# Active trailer steering for robotic tractor-trailer combinations

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Abstract—Active trailer steering control can improve the manoeuvrability of (long) truck-trailer combinations during cornering. To assess the effectiveness of trailer steering control, we formulate the problem of reducing the so-called sweptpath width during cornering, and that of eliminating unsafe tail swing of the trailer, as a tracking control problem. We present a kinematic tractor-trailer model including off-axle hitching, on the basis of which nonlinear control strategies solving this tracking problem are developed. The effectiveness of the proposed approach is evidenced by means of a benchmark simulation study.

#### I. INTRODUCTION

Long combination truck-trailer vehicles have advantages related to reduced costs for goods transportation and reduced fuel consumption (i.e. reduced impact on the environment). However, drawbacks related to inferior vehicle manoeuvrability hamper widespread introduction of such vehicles. In particular, the space needed by a (conventional) tractortrailer combination in a turning manoeuvre, the so-called swept path, is an important manoeuvrability/safety aspect [1]. Especially in urban areas or on narrow roads the available space is limited and performing a turning manoeuvre, such as a 90 degree turn or taking a roundabout, can be a difficult and even an unsafe task for long combination (truck-trailer) vehicles.

Existing trailer steering systems [2], [3] evidence the fact that the application of trailer axle steering can reduce the swept path width of a tractor-trailer combination. Although these systems reduce the swept path width, a further reduction in swept path width can be obtained by using more advanced control strategies for trailer axle steering. Furthermore, these systems generate tail swing during transient cornering, i.e. during entering and exiting a turn, which represents a serious safety hazard.

Recent works on trailer steering control [4], [5], [6], [7], [8] *kinematically* model a tractor-trailer vehicle with trailer steering, in which a nonholonomic velocity constraint is used to ideally model the absence of lateral tyre slipping. Especially during low-speed manoeuvres, such kinematic

models can be considered to be accurate since inertial effects can then be neglected (provided that tyre slip effects are indeed negligible). In [5], [9], [10], [11], [12], [13], [14], [15], [16], a linearised dynamic model is used to model an articulated vehicle, while [17], [18], [19] use a nonlinear dynamic model. In this way, dynamic effects can be evaluated which significantly affect the behaviour of the vehicle, although mainly during high speeds. Furthermore, in [4], [5], [6], [9], [10], [11], [12], [13], [14], [15], [18], [19], off-axle hitching is included in the model, while [4], [7] consider on-axle hitching. Especially during sharp turning manoeuvres, the presence of off-axle hitching significantly affects the behaviour of the tractor-trailer combination and hence should be included in the model description. Since the current paper aims to improve low-speed manoeuvrability by axle steering control, we pursue a kinematic modelling approach including off-axle hitching.

Different control problem formulations for active trailer steering have been considered in the literature [20]. In [7], [9], [10], [12], [13], [18], [21], the steering angle(s) of (multiple) *tractor* axle(s) is considered as a control input(s) and a reference path for the front tractor axle is prescribed. Such control problem formulation would require fully automated vehicles in order to achieve path-following. The problem considered in the current paper is that of active *trailer* steering control for truck-trailer combinations in which a *human driver* determines the path (and speed) driven by the tractor, see also [6], [11], [14], [17], [19], [22]. In this setting, we consider 1) the trailer axle steering speed as the control input and 2) the problem of reducing the swept-path width (and tail swing) by ensuring that the trailer axle.

In the literature, a range of different control strategies for trailer steering control have been proposed: an adaptive approach in [9], [13], an  $H_{\infty}$  approach in [10], LQRcontrol in [11], classical linear control in [6], [14], a fuzzy control approach in [17], a Lyapunov-based approach in [7] and an approach based on backstepping in [18]. In these papers, the considered problems are formulated as a pathfollowing problem, while in the current paper, the pathfollowing problem is reformulated as a tracking problem, for which a nonlinear control solution is proposed.

The main contributions of this work are, firstly, the fact that we cast the problem of reducing the swept-path width and avoiding tail swing into a tracking problem for a kinematic (reference) model of the tractor-trailer system including off-axle hitching, secondly, the design of nonlinear controllers solving this tracking problem including related stability results and, thirdly, the simulation-based validation

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of the proposed control strategies evidencing the effectiveness of the approach in reducing the swept-path width and avoiding tail swing.

