Model-Based Robust Control of Directional Drilling Systems

Niek Antonius Henricus Kremers, Emmanuel Detournay, and Nathan van de Wouw

Abstract-To enable access to unconventional reservoirs of oil and (shale-) gas, geothermal energy and minerals, complex curved boreholes need to be drilled in the earth's crust. Directional drilling techniques, incorporating down-hole robotic actuation systems called rotary steerable systems, are used to generate these curved boreholes. In practice, however, boreholes drilled with such systems often show instability-induced borehole spiraling, which negatively affects the borehole quality and increases drag losses while drilling. As a basis for controller synthesis, we present a directional drilling model in terms of delay differential equations. Next, the problem of curved well-bore generation is formulated as a tracking problem and a model-based robust control strategy is developed, solving this tracking problem while guaranteeing the prevention of borehole spiraling. The effectiveness of the proposed approach is illustrated by representative case studies for the generation of curved boreholes.

Index Terms—Delay differential equations (DDEs), directional drilling, output feedback, robust control.

I. INTRODUCTION

E NHANCED access to underground energy resources (such as oil and gas) requires drilling complex curved boreholes. Drill rigs, as schematically shown in Fig. 1, are employed to generate boreholes targeting resource locations in the earth's crust. As part of these so-called directional drilling rigs, down-hole robotic systems, known as rotary steerable systems (RSS), are used to drill such curved boreholes. This paper focuses on a push-the-bit RSS, which steers the borehole propagation by exerting a force on the borehole using extendable pads. The propagation direction of a borehole is typically controlled using three control loops. The inner control loop regulates the force delivered by the RSS actuator on the borehole. The second control loop controls the propagation

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N. A. H. Kremers is with the Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven 5612 AZ, The Netherlands (e-mail: nah.kremers@gmail.com).

E. Detournay is with the Department of Civil, Environmental and Geo-Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: detou001@umn.edu).

N. van de Wouw is with the Department Mechanical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands, the Department of Civil, Environmental and Geo-Engineering, University of Minnesota, Minneapolis, MN 55455 USA, and also with the Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands (e-mail: n.v.d.wouw@tue.nl).

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Rig Drillstring BHA RSS Bit

Fig. 1. Schematic of a directional drilling system [1].

Fig. 2. Down-hole measurements showing undesired spiraling in the borehole wall [3]. direction of the borehole by giving force commands to the RSS. Finally, the outer control loop often involves a human

the RSS. Finally, the outer control loop often involves a human operator in the loop that generates desired borehole trajectories on the basis of complex data sets involving the target destination, rock layer geometries and properties, and so on. This paper focuses on the development of novel control strategies for the second control loop.

Although RSSs are extensively used in drilling practice, it is known from experimental evidence that their usage can induce borehole oscillations (see [2]–[4]) and Fig. 2. These oscillations in the borehole geometry are undesirable as such oscillations: 1) endanger borehole stability; 2) induce increased drag while drilling (thereby reducing drilling efficiency); 3) reduce target accuracy; 4) make it more difficult to insert the borehole casing to prepare for production; and 5) reduce the rate-of-penetration (i.e., the speed of the drilling process). Current control techniques seem unable to prevent this so-called borehole spiraling. In this paper, we aim to

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develop a model-based robust controller synthesis approach, which enables the drilling of complex borehole geometries while preventing borehole spiraling.

The entire drill string can be considered as an elastic rod, constrained inside the borehole (see Fig. 1). Torque-and-drag models exist that consider the entire drill string including the complexity of all (unilateral) contacts of the string with the borehole [5], [6]. However, such detailed modeling of the drill string is not needed in the scope of describing the directional propagation of the borehole and would make the resulting model unnecessarily complex. For this reason, only the lower part of the drill string, the bottom hole assembly (BHA), is considered. The effect of the upper part of the drill string is taken into account as a force boundary condition for the lower part. To prevent buckling of the BHA and to influence the directional tendencies of the borehole propagation, several stabilizers are included in the BHA (see Fig. 1), which are in constant contact with the borehole wall. Due to the fact that the BHA (with stabilizers) has to fit inside the borehole that has already been drilled, the existing borehole geometry affects the directional tendencies of the borehole propagation in a spatially delayed manner.

There exist many numerical directional drilling models [7]-[14], in which a finite-element model of the BHA is used to compute the forces and moments acting on the drill bit. These forces and moments are then related to the propagation of the bit into the rock, using a bit-rock interaction law. In these models, the evolution of the borehole is propagated in a stepwise fashion by assuming that the forces and moments are constant during such a step. These models do not lead to a closed-form dynamic model description for borehole propagation in directional drilling. To design a model-based controller for the directional drilling system, a closed-form dynamic model is needed to predict the bit trajectory given RSS actuation commands.

Such a closed-form model, which considers actuation based on the eccentricity of an adjustable stabilizer, was first developed by Neubert and Heisig [15], [16]. This model, based on a linear beam description of the BHA, kinematics of the bit, and bit/rock interface laws, leads to a set of (nonlinear) delay differential equations (DDEs) governing the borehole propagation. The next model development is due to Downton [17], who formulated the borehole propagation equations for a class of directional drilling systems (either completely rigid or flexible with the addition of an equivalent spring) and analyzed the stability of the resulting (linear) DDE. The papers of Detournay and Perneder [1], [18]–[21] and Downton and Ignova [22] treat the BHA as an Euler-Bernoulli beam, similarly to [15], and consider a force actuation of a push-the-bit RSS. Although these two models describe the same physics, their formulation is different. The PD model [1], [18]–[21] is based on an angular description of the BHA and borehole tendencies and can thus naturally be used for describing boreholes undergoing large rotations, while the directional propagation of the borehole in the formulation of Downton [17] and Downton and Ignova [22] is described using the lateral displacement of the BHA with respect to an initial configuration, which needs to be regularly updated.

Recently, it has been shown, using field data, that the PD model can predict the effect of borehole spiraling [4], which further motivates to adopt this type of modeling approach as a basis for controller design in this paper.

Several works exist on the topic of the control of borehole propagation using an RSS. In [23]-[25], controllers are developed based on empirical models of the borehole propagation process in which a direct link between the force applied by the RSS and the curvature of the borehole is assumed. This approach ignores (physically relevant) transient behavior of the borehole propagation, which is essential in preventing borehole spiraling. In [26], a state-space model for borehole propagation is derived and on the basis of this model, a controller is designed. However, the essential delay nature of the borehole propagation dynamics is not captured in this model. In [27], an \mathcal{L}_1 adaptive controller is designed, based on the directional drilling model in [17]. In this approach, it assumed that the inclination of the borehole at the bit can be measured directly, which is generally not the case (as also acknowledged in [22]). The same restrictive assumption is made in most of the works above. This assumption is invalid in practice, since an inclination sensor cannot be placed at (close to) the bit. In addition, even if available in practice, such an inclination sensor would measure the local inclination of the deformed BHA at the bit, which is not necessarily equal to the borehole inclination at the bit. Indeed, the bit is often tilted with respect to the borehole due to sideways cutting by the bit gauge.

The main contribution of this paper is the development of a synthesis strategy for robust model-based output feedback controllers for directional drilling systems. Underlying, more detailed contributions are as follows. First, this synthesis method is based on a closed-form model description of the borehole propagation, unlike some of the works mentioned above. In addition, this is the first controller synthesis strategy based on the PD model, which captures the essential, physically relevant, behavior of a directional drilling system. Second, the resulting controllers can be used to drill complex borehole geometries. Unlike existing control methods, the goal of the controller synthesis method is to design a controller that also reduces borehole spiraling and prevents oscillations in the transient closed-loop response (both of which are detrimental to borehole quality). Third, we assume that only local inclination measurements of the deformed BHA are available at discrete locations other than the bit. For this reason, an observer-based output feedback control strategy is developed. The observer is used to reconstruct the borehole inclination at the bit based on the sensor measurements on local BHA inclinations. Fourth, in existing works, the effect of parametric uncertainties on the stability and performance of the closed-loop drilling system is not investigated, [27] being an exception. Here, the robustness of the proposed controller for uncertainties in the weight-on-bit is verified explicitly as this drilling parameter is subject to significant uncertainty in practice. Finally, the influence of (quasi-)constant disturbances, such as the influence of gravitational effects, on the accuracy of borehole propagation is reduced by dedicated designs of both the controller and observer.



Fig. 3. Three components of the model and their interaction [18].

The remainder of this paper is organized as follows. Section II presents a state-space description of the directional drilling model in terms of a DDE describing the borehole propagation. In Section III, it is shown that (open-loop) control techniques, still used in practice, are not suitable for drilling complex boreholes, in particular due to the occurrence of borehole spiraling, and the influence of parametric model uncertainty and disturbances in directional drilling performance. Next, the problem of accurate borehole propagation while avoiding borehole spiraling is formulated as a robust tracking control problem with transient performance specifications. In Section IV, we propose an observerbased control strategy which is able to solve this robust tracking control problem. In addition, a robustness analysis of the controller for parametric uncertainty is performed. In Section V, simulation results illustrating the effectiveness of the proposed control strategy for two case studies of desired curved borehole geometries are shown. Finally, the conclusion is given in Section VI.

