 and 2. Therefore, we write the reducedorder drill-string system in perturbation states, i.e. $\xi := x - x_{eq}$. Furthermore, we apply a linear loop transformation to change the properties of the nonlinearity φ . This results in the following state-space representation

$$\dot{\xi} = A_t \xi + B u_{fb} + G \tilde{w} \tilde{z} = H \xi \tilde{w} \in -\tilde{\varphi}(\tilde{z})$$
(16)

with $A_t := A + \delta GH$, $\delta > 0$ a constant to apply the linear loop transformation, $\tilde{\varphi}(\tilde{z}) := \varphi(\tilde{z} + Hx_{eq}) - \varphi(Hx_{eq}) + \delta \tilde{z}$ and $\tilde{w} = w - w_{eq} - \delta \tilde{z}$. The transformed nonlinearity $\tilde{\varphi}(\tilde{z})$ is shown in Fig. 7. As can be seen in this figure, $\tilde{\varphi}(\tilde{z})$ belongs locally to the sector [0, k] with k = 570 Nms/rad (note $\delta =$ 29.2 Nms/rad in this case). The physical meaning of this condition is that the amount of velocity weakening in the bit-rock interaction is limited. A larger sector, including the total nonlinearity $\tilde{\varphi}(\tilde{z})$, would result in high control gains K. Such high gains result in high control torques u that can not be realized by the top drive and are therefore infeasible in practice. In Fig. 7, we have also indicated the point $\tilde{z}_{a1} =$ -28.9 rpm for which it holds that for $\tilde{z}_{a1} < \tilde{z} < \tilde{z}_{a2}$ the sector condition is satisfied (i.e. \tilde{z}_{a2} can be chosen arbitrarily large in this case) and the point $\tilde{z}_{b1} = -20.1$ rpm such that for $\tilde{z}_{b1} < \tilde{z} < \tilde{z}_{b2}$ it holds that $\tilde{\varphi}$ is monotonically increasing (i.e. \tilde{z}_{b2} can also be chosen arbitrarily large in this case), as



Fig. 8. Simulation result of the reduced-order model with an existing industrial controller.

stated in Assumption 1 and 2, respectively.

The controller and observer gains are designed according to the conditions given in Theorem 1 and Theorem 2, respectively. The results are obtained by using SeDuMi 1.3 [22], a linear matrix inequality (LMI) solver. Hence, the controller gains K are determined by finding a solution such that $(A_t + BK, G, \breve{H}, \breve{D})$ is strictly passive. To find the observer gains L and N we have to satisfy the strict passivity conditions for $(A_t - LC, G, H - NC, 0)$.

First, we will show a simulation result of the reduced-order drill-string system in closed-loop with an existing industrial controller (based on [5]). For the simulations, we introduce a so-called startup scenario, which is based on practical startup procedures for drilling rigs. Herein, the drill-string is first accelerated to a low constant rotational velocity with the bit above the formation (off bottom) and, subsequently, the angular velocity and weight-on-bit (WOB) are gradually increased to the desired operating conditions. The increase in WOB is modelled as a scaling of the bit-rock interaction torque. For WOB = 0 (off bottom) there is no velocity weakening in the TOB and increasing the WOB relates to an increase in both the static and dynamic torque until the nominal bit-rock interaction is obtained (see [21] for details).

A simulation result of the reduced-order model with the industrial controller is shown in Fig. 8. The controller is a properly tuned active damping system (i.e. PI-control of the angular velocity) which aims at damping the first torsional mode of the drill-string dynamics. In the upper plot the top drive velocity (ω_{td}) is shown along with the reference velocity ω_{ref} that starts at a velocity of approximately 20 rpm and is gradually increased to the desired equilibrium velocity, ω_{eq} , of 50 rpm. From the bit response, in the bottom plot, we can clearly recognize stick-slip oscillations. The increasing amplitude of the oscillations in the top drive velocity, demonstrates that these vibrations arise when the WOB is increased ($20 \le t < 40$ s).

For the designed output-feedback controller, we immediately activate (at t = 0) the observer to obtain the state estimate $\hat{\xi}$; however, this estimate is not used by the industrial



Fig. 9. Simulation result of the reduced-order model with the designed output-feedback controller.

PI-controller that is used in the first 20 seconds (since this controller only uses the top drive velocity as a measured output). When the state-feedback controller is switched on at t = 20, it uses the state estimate $\hat{\xi}$, based on the surface measurements ω_{td} and T_{pipe} only. Fig. 9 shows a simulation result of the closed-loop system with output-feedback controller, where we used the same initial conditions ξ_0 as for the previous simulation (Fig. 8). Furthermore, the initial states for the observer ξ_0 have a 10% offset from the initial states ξ_0 . It can be seen that after some transient behavior, the observer estimates converge to the actual states within approximately 5 seconds. Moreover, the simulation results show that the top drive and bit velocity converge to their equilibrium value and stick-slip oscillations are avoided. The equilibrium velocity of the bit $\omega_{bit,eq} := Hx_{eq}$ is slightly higher than the equilibrium velocity of the top drive $\omega_{td,eq} =$ $\omega_{eq} = 50$ rpm. This small mismatch is due to the reduction as the outputs z and y of the reduced-order system slightly differ from the original outputs \bar{z} and \bar{y} and the feedforward is designed such that in equilibrium the top drive velocity of the reduced-order model matches the desired velocity. The lag between the desired velocity and the top drive velocity between 25 and 45 s is because we designed a low-gain controller aiming at stabilization of the desired equilibrium instead of achieving a high bandwidth. Most importantly, it can be concluded that the stick-slip vibrations are eliminated with the designed controller.

V. CONCLUSIONS

In this work, an observer-based output-feedback control strategy is proposed to eliminate torsional stick-slip vibrations in drilling systems. Particular benefits of the proposed approach with respect to existing ones are, firstly, the fact that a realistic multi-modal model of the drill-string dynamics is taken into account, secondly, that severe velocity weakening in the bit-rock interaction is taken into account, thirdly, that only surface measurements are employed and, finally, that a guarantee for (local) asymptotic stability of the closedloop system is given for bit-rock interaction laws lying within a certain sector (which is beneficial as the bit-rock interaction is subject to uncertainty in practice). Simulation results of applying the proposed controller to a realistic drill-string model show that stick-slip oscillations can be eliminated, while under the same conditions the existing industrial controller is unable to do so.

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