The remainder of the paper is organised as follows. In Section II, we derive the kinematic tractor-trailer model. In Section III, the control problem is formulated. In Section IV, we propose a controller design, solving this problem, and present related stability results. In Section V, a simulation study is presented to evidence the effectiveness of the proposed approach. Finally, in Section VI, we present conclusions.

## II. MODEL OF THE TRACTOR-TRAILER ROBOT

As a stepping-stone towards full-scale trailer steering control for trucks, we consider an off-axle tractor-trailer robot as in Figure 1. This section presents a kinematic model is derived for this robot. We aim to construct a statespace model formulation such that the front wheel steering angle and its forward velocity are given time-varying (driver) inputs and the trailer axle steering velocity is the (only) control input. Hence, in this formulation, the tractor is steered and driven by an (emulated) *driver* and the trailer axle is steered automatically, improving manoeuvrability.

As we consider low-speed turning manoeuvres in this work, and given the fact that the robot has single rear axles both at the tractor and trailer, the assumption of no sideways slip (between the wheels and the floor) is justified. Consequently, a kinematic model can accurately describe the behaviour of the tractor-trailer robot and further dynamic effects, related to wheel slip and inertial effects, can be neglected.

Consider a bicycle-like model representation as shown in Figure 2. Each wheel represents the midpoint of an axle (from right to left: the tractor front axle, the tractor rear axle and the trailer axle). Furthermore, the trailer rear point (tail) and the articulation (hitch) point between tractor and trailer are illustrated as well. The lengths characterising the tractor, trailer and axle locations are defined as follows: the tractor body length  $l_1$ , the off-axle distance  $l_{\text{off}}$ , the trailer length  $l_2$  and the rear overhang  $l_{\text{ro}}$ . The front wheel steering angle  $\phi_1$  and forward velocity  $v_1$  as well as the articulation angle  $\alpha$  and the trailer wheel steering angle  $\phi_3$  are depicted in Figure 2. Also, the relative heading angle  $\phi_4$  of the trailer rear point (tail) is illustrated, which characterises the direction of the velocity  $v_4$  of the trailer rear point (tail).



Fig. 1. Tractor-trailer robot.

The driver input  $\mathbf{d}(t) := [v_1(t), \phi_1(t)]$  consists of the forward velocity  $v_1(t)$  and the steering angle  $\phi_1(t)$  of the tractor front wheel, which are both considered to be given and to depend explicitly on time.

The posture of the tractor with respect to the fixedworld frame (x, y) can be characterised by the coordinates  $(\theta_1, \delta_1, X_1, Y_1)$ , see Figure 3. This posture is completely determined by the driver input  $\mathbf{d}(t)$  (and the initial posture of the tractor) and, therefore, can also be considered as a given (though a priori unknown) function of time:

$$\theta_{1}(t) = \phi_{1}(t) + \frac{1}{l_{1}} \int_{0}^{t} v_{1}(\sigma) \sin \phi_{1}(\sigma) d\sigma,$$
  

$$\delta_{1}(t) = \frac{1}{l_{1}} \int_{0}^{t} v_{1}(\sigma) \sin \phi_{1}(\sigma) d\sigma,$$
  

$$X_{1}(t) = \int_{0}^{t} v_{1}(\sigma) \cos \theta_{1}(\sigma) d\sigma,$$
  

$$Y_{1}(t) = \int_{0}^{t} v_{1}(\sigma) \sin \theta_{1}(\sigma) d\sigma.$$
  
(1)

Herein, we presume, without loss of generality, that:  $\theta_1(t_0) = \phi_1(t_0) = X_1(t_0) = Y_1(t_0) = 0$  with  $t_0 = 0$ . The location  $(X_2(t), Y_2(t))$  of the hitch point can be expressed as a given function of time as well:

$$X_{2}(t) = X_{1}(t) - (l_{1} + l_{\text{off}}) \cos \delta_{1}(t),$$
  

$$Y_{2}(t) = Y_{1}(t) - (l_{1} + l_{\text{off}}) \sin \delta_{1}(t).$$
(2)

Note that, effectively, the motion of the entire tractor is given as a function of time (determined by the driver input d(t)).

The articulation angle  $\alpha$  is equal to the difference between the orientation of the tractor body  $\delta_1(t)$  and that of the trailer body  $\delta_2$ :

$$\alpha = \delta_1(t) - \delta_2. \tag{3}$$



Fig. 2. Bicycle-like schematic model representation.