II. DIRECTIONAL DRILLING MODEL

This paper only considers the directional propagation of the borehole in a vertical plane, i.e., the borehole is assumed to have a constant azimuth. Clearly, borehole spiraling is a 3-D phenomenon; still, the results on the prevention of borehole spiraling in the (2-D) inclination dynamics by means of control, as proposed in this paper, serve as a stepping stone to developing control strategies for the full-fledged 3-D problem. Moreover, these results also directly bear relevance for the case of zero bit walk.

The directional drilling model used here is build upon the work in [1] and [18]–[21] and generally consists of three components, as shown in Fig. 3 and concisely discussed in Section II-A: 1) a model for the deformation of the BHA inside the borehole, 2) the bit–rock interface laws, and 3) kinematics relating the bit motion to the propagation of the borehole geometry.

A. Borehole Evolution Equations

To arrive at a closed-form dynamic model for borehole propagation, four main modeling assumptions are adopted.

- 1) The bit–rock interface laws are rate independent [28]. For this reason, the borehole propagation is described as a function of the scaled distance drilled $\xi = (L/\lambda_1)$, where *L* is the length of the borehole and λ_1 is the distance between the bit and the first stabilizer (see Fig. 4).
- 2) BHA vibrational effects are ignored as such vibrational phenomena take place on a much faster time scale than



Fig. 4. Overview of BHA inside the borehole (after [18]).

the time scale relevant to borehole propagation. The directional drilling model relies therefore on forces and penetrations that are averaged over several revolutions of the bit.

- 3) It is assumed that the stabilizers of the BHA are in constant contact with the borehole wall.
- The deformations of the BHA inside the borehole are presumed to be small. Hence, the BHA can be modeled statically as an Euler–Bernoulli beam.

The borehole is described by its inclination $\overline{\Theta}(S)$ with respect to the vertical for $S \in [0, L]$, where S is a curvilinear coordinate measured along the borehole (see Fig. 4). The inclination of the deformed BHA, at a particular value of the distance drilled L, is given as $\theta(L, s)$ for $s \in [0, L_{BHA}]$, where s is a curvilinear coordinate representing the distance measured from the drill bit (with s = 0 at the bit) and L_{BHA} is the length of the BHA. We introduce the dimensionless length drilled $\xi = (L/\lambda_1)$, which is the independent variable with respect to which the dynamics of the borehole propagation will be formulated. We also define $\Theta(\xi) := \overline{\Theta}(\xi \lambda_1)$ and $\theta(\xi, s) := \overline{\theta}(\xi \lambda_1, s)$ being, respectively, the inclination of the borehole at the bit and the inclination of the BHA, depending on the dimensionless drilled distance. Note that, in general, the inclination of the borehole at the bit $\Theta := \Theta(\xi)$ is not necessarily equal to the inclination of the BHA at the bit $\hat{\theta} := \theta(\xi, 0)$, due to lateral cutting of the bit.

1) BHA Modeling: The BHA is modeled as a linear beam with a constant bending stiffness EI, which leads to the following equation for the deflection of the BHA $EI(\partial^3\theta/\partial s^3) = w \sin\langle\Theta\rangle_1$, where w is the distributed gravity force of the beam, assumed to have a constant relative direction along the BHA. Herein, $\langle\Theta\rangle_1$ denotes the average inclinations of the BHA between the bit and the first stabilizer. Similarly, the average inclinations of the BHA between the (i - 1)th and *i*th stabilizer are denoted by $\langle\Theta\rangle_i$, i = 2, ..., n, where n is the number of stabilizers. The stabilizers are modeled as hinges, which only apply a force on the BHA perpendicular to its axis. Given first, the RSS force F_{rss} induced by the down-hole robotic RSS actuator, second, the average inclinations of the BHA sections, and third, the inclination of the bit $\hat{\theta}$, the deflection profile of the BHA can be solved for by splitting up the BHA up into n + 1 linear beam sections, which are connected to each using the appropriate constraints [1]. The inclination of the BHA within these sections is given as

$$\begin{aligned} \theta(\xi, s) &= \theta_i(\xi, s), & \text{for } s \in [s_{i-1}, s_i] \text{ for } i \in [2, \dots, n] \\ \theta(\xi, s) &= \theta_1(\xi, s), & \text{for } s \in [\Lambda \lambda_1, s_1] \\ \theta(\xi, s) &= \theta_0(\xi, s), & \text{for } s \in [s_0, \Lambda \lambda_1], \end{aligned}$$

where $\Lambda \lambda_1$ is the distance between the bit and the RSS actuator, $s_i := \sum_{j=1}^i \lambda_j$, i = 1, ..., n, and λ_i , i = 2, ..., n, is the distance between the (i - 1)th and *i*th stabilizer. The functions $\theta_i(\xi, s)$, characterizing the inclination within the *i*th BHA sections, are parameterized as

$$\theta_i(\xi, s) = \mathcal{A}_{i3}s^3 + \mathcal{A}_{i2}s^2 + \mathcal{A}_{i1}s + \mathcal{A}_{i0}$$
(2)

where \mathcal{A}_{i0} , \mathcal{A}_{i1} , \mathcal{A}_{i2} , and \mathcal{A}_{i3} are functions of the RSS force F_{rss} , the bit inclination $\hat{\theta}$ and the average inclinations $\langle \Theta \rangle_i$ and the BHA configuration (which hence implicitly depend on ξ). The bending moment on the bit \hat{M} and the side force on the bit \hat{F}_2 are then given as: $\hat{M} = -\text{EI}(\partial \theta_0 / \partial s)|_{s=0}, \hat{F}_2 = -\text{EI}(\partial^2 \theta_0 / \partial s^2)|_{s=0}$. This results in the following (scaled) expressions for the force and moment acting on the bit:

$$\frac{\hat{F}_2}{F_*} = \mathcal{F}_b(\hat{\theta} - \langle \Theta \rangle_1) + \mathcal{F}_w \Upsilon \sin \langle \Theta \rangle_1 + \mathcal{F}_r \Gamma + \sum_{i=1}^{n-1} \mathcal{F}_i(\langle \Theta \rangle_i - \langle \Theta \rangle_{i+1})$$
(3)

$$\frac{\hat{M}}{F_*\lambda_1} = \mathcal{M}_b(\hat{\theta} - \langle \Theta \rangle_1) + \mathcal{M}_w \Upsilon \sin \langle \Theta \rangle_1 + \mathcal{M}_r \Gamma + \sum_{i=1}^{n-1} \mathcal{M}_i(\langle \Theta \rangle_i - \langle \Theta \rangle_{i+1}), \qquad (4)$$

where $F_* := (3\text{EI}/\lambda_1^2)$, the scaled RSS force $\Gamma := (F_{\text{rss}}/F_*)$ and the scaled measure of the BHA weight is given as $\Upsilon := (w\lambda_1/F_*)$. The factors \mathcal{F} and \mathcal{M} in (3) and (4) (with appropriate indices) denote the dimensionless coefficients of influence, which only depend on the specific configuration of the BHA. These coefficients are given for a two-stabilizer BHA (also used in Section V) in Appendix A.

2) Kinematic Relationships Relating Bit Motion to Borehole Geometry: The motion of the bit is described using three penetration variables that reflect the penetration of the bit into the rock per revolution of the bit: 1) the axial penetration d_1 ; 2) the lateral penetration d_2 ; and 3) the angular penetration φ . Noting that the axial penetration is much larger than the lateral penetration, the bit tilt $\psi := \hat{\theta} - \hat{\Theta}$ can be expressed as

$$\psi = \hat{\theta} - \hat{\Theta} = \operatorname{atan}\left(-\frac{d_2}{d_1}\right) \approx -\frac{d_2}{d_1}.$$
(5)

3) Bit/Rock Interface Laws: The link between the axial force \hat{F}_1 , lateral force \hat{F}_2 and moment \hat{M} acting on the bit and the penetration variables d_1 , d_2 , and φ is defined by the bit–rock interface laws. For a planar borehole, a general linear

form for the bit–rock interface laws exists, which is derived from the bilinear law for a single cutter/rock interaction [29]:

$$F_1 = -G_1 - H_1 d_1 (6)$$

$$\hat{F}_2 = -H_2 d_2 \tag{7}$$

$$\hat{M} = -H_0\varphi,\tag{8}$$

where the coefficients H_1 , H_2 , and H_0 depend on the properties of the bit and the strength of the rock. The coefficient G_1 represents a part of the axial force, which is transmitted to the wearflats of the bit and does not contribute to the penetration of the bit into the rock (i.e., does not contribute to rock cutting). Let us now introduce the scaled active weight-on-bit

$$\Pi := -\frac{\hat{F}_1 + G_1}{F_*}.$$
(9)

This parameter, which ultimately determines the rate-ofpenetration of the bit according to (6), is here assumed to be constant. Equations (6)–(9) can be combined with the kinematics of the bit (5) to yield expressions for the force and moment acting on the bit:

$$\frac{\hat{F}_2}{F_*} = \eta \Pi (\hat{\theta} - \hat{\Theta}) \tag{10}$$

$$\frac{M}{F_*\lambda_1} = -\chi \Pi \hat{\theta}',\tag{11}$$

where η and χ are, respectively, the lateral and angular steering resistance defined as $\eta := (H_2/H_1)$ and $\chi := (H_3/H_1\lambda_1^2)$. These parameters indicate the relative difficulty of imposing lateral or angular penetration of the bit with respect to axial penetration. Typical values of η for a polycrystalline diamond compact bit range from 5 for bits with a short active gauge to 100 for bits with a long passive gauge [30]. The angular steering resistance χ is generally one or two orders of magnitude smaller than η .

From now on, for notational simplicity the hat character will no longer be explicitly used, i.e., we write the inclination of the borehole at the bit as $\Theta(\xi)$ (instead of as $\hat{\Theta}(\xi)$). By combining (3), (4), (10), and (11), in addition to making the assumption that the active weight-on-bit Π is constant, the following evolution equation for the borehole inclination in terms of a single differential equation can be obtained:

$$\chi \Pi \Theta' = \mathcal{M}_{b} \left(\langle \Theta \rangle_{1} - \Theta \right) + \frac{\chi}{\eta} \mathcal{F}_{b} \left(\Theta - \Theta_{1} \right) \\ + \sum_{i=1}^{n-1} \left(\frac{\mathcal{F}_{b} \mathcal{M}_{i} - \mathcal{F}_{i} \mathcal{M}_{b} - \mathcal{M}_{i} \eta \Pi}{\eta \Pi} \right) \left(\langle \Theta \rangle_{i} - \langle \Theta \rangle_{i+1} \right) \\ - \frac{\chi}{\eta} \sum_{i=1}^{n-1} \mathcal{F}_{i} \left(\frac{\Theta_{i-1} - \Theta_{i}}{\varkappa_{i}} - \frac{\Theta_{i} - \Theta_{i+1}}{\varkappa_{i+1}} \right) \\ + \frac{\mathcal{F}_{b} \mathcal{M}_{w} - \mathcal{F}_{w} \mathcal{M}_{b} - \mathcal{M}_{w} \eta \Pi}{\eta \Pi} \Upsilon \sin \langle \Theta \rangle_{1} \\ - \frac{\chi}{\eta} \mathcal{F}_{w} \Upsilon (\Theta - \Theta_{1}) \cos \langle \Theta \rangle_{1} \\ + \frac{\mathcal{F}_{b} \mathcal{M}_{r} - \mathcal{F}_{r} \mathcal{M}_{b} - \mathcal{M}_{r} \eta \Pi}{\eta \Pi} \Gamma - \frac{\chi}{\eta} \mathcal{F}_{r} \Gamma'.$$
(12)

In (12), the inclination at the delayed location of the *i*th stabilizer is given as $\Theta_i := \Theta(\xi_i)$, with $\xi_i := \xi - \sum_{j=1}^i \varkappa_j$ for i = 1, 2, ..., n and $\xi_0 := \xi$. Herein, \varkappa_i is the dimensionless length of the *i*th BHA segment ($\varkappa_i := \lambda_i/\lambda_1$). The average inclination of the *i*th BHA segment $\langle \Theta \rangle_i$ is given as

$$\langle \Theta \rangle_i := \frac{1}{\varkappa_i} \int_{\zeta_{i-1}}^{\zeta_i} \Theta(\sigma) d\sigma, \qquad (13)$$

which induces terms with distributed delays in (12). The model (12) contains two nonlinear (trigonometric) terms related to the influence of gravity on the BHA. The first term $((\mathcal{F}_b\mathcal{M}_w - \mathcal{F}_w\mathcal{M}_b - \mathcal{M}_w\eta\Pi)/(\eta\Pi))\Upsilon$ sin $\langle\Theta\rangle_1$ depends on the average inclination of the first section of the BHA. Since the average inclinations only change slowly with the distance drilled, this term can be seen as a quasi-constant disturbance $\mathcal{W} := \Upsilon \sin \langle \Theta \rangle_1$. Moreover, it can be shown that the second term is small with respect to the other terms in the DDE [1].

Remark 1: We care to stress that the composite parameter $\eta\Pi$ is key in the stability of the system dynamics (12), see [4]. $\eta\Pi$ can be interpreted as a coefficient of proportionality between the (scaled) lateral force on the bit \hat{F}_2/F_* and the bit tilt ψ (see (10)). This composite parameter depends on both the active weight-on-bit Π and the lateral steering resistance η of the bit, both of which are subject to significant levels of uncertainty in practice. The active weight-on-bit Π depends on the hookload (the axial force applied to the drill string at the surface), which is generally well known. However, Π ultimately depends on the downward force on the BHA on the interface with the drill-string above, which, in turn, depends on the (uncertain) drag forces caused by contacts between the drill-string and the borehole wall. This interaction is particularly complex in highly curved boreholes typical for directional drilling. Moreover, Π depends on the current wear-state of the bit (which is also unknown while drilling), since a part of the axial force is transmitted by the wearflats of the cutters on the bit. Also the lateral steering resistance η is subject to uncertainty, for example, due to bit wear. Therefore, we consider the composite parameter $\eta \Pi$ as an essential, though uncertain, one in the remainder of this paper.

B. State-Space Model Formulation

In this section, the DDE (12) with distributed delays is transformed into the following first-order state-space formulation with point-wise delays by considering the borehole inclination at the bit Θ and the average inclinations $\langle \Theta \rangle_i$ as states:

$$x'(\xi) = A_0 x(\xi) + \sum_{i=1}^{n} A_i x(\xi_i) + B_0 \Gamma + B_1 \Gamma' + B_2 \mathcal{W}$$
(14)

where the state vector is given as $x := [\Theta, \langle \Theta \rangle_1, \dots, \langle \Theta \rangle_n]^T$. Note once more that \mathcal{W} is a gravity related, quasi-constant disturbance (to be treated as unknown in the remainder of this paper). The expressions for the derivatives $\langle \Theta \rangle'_i$ in (14) are obtained by differentiation of (13) with respect to ξ . As a consequence, *n* additional poles at zero have been added to (14). It can be shown that these poles are inconsequential for the stability of the original DDE in (12), and hence can be ignored in the stability analysis of (14) (and closed-loop variants of (14) to be discussed in Section IV) [31].

Let us now introduce the following notational conventions. Given a function $x(\xi) : \mathbb{R} \to \mathbb{R}^{n+1}$ and $\xi \in \mathbb{R}$, we define $x_d(\xi)(\cdot)$ such that $x_d(\xi)(\sigma) := x(\xi + \sigma)$ for all $\sigma \in [-\varkappa_{tot}, 0]$ with $\varkappa_{tot} = \sum_{i=1}^{n} \varkappa_i$. Therefore, $x_d(\xi) \in \mathcal{C}([-\varkappa_{tot}, 0], \mathbb{R}^{n+1})$, where \mathcal{C} is the Banach space of continuous functions mapping the interval $[-\varkappa_{tot}, 0]$ to \mathbb{R}^{n+1} .

Without loss of generality, from now on, a BHA model with two stabilizers is considered. For this two-stabilizer BHA, the system matrices of the model in (14) are given in Appendix A.