Fig. 3. Tractor-trailer posture in fixed-world coordinates.

Using angular kinematics, the rotational velocity  $\delta_2$  of the trailer body can be expressed as

$$\dot{\delta}_2 = \frac{1}{l_2} c_2(t) c_3(\mathbf{x}) - \frac{l_{\text{off}}}{l_1 l_2} c_1(t) c_4(\mathbf{x}), \tag{4}$$

in which

$$c_{1}(t) := v_{1}(t) \sin \phi_{1}(t),$$

$$c_{2}(t) := v_{1}(t) \cos \phi_{1}(t),$$

$$c_{3}(\mathbf{x}) := \sin \alpha - \cos \alpha \tan \phi_{3},$$

$$c_{4}(\mathbf{x}) := \cos \alpha + \sin \alpha \tan \phi_{3},$$
(5)

for  $\phi_3 \in (-\frac{\pi}{2}, \frac{\pi}{2})$ , and where  $\mathbf{x} := \begin{bmatrix} \alpha & \phi_3 \end{bmatrix}^T$ . Then, we can express the position of the rear point of the trailer, characterised by coordinates  $(X_4, Y_4)$ , as

$$X_4 = X_1(t) - (l_1 + l_{\text{off}}) \cos \delta_1(t) - (l_2 + l_{\text{ro}}) \cos \delta_2,$$
  

$$Y_4 = Y_1(t) - (l_1 + l_{\text{off}}) \sin \delta_1(t) - (l_2 + l_{\text{ro}}) \sin \delta_2.$$
(6)

Furthermore, the heading angles  $\theta_3$ ,  $\theta_4$  of the trailer wheel and the rear point are, respectively, given by:

$$\theta_3 = \delta_2 + \phi_3,\tag{7}$$

$$\theta_4 = \delta_2 + \phi_4. \tag{8}$$

Using the kinematic relations derived in (1) - (4), the trailer dynamics can be expressed in terms of the state  $\mathbf{x} = [\alpha \quad \phi_3]^T$  and can be written in state-space form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{d}(t)) + \mathbf{g}u \tag{9}$$

with the control input  $u = \dot{\phi}_3$  and

$$\mathbf{f}(\mathbf{x}, \mathbf{d}(t)) := \begin{bmatrix} f_{\alpha}(\mathbf{x}, \mathbf{d}(t)) \\ 0 \end{bmatrix}, \quad \mathbf{g} := \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$
(10)

Herein,

$$f_{\alpha}(\mathbf{x}, \mathbf{d}(t)) := \frac{1}{l_1} c_1(t) - \frac{1}{l_2} c_2(t) c_3(\mathbf{x}) + \frac{l_{\text{off}}}{l_1 l_2} c_1(t) c_4(\mathbf{x})$$
(11)

with  $c_1(t), c_2(t), c_3(x)$  and  $c_4(x)$  as in (5).

The state-space model (9) - (11), (5) shows that the only control input u is the trailer wheel steering velocity  $\dot{\phi}_3$  and the driver input  $\mathbf{d}(t)$  is considered to be an a priori unknown function of time.

#### **III. CONTROL PROBLEM FORMULATION**

The main control goal considered in this paper is to reduce the swept path width of the tractor-trailer robot while avoiding tail swing. The latter objective can be achieved by ensuring that the rear point of the trailer follows the path driven by the front wheel of the tractor [8], [23]. Hence, in this section, we consider a path-following problem in which a follow-point (rear point of the trailer) is required to follow the path driven by a lead-point (front wheel of the tractor), see Fig. 4. This path-following problem, described in the x - y plane, is reformulated into a state tracking problem, described in terms of the state  $\mathbf{x} = \begin{bmatrix} \alpha & \phi_3 \end{bmatrix}^T$ .

A schematic overview of the resulting tracking control problem is illustrated in Fig. 5. The driver inputs  $(\phi_1(t), v_1(t))$  are considered to be given (although not a



Fig. 4. Reference trailer configuration.

priori known) and are used to construct the reference trailer kinematics, i.e. the desired state trajectory  $(\alpha_d(t), \phi_{3d}(t))$ . Furthermore, the behaviour of the tractor-trailer robot is affected by the driver inputs and the control input u, which is the trailer wheel steering velocity, see the model in (9) - (11). In Section IV, we will propose control strategies to design controllers using information on the measured state (articulation angle and trailer steering angle), the driver input and the reference trailer kinematics.