In practice, the states of the DDE model in (14) cannot be measured. Only sensors measurements of the local BHA inclination are available (not of the borehole inclination). We consider a representative scenario in which one inclination sensor at the RSS and another inclination sensor at a location between the first and second stabilizer is available. The (measured) output vector is then given by $y_m = [\theta_0(\xi, \lambda_1 \Lambda), \theta_2(\xi, s_m)]^T$, where $s_m \in [\lambda_1, \lambda_2]$ defines the location of the second inclination sensor and the functions $\theta_i(\xi, s)$ are given in (2). Since the coefficients $\mathcal{A}_{i1}, \mathcal{A}_{i2}, \mathcal{A}_{i3}$, and \mathcal{A}_{i4} of the functions $\theta_i(\xi, s)$ depend linearly on $\hat{\theta}, \Gamma, \langle \Theta \rangle_1$, $\langle \Theta \rangle_2$, and Υ , the functions $\theta_i(\xi, s)$ in (2) can also be written as

$$\theta_{i}(\xi, s) = O_{i1}(s)\theta + O_{i2}(s)\Gamma + O_{i3}(s)\langle\Theta\rangle_{1} + O_{i4}(s)\langle\Theta\rangle_{2} + O_{i5}(s)\mathcal{W}$$
(15)

where the coefficients O_{i1} , O_{i2} , O_{i3} , O_{i4} , and O_{i5} only depend on the configuration of the BHA and the location of the inclination sensor. We exploit an expression for the bit inclination θ (to be used in (15)) that can be obtained by combining (3) with (10):

$$\theta = \frac{1}{\eta \Pi - \mathcal{F}_b} \left(\eta \Pi \Theta - \mathcal{F}_b \langle \Theta \rangle_1 + F_w \mathcal{W} + \mathcal{F}_r \Gamma + \sum_{i=1}^{n-1} \mathcal{F}_i (\langle \Theta \rangle_1 - \langle \Theta \rangle_2) \right).$$
(16)

Now an expression for the measured output y_m as a function of the state vector x and the input force Γ can be obtained by substituting (16) into (15):

$$y_m = Cx + D\Gamma + E\mathcal{W},\tag{17}$$

where the matrices *C*, *D*, and *E* depend on the configuration of the BHA and the location of the inclination sensors. We note that the influence of the gravity term W on the measured output y_m is generally very small. The total state-space model is now given by (14) and (17), with state *x*, input Γ , and measured output y_m .

III. CONTROL PROBLEM FORMULATION

The main purpose of a directional drilling system is to drill a borehole with some predetermined geometry. It is common practice that, this desired borehole geometry consists of multiple constant curvature segments. In addition, it is often assumed that applying a constant RSS force leads to a constant curvature borehole. Under this assumption, a borehole consisting of multiple constant curvature segments can be

TABLE I GEOMETRY OF THE BENCHMARK BHA

λ_1	\varkappa_1	λ_2	\varkappa_2	Λ	Ir	Or
3.66 m	1	6.10 m	$\frac{2}{1.2}$	$\frac{1}{6}$	0.053 m	0.086 m

drilled by applying the correct constant RSS force in each segment. In Section III-A, we analyze the open-loop dynamics to gain insight into the deficiencies of such open-loop actuation strategies of the directional drilling process and to support the problem formulation for closed-loop control. In particular, it is shown that such an open-loop actuation method may lead to (directional) instability, undesired borehole oscillations, and lacks robustness to parameter uncertainties and disturbances. In Section III-B, the control problem is formulated as a robust tracking problem with transient performance specifications.

A. Analysis of the Open-Loop Dynamics

It can be verified whether a constant RSS force results in a constant curvature borehole in the PD model by analyzing the steady-state solutions of the open-loop dynamics [21] (see also [17]). The stability properties of such steady-state solutions can be assessed by analyzing the poles of (14). Due to the delay nature of (14), there exists an infinite number of system poles. The poles p_k , for $k \in [1, \infty]$, of the open-loop system are computed by solving the characteristic roots of the following equation [32]:

 $\det \Delta(p) = 0, \tag{18}$

where

$$\Delta(p) = pI - A_0 - \sum_{i=1}^{n} A_i e^{-p \sum_{j=1}^{i} \varkappa_j}$$
(19)

is the characteristic matrix. Although for the DDE under consideration there exist an infinite number of poles, it can be shown that $\lim_{k\to\infty} |p_k| \to +\infty$, and $\lim_{k\to\infty} \mathcal{R}(p_k) \to -\infty$ since the DDE (14) is of retarded type [32]. Consequently, there only exist a finite number of poles in a vertical strip of the complex plane. The poles of the DDE can be calculated using the MATLAB toolbox described in [33]. This toolbox allows the computation of the finite number of poles with real value exceeding some bounded value α (i.e., for $\Re(\lambda_k) \ge \alpha$).

Both the model in Section II and the controller synthesis approach in Section IV are fully generic and can deal with scenarios with any number of stabilizers. It has been shown in [4] that the dynamics of directional drilling system is dominated by the effects induced by the first two stabilizers and the inclusion of additional stabilizers in the model does not significantly change the dynamics. This fact motivates the consideration of a benchmark study with two stabilizers. In particular, we consider a particular BHA with two stabilizers and characterized by geometric properties listed in Table I. (The inner and outer radius Ir and Or of the BHA are needed to compute the weight w and the second moment of area Iused in the scaling of the forces and moments.) It is assumed the entire BHA is made of steel with $E = 2e11 \text{ N/m}^2$ and density $\rho = 7800 \text{ kg/m}^3$.

The poles of the corresponding benchmark BHA model are shown in Fig. 5 for $\Pi = 0.0093$, $\eta = 30$, and $\chi = 0.1$

Fig. 6. Characterization of the open-loop response for $(\chi/\eta) = (0.1/30) =$

Minimum phase respon

0.0033 depending on the (uncertain) composite parameter $\eta \Pi$.

 $\Pi = 0.0093$, $\eta = 30$, and $\chi = 0.1$ (i.e., $\eta \Pi = 0.279$).

 $\eta \Pi$

0.1125

Fig. 5.

Poles with $\Re(\lambda) > -1.6$ for the benchmark BHA model for

 $\eta \Pi|_{\underset{N=0}{\operatorname{Non}}}$

8.31

minimum phase response

(i.e., $\eta \Pi = 0.279$). There are n + 1 = 3 poles located at zero. As previously mentioned, n = 2 of these poles are caused by the state-space description (14) of the system (in particular by the inclusion of the averaged BHA segment inclinations as states) and do not affect the stability of the system as described by (12). The additional single pole at zero indicates that the system does not exhibit a globally asymptotically stable constant inclination equilibrium solution. This pole acts as an integrator for the system, and leads to the conclusion that for this benchmark system, a constant RSS force results in a constant curvature borehole in steady state. As long as no poles are located in the open right-half complex plane this constant curvature solution is asymptotically stable.

Note that the open-loop poles depend on the parameters of the system, which are not known exactly in practice. In particular, it has been explained in Remark 1 that the composite parameter $\eta \Pi$ is subject to uncertainty for different reasons, while being an essential one from a stability analysis point of view. A characterization of the three possible open-loop responses for varying values of $\eta \Pi$ is shown in Fig. 6. In general, the open-loop poles of the system move further into the right-half complex plane for lower values of $\eta \Pi$. There exists a value $\eta \Pi |^{\star} = 0.1125$ for which a (pair of) pole(s) crosses the imaginary axis for the first time (see Fig. 6). For values $\eta \Pi < \eta \Pi |^*$, a constant RSS force result in growing borehole oscillations. In practice, these instability-induced oscillations may lead to limit cycling when unmodeled nonlinear behavior (such a nonlinear bit-rock characteristics and unilateral contact between the BHA and borehole) is taken into account in the directional drilling model. For values of $\eta \Pi |^{\star} < \eta \Pi < \eta \Pi |_{nmp}$, the right-most open-loop poles are located in the left-half complex plane (apart from the pole(s) at zero). Due to this, oscillations in the response are damped out and a constant borehole



Effect of step in RSS force on borehole inclination 0.16 $\eta \Pi = 0.106$ 0.14 ηΠ = 0.181 $n\Pi = 0.334$ 0.12 $\eta \Pi = 0.557$ $n\Pi = 2.783$ 0.1 $\eta \Pi = 11.13^{\circ}$ 0.08 $\Theta[rad]$ 0.06 0.04 0.02 C -0.02 0 5 10 15 $\xi[-]$

Fig. 7. Simulation results for a step in Γ depending on the (uncertain) composite parameter $\eta \Pi$ (and for $(\chi/\eta) = (0.1/30) = 0.0033$).

curvature is obtained. Finally, for high values of the weight-on-bit ($\eta \Pi > \eta \Pi |_{nmp}$), a non-minimum phase response is obtained, which corresponds to a change of sign of the coefficient b_0 at $\eta \Pi = \eta \Pi |_{nmp}$. Physically, this is caused by the fact that for high values of $\eta \Pi$, the penetration direction of the bit is dominated by the influence of the deformation of the BHA (caused by the RSS force) on the bit inclination, and not by the lateral cutting action of the bit θ [1].

To investigate the dynamic response to a constant RSS force, simulations of the response of system (12) to a step in the RSS force have been performed. The initial condition corresponds to a vertical borehole, characterized by $\Theta_d(0) = 0$. The RSS force $F_{\rm rss}$ jumps from 0 to 10000 N at $\xi = 1$ (this corresponds to $\Gamma = 0.0062$). Fig. 7 shows the inclination response to this step force for several values of the composite parameter $\eta \Pi$. The three different types of openloop responses, mentioned above, can clearly be observed in the simulation results displayed in Fig. 7. Moreover, it can be observed that at $\xi = 1$ there is a kink in the borehole (jump in the inclination). This kink is caused by the fact that the instantaneously applied RSS force leads to a lateral force at the bit, which in turn causes lateral penetration of the bit into the rock. As mentioned before, such borehole spiraling/kinking is undesirable, since it negatively affects borehole quality.