#### A. Reference trailer kinematics

The reference trailer kinematics describe the kinematics of the trailer for which it holds that the rear point of the trailer exactly follows the path driven by the tractor front wheel, also during transient cornering, see Fig. 4. Hence, these kinematics can be employed to generate the state reference trajectory ( $\alpha_d(t), \phi_{3d}(t)$ ) for the control strategy presented in Section IV.

To construct the reference kinematics in terms of  $(\alpha_d(t), \phi_{3d}(t))$ , we compute a feasible position for the rear point of the trailer on the path driven by the front wheel, given the length  $l_2$  of the trailer and the rear overhang  $l_{\rm ro}$ , by solving the following (minimisation) problem:

$$\hat{\tau}(t) := \min_{\tau} \{ \tau \ge 0 | f_{\tau} = 0 \}$$
 (12)

with

$$f_{\tau} := (X_2(t) - X_1(t-\tau))^2 + (Y_2(t) - Y_1(t-\tau))^2 - (l_2 + l_{ro})^2.$$
(13)

In this way, a feasible rear point trailer position on the driven tractor front wheel path can be found, i.e. which is located a distance of  $l_2 + l_{\rm ro}$  from the hitch point  $(X_2(t), Y_2(t))$  (hence the form of  $f_{\tau}$  in (13)). The solution  $\hat{\tau}$  of the problem in (12), (13) is the corresponding time difference between the time instant t at which the tractor front wheel is at a certain position and the time instant  $t - \hat{\tau}$  at which the (desired)



Fig. 5. Control block diagram.

trailer rear point is at the same position. During transient cornering, the time difference  $\hat{\tau}(t)$  is indeed time-varying.

*Remark 1:* In order to prevent non-uniqueness of the solution of (12), (13), certain assumptions on the tractor front wheel path have to be satisfied. These assumptions will be made explicit in Section III-B. Additionally, we assume that for  $t \leq 0$  it holds that  $v_1(t) = v_1(0)$  and  $\phi_1(t) = \phi_1(0) = 0$ , such that there always exists a solution of the (minimisation) problem (12), (13) at t = 0.

For the derivation of the reference kinematics  $(\alpha_d(t), \phi_{3d}(t))$ , we use the following approach:

- 1) As a stepping stone, we construct a reference trajectory in terms of  $\alpha_d(t), \phi_{4d}(t)$ , where we recall that  $\phi_4$ indicates the direction of the velocity of the rear point of the trailer with respect to the trailer body, see Fig. 2.
- 2) Next, we convert the reference trajectory in terms of  $\alpha_d(t), \phi_{4d}(t)$  to a state reference trajectory in terms of  $\alpha_d(t), \phi_{3d}(t)$ .

## Reference trajectory in terms of $\alpha_d(t), \phi_{4d}(t)$ .

To construct the reference kinematics in terms of  $(\alpha_d(t), \phi_{4d}(t))$ , we use the feasible position  $(X_1(t - \hat{\tau}(t)), Y_1(t - \hat{\tau}(t)))$  as the reference trailer rear point position  $(X_{4d}(t), Y_{4d}(t))$ , i.e.

$$X_{4d}(t) := X_1(t - \hat{\tau}(t)),$$
  

$$Y_{4d}(t) := Y_1(t - \hat{\tau}(t)).$$
(14)

Furthermore, the reference heading direction  $\theta_{4d}(t)$  of the rear point of the trailer is set equal to the orientation of the front wheel at time  $t - \hat{\tau}$ , i.e.

$$\theta_{4d}(t) := \theta_1(t - \hat{\tau}(t)), \tag{15}$$

and the desired path curvature  $\kappa_{4d}(t)$  of the path followed by the rear point is set equal to the path curvature  $\kappa_1(t) := \frac{\dot{\theta}_1(t)}{v_1(t)}$ of the tractor front wheel at time  $(t - \hat{\tau}(t))$ :

$$\kappa_{4d}(t) := \kappa_1(t - \hat{\tau}(t)). \tag{16}$$

Then, the desired orientation  $\delta_{2d}(t)$  of the reference trailer body can be expressed as follows:

$$\delta_{2d}(t) = 2 \arctan\left(\frac{\Delta Y}{\sqrt{\Delta X^2 + \Delta Y^2} + \Delta X}\right), \qquad (17)$$

in which

$$\Delta X := X_2(t) - X_{4d}(t), \Delta Y := Y_2(t) - Y_{4d}(t),$$
(18)

where the singularity at  $(\Delta X, \Delta Y) = (0, 0)$  cannot occur for solutions of (12), (13). Using (3), the reference articulation angle  $\alpha_d(t)$  can be expressed as:

$$\alpha_d(t) = \delta_1(t) - \delta_{2d}(t). \tag{19}$$

Then, based on (8), the desired rear trailer point heading angle  $\phi_{4d}(t)$  can be expressed as:

$$\phi_{4d}(t) = \theta_{4d}(t) - \delta_{2d}(t).$$
(20)

Below, the reference trajectory  $(\alpha_d(t), \phi_{4d}(t))$  will be transformed into a state reference trajectory in terms of

 $(\alpha_d(t), \phi_{3d}(t)).$ 

# State reference trajectory $\alpha_d(t), \phi_{3d}(t)$ .

It can be shown that the reference trajectory  $(\alpha_d(t), \phi_{4d}(t))$ can be reformulated into a state reference trajectory  $(\alpha_d(t), \phi_{3d}(t))$  with the resulting  $\phi_{3d}(t)$  described by

$$\phi_{3d}(t) = \arctan\left(\frac{w_3(t) - w_1(t)}{w_2(t)}\right),\tag{21}$$

for  $w_1(t)$ ,  $w_2(t)$  given by

$$w_{1}(t) := \frac{1}{l_{1}}c_{1}(t) - \frac{1}{l_{2}}c_{2}(t)\sin\alpha_{d}(t) + \frac{l_{\text{off}}}{l_{1}l_{2}}c_{1}(t)\cos\alpha_{d}(t),$$
  

$$w_{2}(t) := \frac{1}{l_{2}}c_{2}(t)\cos\alpha_{d}(t) + \frac{l_{\text{off}}}{l_{1}l_{2}}c_{1}(t)\sin\alpha_{d}(t)$$
(22)

with  $w_2(t) \neq 0$  for all t, which is enforced by Assumption 1.2 in Section III-B, and  $w_3(t)$  given by

$$w_{3}(t) := \frac{1}{l_{1}}c_{1}(t) - \frac{1}{l_{2} + l_{\rm ro}}c_{2}(t)c_{5d}(t) + \frac{l_{\rm off}}{l_{1}(l_{2} + l_{\rm ro})}c_{1}(t)c_{6d}(t)$$
(23)

Details on the derivation of (21)-(23) are omitted for the sake of brevity.

### **B.** Assumptions

In this section, we adopt two types of assumptions: 1) assumptions needed to avoid singularities in the description of the reference trailer kinematics, which will ultimately also be needed to avoid singularities in the (real) trailer kinematics, and 2) assumptions guaranteeing that a (physically realisable) solution to the (minimisation) problem in (12), (13) exists.

**Assumption 1** *The reference kinematics satisfy the following conditions:* 

- 1) There exists an  $\varepsilon_1 > 0$  such that  $\phi_{id}(t) \in [\varepsilon_1 \frac{\pi}{2}, \frac{\pi}{2} \varepsilon_1]$  for all t and for i = 3, 4.
- 2) There exists an  $\varepsilon_2 > 0$  such that  $w_2(t) \ge \varepsilon_2$  for all t.

Assumption 1.1 implies that the desired velocity of the trailer wheel (and of the trailer tail) are not allowed to be perpendicular to the trailer body, which are reasonable assumptions in practice. Assumption 1.2 implies that the (desired) longitudinal trailer velocity  $l_2w_2(t)$  (with  $w_2(t)$  given in (22)) should be strictly positive, which is again a reasonable assumption in practice (as the current control strategy is designed for normal forward driving conditions and not for backward driving).

Assumption 2 below is related to the path driven by the tractor front wheel and is adopted to avoid scenarios which are physically infeasible or in which no (or undesirable) solutions to the (minimisation) problem in (12), (13) exist.

**Assumption 2** *The path driven by the tractor front wheel satisfies the following conditions:* 

- 1) the curvature  $\kappa_1(t)$  of the path driven by the tractor front wheel should satisfy  $\kappa_1(t) < \frac{1}{l_2 + l_m}$  for all  $t \ge 0$ .
- 2) the tractor front wheel velocity should be strictly positive, i.e.  $v_1(t) > 0$  for all  $t \ge 0$ .
- 3) the total length of the trailer has to be longer than the total length of the tractor, i.e.  $l_2 + l_{ro} > l_1 + l_{off}$ .