Besides the above problems in the transient response, this open-loop actuation technique is not robust for parameter estimation errors and disturbances acting on the system. The RSS force needed to drill a borehole with some desired curvature can be calculated based on the model. Due to parameter estimation errors in the model (e.g., of the weight-on-bit Π), this force is generally incorrect and a slightly different steady-state borehole curvature is induced. As a result, the borehole inclination. Continual (human-in-the-loop) steering adjustments would be needed to correct for such undesired behavior. In addition, the influence of gravity can be seen as a quasi-constant disturbance on the system. This disturbance also causes a mismatch between desired and obtained steady-state borehole curvature.

B. Robust Tracking Problem Formulation

The main goal of directional drilling is the generation of a borehole with some particular geometry. In terms of the model in (14), this objective can be formulated as a tracking problem. More specifically, we aim to track some inclination reference trajectory $y_r(\xi) = \Theta_r(\xi)$ for $\xi \in [-\varkappa_{tot}, \infty]$. Note that defining $y_r(\xi)$ immediately results in the state reference trajectory $x_r(\xi)$ being fully defined, since the average inclination state references $\langle \Theta \rangle_{r1}, \langle \Theta \rangle_{r2}$ can be obtained by integration over Θ_r . Hence, the problem can be formulated as a state tracking problem. We will assume that $\Theta_r(\xi)$ is continuously differentiable, which is reasonable in practice as it avoids curvature discontinuities. Given the directional drilling model for the two-stabilizer BHA:

$$x'(\xi) = A_0 x(\xi) + A_1 x(\xi_1) + A_2 x(\xi_2) + B_0 \Gamma + B_1 \Gamma' + B \mathcal{W}$$
(20)

$$y_m(\xi) = Cx + D\Gamma + E\mathcal{W},\tag{21}$$

an output-feedback controller needs to be designed such that the control input $\Gamma(\xi)$ renders $x_r(\xi)$ the globally asymptotically stable solution of the closed-loop system.

Besides the above formulation of the control goal as a state tracking problem, certain additional objectives are induced by the fact that the spiraling behavior in the borehole, which is observed in practice, needs to be reduced/eliminated. Such borehole spiraling can either be caused by poles in the righthalf complex plane (i.e., instability, which is avoided if the state tracking problem is solved) or by weakly damped poles (i.e., undesired transient behavior). For this reason, we focus on appropriate placement of the poles of the tracking error dynamics (with the tracking error defined as $e := x - x_r$), to reduce/eliminate borehole spiraling. Another control objective is related to the fact that not all model parameters are known exactly. The controller, which is designed based on estimates of the model parameters, is required to be robust for these parameter uncertainties, such that the tracking error dynamics remain stable under such uncertainties. In this paper, we focus on uncertainties in the active weight-on-bit Π , since, as mentioned before, this parameter is both a key drilling process parameter and subject to relatively high levels of uncertainty. The last control objective is related to the fact that there exist several sources of force disturbances. We focus on the effect of the gravity-induced forces here. Although strictly speaking, the gravity term in (12) acts as a nonlinear term in the DDE, it can be seen as a slowly varying quasi-constant disturbance force (see (14)), since the average inclination $\langle \Theta \rangle_1$, on which the gravitational term in the directional drilling model depends, only changes slowly with the distance drilled ξ . We aim to reduce the influence of this disturbance on the steady-state inclination error $e_{\Theta} := \Theta - \Theta_r$.

IV. CONTROLLER SYNTHESIS APPROACH

In this section, we propose an observer-based output feedback control strategy that solves the tracking problem stated above, while also accounting for the additional performance aspects mentioned in Section III. In Section IV-A, we introduce the controller structure and the resulting tracking error dynamics is presented in Section IV-B. An optimization-based method is employed in Section IV-C for the parametric tuning of the controller. Finally, in Section IV-D, we analyze the robustness of the control strategy in the presence of parameter uncertainties in the weight-on-bit.

A. Controller Structure

The directional drilling model for the case of a two-stabilizer BHA (20) contains terms in both the RSS force Γ and its derivative Γ' . In support of the controller design, we introduce a control input *u* defined as $Bu(\xi) = B_0\Gamma(\xi) + B_1\Gamma'(\xi)$, with $B = [1, 0, 0]^T$, which is well defined by the grace of the specific structure of *B* and that of B_0 and B_1 (see Appendix B). Substituting this expression for *u* in (20) results in the following DDE model:

$$x'(\xi) = A_0 x(\xi) + A_1 x(\xi_1) + A_2 x(\xi_2) + B u(\xi) + B_2 \mathcal{W}.$$
 (22)

Note that the state-dependent influence of gravity is modeled here as a quasi-constant disturbance B_2W . The input force Γ , supplied to the RSS actuator, now satisfies the following differential equation:

$$\Gamma' = -\frac{b_0}{b_1}\Gamma + \frac{1}{b_1}u$$
(23)

where b_0 and b_1 are the only nonzero terms in, respectively, i.e., $B_0 = [b_0, 0, 0]^T$ and $B_1 = [b_1, 0, 0]^T$ (see Appendix B). For the filter (23) to be asymptotically stable, b_0/b_1 needs to be positive (this holds for minimum phase situations, which are under consideration here). The control input u is decomposed as $u(\xi) = v(\xi) + u_r(\xi)$, where u_r is a model-based feedforward signal and v is the control input used for feedback. The influence of the quasi-constant gravity disturbance will not be taken into account in the feedforward. Hence, the feedforward input u_r is obtained by solving

$$Bu_r(\xi) = x'_r(\xi) - A_0 x_r(\xi) - A_1 x_r(\xi_1) - A_2 x_r(\xi_2).$$
(24)

To obtain $u_r(\xi)$ satisfying (24), it suffices to ensure the satisfaction of the first scalar equation of the vector (24), since by definition if $\Theta(\xi) = \Theta_r(\xi)$ then $\langle \Theta \rangle_1(\xi) = \langle \Theta \rangle_{1r}(\xi)$ and $\langle \Theta \rangle_2(\xi) = \langle \Theta \rangle_{2r}(\xi)$. The solution for u_r that solves (24) is thus given as

$$u_r(\xi) = B^T(x'_r(\xi) - A_0 x_r(\xi) - A_1 x_r(\xi_1) - A_2 x_r(\xi_2)).$$
(25)

Next, we propose a feedback control strategy (for v) that consists of a model-based observer with integral action in combination with a dynamic state-feedback controller including a low-pass filter and integral action. The observer provides a state estimate \hat{x} to be employed by the state-feedback controller. The following observer is proposed:

$$\hat{x}' = A_0 \hat{x}(\xi) + A_1 \hat{x}(\xi_1) + A_2 \hat{x}(\xi_2) + L(y_m - \hat{y}_m) + B(q + u)
q' = \zeta_o [l_1, l_2](y_m - \hat{y}_m)
\hat{y}_m = C \hat{x} + D\Gamma,$$
(26)

where a hat is now used to denote an estimate.

The observer gain matrix L is defined as

$$L = \begin{bmatrix} l_1 & l_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$
 (27)

This structure in *L* ensures that the estimates of the average inclinations $\langle \hat{\Theta} \rangle_1$ and $\langle \hat{\Theta} \rangle_2$ are simply given by integration of the inclination estimate $\hat{\Theta}$ (i.e., are obtained purely by model-based prediction). This choice reduces the number of observer parameters that needs to be determined. The strength of the weak integral action is determined by the parameter ζ_o . This integral action is included to ensure convergence of the observer error to zero in the presence of the gravitational disturbance acting on the system (which is not modeled in (26)). The dynamic state-feedback controller is designed as follows:

$$z'_{1} = \zeta[k_{1}, 0, 0](\hat{x} - x_{r})$$

$$z'_{2} = -\gamma z_{2} + \gamma (z_{1} + K(\hat{x} - x_{r}))$$

$$v = z_{2},$$
(28)

where the control gain matrix $K = [k_1, k_2, k_3]$. Weak integral action is included to remove the influence of constant disturbances (such as gravitational effects) on the steady-state tracking error. The cut-off frequency of the weak integral action is determined by the control parameter ζ . Moreover, the controller contains a low-pass filter to reduce oscillations in the transient borehole inclination response (to further reduce borehole oscillations). The controller parameter γ determines the cut-off frequency of the low-pass filter. Note that indeed the observer-controller combination (26) and (28) (and in particular the inclusion of the low-pass and integrals actions) aims at addressing the additional performance aspects discussed in Section III: 1) robustness to (quasi-)constant disturbances and 2) improving the transient response in an attempt to reduce undesired borehole oscillations.