Assumption 2.1 is adopted to avoid a.o. a scenario in which a physically infeasible trajectory would result in the sense that the trailer body would collide with that of the tractor. Assumption 2.2 avoids a scenario in which  $v_1(t)$  changes sign resulting in a non-smooth tractor front wheel path, which is infeasible for the trailer. Assumption 2.3 avoids geometric scenarios in which 1) the trailer is too short to be put back on the tractor front wheel path or 2) multiple feasible positions on the tractor front wheel path exist for the trailer wheel and the solution to the (minimisation) problem would be an undesirable one, see Fig. 6 for a geometric scenario in which Assumption 2.3 is violated.

## IV. TRACKING CONTROLLER DESIGN

In this section, a controller design will be proposed for the tractor-trailer robot, described by (9) - (11), (5), which exponentially stabilises the desired state trajectory  $(\alpha_d(t), \phi_{3d}(t))^1$ . Since the desired trajectory  $(\alpha_d(t), \phi_{3d}(t))$ may involve large angle trajectories (for realistic manoeuvres such as 90 degree turn or driving part of a roundabout), a small-angle model approximation is inappropriate. Hence, we will propose a nonlinear controller design based on feedback linearisation [26].

Consider the tractor-trailer dynamics in (9) - (11), (5) and the reference trailer kinematics as described in Section III-A. We propose the following control law:

$$u = \zeta_2(\mathbf{x}, t)^{-1} \left( -\zeta_1(\mathbf{x}, t) + v \right)$$
(24)

with

$$\begin{aligned} \zeta_{1}(\mathbf{x},t) &= \zeta_{11}(\mathbf{x},t) + \zeta_{12}(\mathbf{x},t) + \zeta_{13}(\mathbf{x},t), \\ \zeta_{2}(\mathbf{x},t) &= \frac{c_{2}(t)\cos\alpha}{l_{2}\cos^{2}\phi_{3}} + \frac{c_{1}(t)l_{\text{off}}\sin\alpha}{l_{1}l_{2}\cos^{2}\phi_{3}}, \\ v &= -k_{1}\left(\alpha - \alpha_{d}(t)\right) - k_{2}\left(\dot{\alpha} - \dot{\alpha}_{d}(t)\right), \end{aligned}$$
(25)

in which the controller gains  $k_1, k_2 > 0$  and

$$\zeta_{11}(\mathbf{x},t) = c_7(t) - c_8(t)c_3(\mathbf{x}) + \frac{\iota_{\text{off}}}{l_2}c_7(t)c_4(\mathbf{x}),$$
  

$$\zeta_{12}(\mathbf{x},t) = -\frac{1}{l_2}c_2(t)c_4(\mathbf{x}) - \frac{l_{\text{off}}}{l_1l_2}c_1(t)c_3(\mathbf{x}),$$
  

$$\zeta_{13}(\mathbf{x},t) = -\frac{c_2(t)\cos\alpha}{l_2\cos^2\phi_3} - \frac{c_1(t)l_{\text{off}}\sin\alpha}{l_1l_2\cos^2\phi_3}$$
(26)

<sup>1</sup>See [25] for background information on tracking control in a mobile robotic context.



Fig. 6. Situation in which Assumption 2.3 is violated.

with  $c_1(t), c_2(t), c_3(\mathbf{x})$  and  $c_4(\mathbf{x})$  as in (5) and

$$c_{7}(t) := \frac{1}{l_{1}} \left( \dot{v}_{1}(t) \sin(\phi_{1}(t)) + v_{1}(t) \dot{\phi}_{1}(t) \cos \phi_{1}(t) \right),$$
  

$$c_{8}(t) := \frac{1}{l_{2}} \left( \dot{v}_{1}(t) \cos \phi_{1}(t) - v_{1}(t) \dot{\phi}_{1}(t) \sin \phi_{1}(t) \right).$$
(27)

Theorem 1 below shows that this control law (locally) exponentially stabilises the desired trajectory  $(\alpha_d(t), \phi_{3d}(t))$  and, hence, solves the tracking problem formulated in Section III.

Theorem 1: Consider the tractor-trailer dynamics in (9)-(11), (5) and the reference trailer kinematics in Section III-A. Adopt Assumptions 1 and 2 and consider the controller in (24)-(27). Then, the desired trajectory  $(\alpha_d(t), \phi_{3d}(t))$  is a (locally) exponentially stable solution of the closed-loop system (9)-(11), (5), (24)-(27).

**Proof:** Consider the dynamics in (9) - (11), (5), and the reference trailer kinematics described in Section III-A, for which a physically feasible solution exist by the adoption of Assumptions 1 and 2. We pursue a feedback linearisation approach towards stabilising controller design for control input  $u = \phi_3$  and, in doing so, we choose the following output function  $h(\mathbf{x}, t) := \alpha - \alpha_d(t)$ . It can be shown that the relative degree of this output equals two if

$$v_1(t)\cos\phi_1(t)\cos\alpha + v_1(t)\frac{l_{\text{off}}}{l_1}\sin\phi_1(t)\sin\alpha \neq 0.$$
 (28)

We will address later how the satisfaction of (28) is guaranteed. Next, we employ the following time-varying coordinate transformation:  $z_1 = \alpha - \alpha_d(t)$ ,  $z_2 = \dot{\alpha} - \dot{\alpha}_d(t)$ . The system dynamics in these new coordinates with the feedback linearising control law as in (24) and (25) yields the following linearised dynamics:

$$z_1 = z_2,$$
  
 $\dot{z}_2 = v.$  (29)

The stabilising control law v as in (25) then exponentially stabilises the origin of the dynamics in (29). Note that the convergence of  $(z_1, z_2)$  to the origin implies that  $(\alpha(t), \dot{\alpha}(t))$ converges to  $(\alpha_d(t), \dot{\alpha}_d(t))$ , and hence  $\phi_3(t)$  converges to  $\phi_{3d}(t)$ . Therefore, exponential stability of the origin of  $(z_1, z_2)$  implies exponential stability of the desired trajectory  $(\alpha, \phi_3) = (\alpha_d(t), \phi_{3d}(t))$ .

In order to avoid singularities in the control law in (24)-(27), we require that  $\phi_3(t) \in (-\frac{\pi}{2}, \frac{\pi}{2})$  for all  $t \ge 0$  and that the condition in (28) is satisfied for all  $t \ge 0$ . Using the fact that (0,0) is an exponentially stable equilibrium point of the  $(z_1, z_2)$ -dynamics, we have that for any  $\varepsilon_z > 0$  there exists a  $\delta_z > 0$  such that  $||(z_1(t), z_2(t))^T|| \le \varepsilon_z$  for all  $t \ge 0$  if  $||(z_1(0), z_2(0))^T|| \le \delta_z$ . Now, using 1) the continuity of the coordinate transformation to  $(z_1, z_2)$ -coordinates and 2) the continuity of the expression for  $\phi_3$  in terms of  $\alpha$  and  $\dot{\alpha}$ :

$$\phi_{3} = \arctan\left(\frac{l_{2}\dot{\alpha} - \frac{l_{2}}{l_{1}}c_{1}(t) + c_{2}(t)\sin\alpha - \frac{l_{\text{off}}}{l_{1}}c_{1}(t)\cos\alpha}{c_{2}(t)\cos\alpha - \frac{l_{\text{off}}}{l_{1}}c_{1}(t)\sin\alpha}\right)$$
(30)

under the condition in (28), which guarantees that the denominator in (30) is non-zero, we have that for any  $\varepsilon_x > 0$  there exists a  $\delta_x > 0$  such that  $\|(\alpha(t) - \alpha_d(t), \phi_3(t) - \phi_{3d}(t))^T\| \le \varepsilon_x$  for all  $t \ge 0$  if  $\|(\alpha(0) - \alpha_d(0), \phi_3(0) - \phi_{3d}(0))^T\| \le \delta_x$ . By choosing  $\delta_x$  small enough, we can ensure that Assumption 1 implies that  $\phi_3(t) \in (-\frac{\pi}{2}, \frac{\pi}{2})$  for all  $t \ge 0$  and that Assumption 1.2 implies that indeed the condition in (28) is satisfied for all  $t \ge 0$ .

Based on Theorem 1, we can conclude that the tracking problem posed in Section III is solved by the controller (24)-(27), implying that the trailer rear point indeed follows the path driven by the tractor front wheel.