Remark 2: The dynamics of the directional drilling model exhibits three essential length scales: 1) short range, $\xi = O(10^{-1})$; 2) medium range, $\xi = O(10^0-10^1)$; and 3) long range, $\xi = O(10^2-10^3)$ [1]. The structural design of the proposed above controller targets these different length scales in the following way:

- Short Range: The low-pass filtering properties in the feedback controller (28) ensure that excitation of the short-range (boundary layer) dynamics is avoided, therewith avoiding severe borehole kinking.
- Medium Range: The design of both the observer in (26) and the controller in (28) aims at the stabilization of the medium-range dynamics (through design of the gains K and L to be addressed in Section IV-C), therewith guaranteeing the absence of instabilities related to borehole oscillations (as observed in the open-loop behavior in Fig. 7).
- 3) *Long Range:* The inclusion of integral action in both the observer in (26) and the controller in (28) ensure the long-range tracking error to be zero in the presence of (e.g., gravity related) disturbances.

Remark 3: The model in (14) and the proposed controller are formulated with ξ (associated with the length of the borehole) as an independent variable. To implement such controllers in practice, information on the rate of penetration would be needed to translate control action as a function of borehole length to control action as a function of time.

B. Error Dynamics

In support of the optimization-based tuning of the controller and observer parameters, we now construct the tracking and observer error dynamics. We define the tracking error as $e := x - x_r$ and the observer error as $\delta := x - \hat{x}$. Applying the control decomposition $u = u_r + v$ and observer-based controller (26), (28) to (20), we obtain the following closedloop error dynamics:

Note that the quasi-constant disturbance of the gravity is neglected in (34), since the error dynamics are constructed to support the design of a stabilizing controller. The effect of gravity-induced disturbances on the closed-loop response is further investigated in Section V. The origin (corresponding to zero tracking and observer errors) is an asymptotically equilibrium point of (34) if all poles of these closed-loop dynamics are located in the open left-half complex plane.¹ Note that due to the block-diagonal structure of the system matrices in (34), the separation principle holds. This means that the poles of the closed-loop system are given by the union of the poles of the tracking error subsystem, with state $[e, z_1, z_2]^T$ and input $[\delta, q]^T$, and the observer error subsystem, with state $[\delta, q]^T$. This allows the controller parameters K, ζ , and γ and the observer parameters l_1, l_2 , and ζ_0 to be designed separately, such that the poles of the respective subsystems are properly placed in the left-half complex plane and stabilization is achieved.

Remark 4: Besides the stabilization of the desired inclination trajectory as pursued here, in practice also the Cartesian position of the borehole should be controlled, which is typically performed by a human directional driller on the basis of complex data sets involving the target destination, rock layer geometries and properties, etc.

C. Optimization-Based Tuning for Stabilization

To guarantee asymptotic stability of the closed-loop system, the controller and observer parameters need to be tuned such that the poles of the error dynamics are located in the left-half

complex plane. An optimization-based approach is taken to design such stabilizing controller and observer parameters. Herein, we aim to minimize the real part of the right-most pole of both the closed-loop tracking error dynamics and the observer error dynamics. By the grace of the separation principle, mentioned above, if these right-most poles have negative real part, then the closed-loop system is globally asymptotically stable. Moreover, this eigenvalue- and optimization-based approach toward controller/observer tuning also aims to improve transient performance to limit transient borehole oscillations. Here, we only design the controller gain matrix K and the observer gains l_1, l_2 using such an optimization-based tuning approach. The parameters ζ , γ , and ζ_o , corresponding to the properties of the dynamic low-pass and integrating filters, are designed a priori as: 1) these effectuate the desired controller properties at different length scales (see Remark 2) and 2) this reduces the number of parameters that need to be optimized.

The optimization problem to be solved to design K and L can now be described as follows:

$$\min_{K} \alpha_{c}(K)$$

$$\min_{L} \alpha_{o}(L), \qquad (30)$$

where the objective functions $\alpha_c(K)$ and $\alpha_o(L)$ are given as

$$\alpha_{c}(K) = \sup_{i \in [1, 2, ..., \infty]} \{ \Re(p_{Ci}(K) \}$$
(31)

$$\alpha_o(L) = \sup_{i \in [1,2,...,\infty]} \{ \Re(p_{Oi}(L) \}.$$
 (32)

Herein, $p_{Ci}(K)$ indicates poles relating to the tracking error dynamics

$$\begin{bmatrix} e'\\ z'_1\\ z'_2 \end{bmatrix} = \begin{bmatrix} A_0 & 0 & B\\ \zeta[k_1, 0, 0] & 0 & 0\\ \gamma & K & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} e(\zeta)\\ z_1(\zeta)\\ z_2(\zeta) \end{bmatrix} + \begin{bmatrix} A_1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e(\zeta_1)\\ z_1(\zeta_1)\\ z_2(\zeta_1) \end{bmatrix} + \begin{bmatrix} A_2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e(\zeta_2)\\ z_1(\zeta_2)\\ z_2(\zeta_2) \end{bmatrix}$$
(33)

for controller gain K, and $p_{Oi}(L)$ indicates the poles corresponding to the observer error dynamics

$$\begin{bmatrix} \delta' \\ q' \end{bmatrix} = \begin{bmatrix} A_0 - LC & -B \\ \zeta_0[l_1, l_2]C & 0 \end{bmatrix} \begin{bmatrix} \delta(\xi) \\ q(\xi) \end{bmatrix} + \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(\xi_1) \\ q(\xi_1) \end{bmatrix} + \begin{bmatrix} A_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(\xi_2) \\ q(\xi_2) \end{bmatrix}$$
(34)

for observer gain L.

Since the separation principle holds, the synthesis method consists of separately minimizing the objective function $\alpha_c(K)$, describing the right-most pole of the tracking error dynamics and minimizing the objective function $\alpha_o(L)$, describing the right-most pole of the observer error dynamics. This allows us to place the poles of the observer further

¹As mentioned before, the poles at zero, caused by the inclusion of the average inclination states in the state-space description, can be disregarded for the stability analysis of the system.

into left-half complex plane, such that the observer error δ converges to zero faster than the inclination error *e*.

Remark 5: As shown in [32], the objective functions in (31) and (32) are typically nonsmooth. In Section V, we employ a hybrid algorithm for nonsmooth optimization [34], [35] combining: 1) the Broyden–Fletcher–Goldfarb–Shanno (BFGS) method, a quasi-Newton algorithm, with an inexact line search algorithm based on weak Wolfe conditions, and 2) a particular gradient bundle method, called gradient sampling, near nonsmooth manifolds of the objective function. Moreover, stopping criteria $\alpha_c(K) \leq \alpha_{c,\max} < 0$ (for the design of K) and $\alpha_o(L) \leq \alpha_{o,\max} < 0$ (for the design of L) are used to terminate the optimization-based gain tuning algorithm, where $\alpha_{c,\max}$ and $\alpha_{o,\max}$ reflect certain transient performance specifications.

D. Robustness Analysis

Above, we assumed that the system matrices A_0 , A_1 , A_2 , B, C, and D are known exactly. In practice, these matrices are subject to uncertainty due to parameter estimation errors. Here, in particular, parametric uncertainty in the essential composite parameter $\eta \Pi$ (see Remark 1) is considered. We note that the parameters related to the dimensions of the BHA are typically accurately known.

The actual value of $\eta\Pi$ is given as $\eta\Pi = \overline{\eta\Pi} + \eta\Pi$, where $\overline{\eta\Pi}$ is the nominal value and $\eta\Pi$ reflects the uncertainty. Now, the controller and observer are based on the nominal system matrices \overline{A}_0 , \overline{A}_1 , \overline{A}_2 , \overline{B} , \overline{C} , and \overline{D} corresponding to the nominal value $\eta\Pi$. Next, we will analyze the consequences of the inclusion of such parametric model uncertainty on two levels. First, we assess closed-loop stability in the presence of such uncertainty. Robust stability in the presence of uncertainty on the weight-on-bit is essential in practice to guarantee stable directional drilling operations without borehole oscillations for a wide range of operational conditions. Second, the feed forward input u_r is calculated based on the nominal system matrices, and is thus prone to errors in the presence of uncertainty. We will investigate the influence of such feed-forward error on the closed-loop performance in Section V.

Note that the input-transformation in (23) depends on the uncertain weight-on-bit (as the parameters b_0 and b_1 do). As a consequence, the corresponding (nominal) filter

$$\Gamma' = -\frac{b_0}{\bar{b}_1}\Gamma + \frac{1}{\bar{b}_1}u$$
(35)

needs to be included in an uncertain closed-loop model description for the purpose of robustness analysis. The resulting uncertain closed-loop dynamics can be written as follows:

$$z'(\xi) = Q_0 z(\xi) + Q_1 z(\xi_1) + Q_2 z(\xi_2) + U(\xi), \quad (36)$$

where the state vector $z = [e, \Gamma, z_1, z_2, \delta, q]^T$ (see [31] for details). Note that the influence of all gravity-related effects are neglected from this analysis (as these do not affect stability). The system matrices Q_1 , Q_2 , and Q_3 and the additional perturbation U are given in Appendix C. It can be observed that the block-triangular structure of the closed-loop system matrices is destroyed which invalidates the separation principle



Fig. 8. Comparison of open-loop poles, tracking error dynamics poles, and observer error dynamics poles.

for the purpose of robustness analysis. In addition, due to the perturbation terms U the observer error δ no longer converges to zero exactly, since it influences the integrator state q.

The robust stability of a specific controller/observer combination can be analyzed by computing the right-most pole of (36) for an uncertainty range of $\eta \Pi \in [-\eta \Pi_{max}, \eta \Pi_{max}]$, where $\eta \Pi_{max}$ is the maximum allowable uncertainty on the composite parameter $\eta \Pi$. If the right-most pole of the system is strictly negative for this entire uncertainty range, robust stability is guaranteed.

V. ILLUSTRATIVE BENCHMARK STUDIES

In this section, several simulations studies are performed to confirm that the proposed control strategy solves the tracking (borehole generation) problem at hand. We begin by validating the control strategy under nominal conditions while drilling a simple curved borehole. Second, we demonstrate that the proposed control strategy is able to drill a complex borehole, consisting of multiple constant curvature sections, while under the influence of a parameter uncertainty in the weight-on-bit.

Let us design a controller and an observer for the benchmark BHA system introduced in Section III using the design strategy proposed in Section IV. The nominal value for $\eta\Pi$ is taken as $\eta\Pi = 0.279$. The controller parameters are chosen as $\gamma = 0.8$ and $\zeta = 0.5$; the optimization for designing the controller gain K is performed until $\alpha_c(K) < -0.5$. The observer objective function is optimized such that $\alpha_o(L) < -0.8$ with $\zeta_o = 0.45$. Optimization of both objective functions results in the controller gain matrix K = [-2565, -742, 161] and observer gains $l_1 = 133$ and $l_2 = 2998$. Fig. 8 shows a comparison of the open-loop poles, the poles of the tracking error subsystem and the poles of the observer error subsystem. It can be observed that the optimization procedure successfully places the poles of the closed-loop system according to the two optimization criteria above.

The performance of this controller/observer combination under nominal conditions can be verified by the means of a simulation of the closed-loop system. In the following simulations, the nonlinear influence of gravity on the system (12) is taken into account. Here, we consider the situation in which we transition from a constant inclination borehole into a constant curvature borehole. The inclination



Fig. 9. Inclination and observer error for the observer with weak integrating action and the dynamic controller. In this simulation, the gravitational influence on measured output is not taken into account in the model.

reference trajectory is given as

$$\Theta_r(\xi) = \frac{\pi}{4}, \quad \text{for } \xi \in [-(\varkappa_1 + \varkappa_2), 5]
\Theta_r(\xi) = \frac{\pi}{4} + 0.01(\xi - 5), \quad \text{for } \xi \in [5, \infty]. \quad (37)$$

The initial borehole inclination is given as $\Theta_d(0) = (\pi/4) +$ 0.01 and the initial inclination estimate is given as $\Theta_d(0) =$ $(\pi/4)$ (i.e., there exist both an initial inclination tracking error and an initial observer error). Let us first investigate the response while neglecting the small gravitational influence on the measured output (17) (the nonlinear gravitational influence on the system is still taken into account). Fig. 9 shows the inclination tracking error $e_{\Theta} := \Theta - \Theta_r$ and the observer inclination error $\delta_{\Theta} := \Theta - \hat{\Theta}$. Note that the nonlinear influence of the gravity disturbance on the system (12) has been successfully compensated for by the integral action in both the controller and observer and as a result the steadystate errors converge to zero. Although the observer error contains some fast transients, the inclination error remains smooth due to low-pass filter included in the controller. Due to the fact that a nominal value for $\eta \Pi$ is considered here, the required feedforward signal u_r is known exactly. As a consequence, no error is induced at the transition between the constant inclination and constant curvature borehole at $\xi = 5$. In other words, during a transition from a constant inclination section to a constant curvature section, no borehole oscillations are induced (as would likely be the case with conventional constant RSS force actuation). Fig. 10 shows the scaled RSS force Γ (i.e., control effort) corresponding to the response in Fig. 9. Note that the peak value of approximately $\Gamma = 0.008$ corresponds to a (real) RSS force of approximately $F_{\rm rss} = 13\,000$ N, which is feasible in practice. We also note that the transient control action hardly overshoots the steady-state RSS force needed to generate the steady-state curvature of 0.0027 1/m. Moreover, Fig. 10 shows that indeed no step in the RSS force is employed at the transition between the straight borehole section and the constant curvature section (at $\xi = 5$), thereby avoiding borehole kinking at such a transition. Still, this transition is clearly visible in the control action. Fig. 11 shows the results of a simulation in which the gravitational effect on the measured output is taken into



Fig. 10. Scaled RSS force Γ for the observer with weak integrating action and the dynamic controller (corresponding to Fig. 9).



Fig. 11. Inclination and observer error for the observer with weak integrating action and the dynamic controller. In this simulation, the gravitational influence on measured output is taken into account in the model.

account (see (17)). Due to this gravitational influence on the measured output, which was not taken into account in the observer design, only a small steady-state error occurs ($<10^{-3}$ rad). By comparing Figs. 9 and 11, we conclude, however, that the transient behavior is hardly affected the influence of gravity on the measured output.

Let us now consider the case in which we aim to drill a borehole consisting of multiple constant curvature sections. The transition from an initially vertical borehole into a horizontal borehole is made over 800 m (this corresponds with a scaled distance of $\xi = 800/\lambda_1 \approx 219$). The inclination reference is shown in Fig. 12 and is given as

$$\begin{split} \Theta_r(\xi) &= 0, & \text{for } \xi \in \left[0, \frac{100}{\lambda_1}\right] \\ \Theta_r(\xi) &= c_1 \left(\xi - \frac{100}{\lambda_1}\right), & \text{for } \xi \in \left[\frac{100}{\lambda_1}, \frac{200}{\lambda_1}\right] \\ \Theta_r(\xi) &= c_1 \frac{100}{\lambda_1}, & \text{for } \xi \in \left[\frac{200}{\lambda_1}, \frac{400}{\lambda_1}\right] \\ \Theta_r(\xi) &= c_1 \frac{100}{\lambda_1} + c_2 \left(\xi - \frac{400}{\lambda_1}\right), & \text{for } \xi \in \left[\frac{400}{\lambda_1}, \frac{800}{\lambda_1}\right] \\ \Theta_r(\xi) &= \frac{\pi}{2}, & \text{for } \xi \in \left[\frac{800}{\lambda_1}, \frac{1000}{\lambda_1}\right], \end{split}$$

where $c_1 = 0.0128$ and $c_2 = 0.0112$.



Fig. 12. Reference trajectory of the complex borehole.



Fig. 13. Real value of the right-most pole of the uncertain closed-loop system for varying parameter uncertainty in the composite parameter $\eta \Pi$.

In this case study, the influence of a parameter uncertainty $\tilde{\eta}\Pi = 0.25 \overline{\eta}\Pi$ is also investigated. Due to this parameter uncertainty, the perfect feedforward is no longer known. In addition, the robust stability of the system needs to be verified. Fig. 13 shows the real value of the right-most pole of the uncertain closed-loop system (36) for a parameter uncertainty up to $\tilde{\eta}\Pi_{max} = 0.5 \overline{\eta}\Pi$. It can be observed that this controller/observer combination possesses excellent robustness properties. A parameter uncertainty of 25% in $\eta\Pi$ only results in a slight movement of the right-most pole and thus it can be concluded that the controller/observer combination is robustly stable.

For this case study, the initial borehole inclination is given as $\Theta_d(0) = 0.01$ and the initial inclination estimate is given as $\hat{\Theta}_d(0) = 0$. Fig. 14 shows the inclination tracking error and observer inclination error. It can be observed that the uncertain closed-loop system is indeed still asymptotically stable. Note that at the transition between the multiple constant curvature sections a peak in the tracking error occurs due to the feedforward errors induced by the 25% parameter error in the weight-on-bit. However, this error quickly damps out and remains small due to the integral action. In addition, due to the perturbation term U in the uncertain closed-loop dynamics (see (36)), the steady-state error does not converge to zero exactly but to a small, practically acceptable error



Fig. 14. Inclination error and observer error while drilling the complex borehole with an observer-based output feedback controller. Left: transient response. Right: response to curvature transitions.