# V. A SIMULATION CASE STUDY

A simulation study of a 270 degree turning manoeuvre (three quarters of a roundabout) is performed to validate the effectiveness of the proposed control strategy. The dimensions of the tractor-trailer (robot) in Figure 1 are given by  $l_1 = 0.209$ ,  $l_{\text{off}} = 0.050$ ,  $l_2 = 0.312$  and  $l_{\text{ro}} = 0$ .

## A. The 270 degree turning manoeuvre

A 270 degree turning manoeuvre is considered that consists of three phases: driving a straight line section, performing a 270 degree turn to the left with a tractor front wheel turning radius of R = 0.4 m, and driving a straight line section again. During the transition between these phases, transient cornering occurs, i.e.  $\dot{\phi}_3 \neq 0$  rad/s. Furthermore, in this type of manoeuvre, also steady-state cornering ( $\dot{\phi}_3 = 0$  rad/s) can be observed. Therefore, this type of manoeuvre is representative for the cornering behaviour observed in many other turning manoeuvres, e.g. a 90 degree turn or a U-turn. In addition, we introduce an initial offset on the articulation angle:  $\alpha(t = 0) = 0.3$  rad. Such initialisation allows us to observe the transient convergence of the trailer rear point from  $\alpha(t = 0) = 0.3$  rad towards  $\alpha_d = 0$  rad.

Fig. 7 displays the driver inputs ensuring that the tractor front wheel performs the described 270 degree turning manoeuvre. Initially, the tractor-trailer accelerates up to a forward velocity  $v_1(t)$  of 0.2 m/s and maintains this velocity during the remainder of the manoeuvre. At t = 5 s, the front wheel starts steering for  $\frac{1.5\pi R}{v_1} = 9.4$  s in order to perform the 270 degree turn. Finally, a straight line section is driven again from t = 14.4 s onward. Note that for the purpose of the control strategy the driver inputs are considered to be a priori unknown and hence, the reference trailer kinematics, see Section III-A, are calculated in an online fashion.

# B. Simulation results of a 270 degree turning manoeuvre

The controller gain settings  $(k_1, k_2)$  are tuned such that the convergence rate towards the desired trajectory is maximised without violating the following additional performance criteria:

1) No overshoot on the articulation angle is allowed. Overshoot could cause dangerous situations (such as those induced by transient tail swing or rearward amplification) and increases the space required by the tractor-trailer robot in transients. 2) The maximum absolute steering angular velocity  $(\phi_3)$  is limited in order to respect actuator constraints.

Using this tuning procedure, we obtain the controller gains  $k_1 = k_2 = 4$ , see [24] for more details.

The effectiveness of the control strategy, with the controller design in (24)-(27), is validated in simulations. The results, illustrated in Fig. 8, show that the trailer rear point indeed converges to the path of the tractor front wheel and



Fig. 7. Driver inputs for the 270 degree turning manoeuvre.



Fig. 8. Closed-loop tractor-trailer response for controller (24)-(27).



Fig. 9. Tractor-trailer response without trailer steering.



Fig. 10. Swept path for the cases with and without active trailer steering.

follows the latter throughout the entire cornering manoeuvre, while avoiding tail swing at all times. Fig. 9 shows the response of the tractor-trailer robot without trailer wheel steering. In both simulations, the hitch point travels the same path according to (2). Fig. 10 shows that the maximum swept path width is indeed significantly reduced by using controlled trailer steering compared to the case without trailer wheel steering. In fact, a reduction of 63% in maximum swept path width is obtained during this turning manoeuvre. Note that the swept path analysis is performed with the bicycle-like representation and, therefore, the width of the vehicle is not taken into account in this analysis.

Experimental evidence of the effectiveness of the proposed approach for active trailer steering, for manoeuvers other than studied in this paper, can be found in [24].

## VI. CONCLUSIONS

A control strategy for active trailer axle steering is proposed to reduce the swept path width of a tractor-trailer (robot) combination. The control goal is formulated such that the rear point of the trailer tracks the path driven by the tractor front wheel. In this way, a significant reduction in swept path width is obtained and tail swing can be prevented. The resulting path following problem is reformulated as a state tracking problem for a kinematic model of the tractor-trailer. Next, a controller design based on feedback linearisation has been proposed that solves this tracking problem. The effectiveness of the proposed approach has been demonstrated in a representative simulation case study.

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