Fig. 15. Scaled RSS force Γ for complex borehole (corresponding to Fig. 14).

level (especially given the relatively high level of uncertainty considered here). Finally, Fig. 15 shows the (scaled) RSS force Γ for this scenario. The difference between the RSS force levels need in constant inclination sections (dominated by the compensation of gravity) and constant curvature sections is clearly visible.

VI. CONCLUSION

In this paper, a model-based control strategy for directional drilling has been proposed. The problem of drilling complex curved boreholes using down-hole robotic systems has been formulated as a robust tracking problem. The benefits of the proposed control strategy are as follows. First, it guarantees the stable generation of complex curved boreholes without the occurrence of undesired borehole oscillations. Second, the observer-based controllers only need limited measurements of the inclination of the BHA. Third, the resulting closed-loop system is robust against both parameter uncertainties and (gravity-induced) perturbations. The effectiveness of the proposed observer-based output feedback control strategy, including its robustness properties, has been illustrated by case studies.

$$Q_{0} = \begin{bmatrix} A_{0} & B_{0} - B_{1} \frac{\bar{b_{0}}}{\bar{b_{1}}} & 0 & \frac{B_{1}}{\bar{b_{1}}} & 0 & 0 \\ 0 & -\frac{\bar{b_{0}}}{\bar{b_{1}}} & 0 & \frac{1}{\bar{b_{1}}} & 0 & 0 \\ \zeta[k_{1}, 0, 0] & 0 & 0 & 0 & -\zeta[k_{1}, 0, 0] & 0 \\ \gamma K & 0 & \gamma & -\gamma & -\gamma K & 0 \\ -dA_{0} + LdC & LdD - dB_{0} + dB_{1} \frac{\bar{b}_{0}}{\bar{b_{1}}} & 0 & -dB_{1} \frac{1}{\bar{b}_{1}} & \bar{A}_{0} - LC - LdC & -B \\ -\zeta_{o}[l_{1}, l_{2}]dC & -\zeta_{o}[l_{1}, l_{2}]dD & 0 & 0 & \zeta_{o}[l_{1}, l_{2}](C + dC) & 0 \end{bmatrix}$$

APPENDIX A COEFFICIENTS OF INFLUENCE

The coefficients of influence for a BHA with two stabilizers are given as

$$\begin{aligned} \mathcal{F}_{b} &= -\frac{6+4\varkappa_{2}}{3+4\varkappa_{2}} \\ \mathcal{F}_{w} &= \frac{6+10\varkappa_{2}-3\varkappa_{2}^{3}}{12+16\varkappa_{2}} \\ \mathcal{F}_{r} &= \frac{-3-4\varkappa_{2}+\Lambda^{2}(9+6\varkappa_{2})-2\Lambda^{3}(3+\varkappa_{2})}{3+4\varkappa_{2}} \\ \mathcal{F}_{1} &= \frac{6}{3+4\varkappa_{2}} \\ \mathcal{M}_{b} &= \frac{4(1+\varkappa_{2})}{3+4\varkappa_{2}} \\ \mathcal{M}_{w} &= \frac{-1-2\varkappa_{2}+\varkappa_{2}^{3}}{12+16\varkappa_{2}} \\ \mathcal{M}_{r} &= \frac{\Lambda(1-\Lambda)[3+4\varkappa_{2}-\Lambda(3+2\varkappa_{2})]}{3+4\varkappa_{2}} \\ \mathcal{M}_{1} &= -\frac{2}{3+4\varkappa_{2}}, \end{aligned}$$

where κ_i is the dimensionless length of the *i*th BHA section $(\kappa_i := (\lambda_i / \lambda_1), i = 1, 2)$.

APPENDIX B System Matrices for Two-Stabilizer BHA

$$A_0 = \frac{1}{\chi \Pi} \begin{bmatrix} -\mathcal{M}_b + \frac{\chi}{\eta} \left(\mathcal{F}_b - \frac{\mathcal{F}_1}{\varkappa_1} \right) & \mathcal{M}_b + E & -E \\ \frac{\chi \Pi}{\varkappa_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with $E = (\mathcal{F}_b \mathcal{M}_1 - \mathcal{F}_1 \mathcal{M}_b - \mathcal{M}_1 \eta \Pi) / (\eta \Pi)$

$$A_{1} = \frac{1}{\chi \Pi} \begin{bmatrix} \frac{\chi}{\eta} \left(\frac{\mathcal{F}_{1}}{\varkappa_{1}} + \frac{\mathcal{F}_{1}}{\varkappa_{2}} - \mathcal{F}_{b} \right) & 0 & 0 \\ -\frac{\chi \Pi}{\varkappa_{1}} & 0 & 0 \\ \frac{\chi \Pi}{\varkappa_{2}} & 0 & 0 \end{bmatrix}$$
$$A_{2} = \frac{1}{\chi \Pi} \begin{bmatrix} -\frac{\chi}{\eta} \frac{\mathcal{F}_{1}}{\varkappa_{2}} & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{\chi \Pi}{\varkappa_{2}} & 0 & 0 \end{bmatrix}$$

$$B_{0} = \frac{1}{\chi \Pi} \left[\frac{\mathcal{F}_{b} \mathcal{M}_{r} - \mathcal{F}_{r} \mathcal{M}_{b} - \mathcal{M}_{r} \eta \Pi}{\eta \Pi}, 0, 0 \right]^{T}$$
$$B_{1} = \frac{1}{\chi \Pi} \left[-\frac{\chi}{\eta} \mathcal{F}_{r}, 0, 0 \right]^{T}.$$

APPENDIX C System Matrices for Uncertain Closed-Loop Dynamics

 Q_0 is given by the matrix shown at the top of this page, and

for i = 1, 2

$$U = \begin{bmatrix} dBu_r(\xi) - dA_0x_r(\xi) - dA_1x_r(\xi_1) - dA_2x_r(\xi_2) \\ \frac{1}{\overline{b_1}}u_r(\xi) \\ 0 \\ -dA_0x_r(\xi) - dA_1x_r(\xi_1) - dA_2x_r(\xi_2) + LdCx_r(\xi) \\ -\zeta_o[l_1, l_2]dCx_r(\xi) \end{bmatrix}$$

where $dA_i = A_i - \overline{A}_i$, $dB_i = B_i - \overline{B}_i$, $dC = C - \overline{C}$, and $dD = D - \overline{D}$.

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Niek Antonius Henricus Kremers was born in Veghel, The Netherlands, in 1989. He received the B.Sc. and M.Sc. degrees in mechanical engineering from the Eindhoven University of Technology, Eindhoven, The Netherlands, in 2010 and 2013, respectively.

He is currently with 4PICO, Sint-Oedenrode, The Netherlands, a high-tech mechatronics company.



Emmanuel Detournay received the Mining Engineering degree from the University of Liège, Liège, Belgium, and the M.Sc. and Ph.D. degrees in geoengineering from the University of Minnesota, Minneapolis, MN, USA.

He joined the Department of Civil, Environmental, and Geo-Engineering, University of Minnesota, in 1993, where he is currently the Theodore W. Bennett Chair Professor of Mining Engineering and Rock Mechanics. Prior to his faculty position, he was a Senior Research Scientist with Schlumberger

Cambridge Research, Cambridge, U.K. His expertise is in petroleum geomechanics. His current research interests include drilling mechanics (bit/rock interaction, self-excited drilling vibrations, drillstring/borehole interaction, and directional drilling) and mechanics of fluid-driven fractures (asymptotic analysis, scaling, and numerical modeling).



Nathan van de Wouw was born in 1970. He received the M.Sc. (Hons.) and Ph.D. degrees in mechanical engineering from the Eindhoven University of Technology, Eindhoven, The Netherlands, in 1994 and 1999, respectively. He was with the Department of Mechanical Engineering, Eindhoven University of Technology, as an Assistant Professor/Associate Professor. He was with Philips Applied Technologies, Eindhoven, in 2000, and the Netherlands Organization for Applied Scientific Research,

Delft, The Netherlands, in 2001. He was a Visiting Professor with the University of California at Santa Barbara, Santa Barbara, CA, USA, in 2006/2007, the University of Melbourne, Melbourne, VIC, Australia, in 2009/2010, and the University of Minnesota, Minneapolis, MN, USA, in 2012 and 2013. He is currently an Adjunct Full Professor with the Delft University of Technology, Delft. He has authored a large number of journal and conference papers and the books entitled *Uniform Output Regulation of Nonlinear Systems: A Convergent Dynamics Approach* (Birkhauser, 2005) with A.V. Pavlov and H. Nijmeijer, and *Stability and Convergence of Mechanical Systems with Unilateral Constraints* (Springer-Verlag, 2008) with R. I. Leine. His current research interests include the analysis and control of nonlinear/hybrid systems, with applications to vehicular platooning, high-tech systems, resource exploration, and networked control systems.

Prof. van de Wouw is an Associate Editor of the journal *Automatica* and the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY.