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Model Order Reduction of Linear Sampled-Data Control Systems*

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Abstract— **Virtually all industrial control systems are implemented digitally by a sample-and-hold device. The design and performance analysis of sampled-data control systems for large-scale systems, represented by a set of high-dimensional differential equations, is challenging. Therefore, commonly, the complex high-dimensional system (plant) model is reduced to 1) support computationally efficient performance analysis for a given digital controller or 2) support (sampled-data) controller synthesis. This paper proposes a novel approach for reduction of such systems taking the sampled-data effects into account in the reduction process. In addition, we propose a condition under which stability of the closed-loop sampled-data control system is preserved in the reduction. This easy-to-check condition depends on 1) reduction error on the continuous-time plant model, 2) the sampling interval, 3) the controller gain. Finally, under this condition also an error bound for the reduced-order system is provided. The proposed methodology is illustrated by numerical examples of a controlled structural dynamical system.**

I. INTRODUCTION

In industry, almost all controllers are implemented digitally. A sample-and-hold (S&H) device prepares the sensor data for the controller logic, the microprocessor calculates the proper control action in discrete time, and the S&H transforms the discrete-time command signal to a continuous-time form [1].

Due to the sample-and-hold effects, and also the time delay between the measured output and the real output, stability analysis of sampled-data systems requires special attention [2]. Different methods have been proposed for this purpose, such as passivity-type stability analysis [7], L_2-stability analysis [20] and Lyapunov-based analysis [14]. Moreover, sampled-data systems can be described by a time-varying delayed system [6]. Stability analysis of such systems have been investigated in [9], [23]. However, these stability analyses rely on solving some linear matrix inequality conditions, which becomes computationally infeasible for large-scale systems.

The dynamics of complex (engineering) systems are usually governed by a set of high-dimensional differential equations. As a consequence, the design of sampled-data control systems for such large-scale systems and the corresponding stability analysis are challenging. This also holds when we are dealing with simulation-based performance analysis of such controllers. For example, when the control system is going to operate under wide range of disturbance scenarios, a large number of simulations are required to evaluate controller performance, which is computationally expensive. Therefore, Model order reduction (MOR) plays a role to act against such *curse of dimensionality*.

MOR of the high-dimensional plant model can be exploited, first, to facilitate controller design, and second, to render such simulation-based performance analysis computationally feasible. The literature on the MOR of sampled-data control systems is rather limited. In [3], an approximate balancing method is proposed, which becomes more accurate when the sampling time decreases. In [4], a moment matching approach is introduced, where the stability of the open-loop system after reduction is analyzed. However in these research works, closed-loop stability of the sampled-data system after reduction is not studied. On the other hand, sampled-data systems have been modelled and analyzed as systems with time-varying (saw-tooth) delays [16], [11], [17]. MOR of time delay systems has been studied in many research works [21], [12], [10], [13], [15], [8]. However, in those works, mostly only the constant delay case is addressed and the delay is an innate property of the dynamical model itself, as opposed to digital controller implementation. Therefore, the nature of the problem considered in this paper intrinsically differs from the problems investigated in those works.

In this paper, we focus on resolving the computational complexity of simulation-based performance analysis of sampled-data systems by addressing two challenges. Firstly, if the plant model of a sampled-data control system is reduced ignoring the fact that it is going to be controlled through a S&H device (we call this *standard* MOR in this paper), the closed-loop sampled-data reduced-order model (ROM) may become unstable and the ROM may not represent the original sampled-data system accurately. Therefore, a new perspective to reduce such systems should be introduced that takes into account the effect of the S&H device during reduction (called *sampled-data aware* MOR in this paper), which is the first contribution of this paper. Secondly, the reduction affects the accuracy of the model, also in terms of its performance characteristics. For the sampled-data aware reduction strategy we propose a condition under which 1) a bound on the reduction error and 2) a guarantee on the performance for the sampled-data ROM can be provided. This easy-to-check condition depends on the reduction error of the continuous-time plant, the sampling interval and the L_2-gain of the designed controller and the full-order model (FOM) with respect to specific input-output channels. This represents the second contribution of this paper.

This paper is organized as follows. Section II defines the problem statement and presents a motivating test-case

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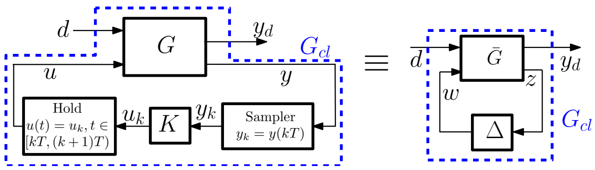

Fig. 1: Left: closed-loop system together with S&H and static controller K; Right: Alternative closed-loop system representation with operator Δ.

to demonstrate the relevance of problems identified above. Section III presents the sampled-data aware MOR strategy solving these problems. Section IV shows an illustrative test-case for our methodology. Finally, Section V concludes the paper.

Notation. The field of real numbers is denoted by $\mathbb{R}$. For a vector $x \in \mathbb{R}^n$, $|x|^2 = x^T x$ with superscript T denoting the transpose operation. The L_2-norm of a signal is defined as $\|u\|^2 = \int_0^\infty |u(t)|^2 \, \mathrm{d}t$. A system has a L_2-gain of κ from any input u to the output y with zero initial condition if $\|y\| \leq \kappa \|u\|$.

II. PROBLEM STATEMENT

In this section, we first formulate the problem and, second, we give a motivating example on the importance of sampled-data effects on model reduction.

A. Problem formulation

Consider linear time-invariant (LTI) open-loop dynamics G described by

$$G : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_d d(t), \\ y(t) = Cx(t), \\ y_d(t) = C_d x(t) \end{cases} \tag{1}$$

with $x \in \mathbb{R}^n$ states of the system, t time, $u \in \mathbb{R}^m$ control input, $d \in \mathbb{R}^{m_d}$ disturbance, $y \in \mathbb{R}^p$ measured output, and $y_d \in \mathbb{R}^{p_d}$ the performance output. Additionally, n, m, m_d, p and p_d are, respectively, the dimension of the states, the input, the disturbance, the measured and the performance outputs. In this setting, n is a large number, which motivates model reduction. Moreover, A, B, B_d, C and C_d are the state matrices of the system with appropriate dimensions.

We consider static output-feedback controller $u = Ky$ that is implemented in a sampled-data fashion. The closed-loop plant controlled by the controller K equipped with a S&H implementation is shown on the left side of Figure 1. The S&H in this study has a fixed sampling-and-hold interval T. The *sampler* block samples a continuous signal, i.e., for the signal $y(t)$, the output of the sampler is $y_k = y(kT)$ with $k \in \{1, 2, 3, \cdots\}$. The *hold* block generates a continuous signal from a discrete signal, i.e., for the discrete input u_k, the output is $u(t) = u_k \; \forall t \in [kT, (k+1)T)$. This value is kept constant until the next sampling and hold action takes place.

The sampled-data-controller introduces the following closed-loop system dynamics G_{cl}:

$$\begin{aligned} G_{cl} &: \begin{cases} \dot{x}(t) = Ax(t) + B_d d(t), \\ y_d(t) = C_d x(t), \end{cases} && \forall t \in [0, T) \\ G_{cl} &: \begin{cases} \dot{x}(t) = Ax(t) + B_d d(t) \\ \qquad + BKCx(kT), \\ y_d(t) = C_d x(t). \end{cases} && \forall t \in [kT, (k+1)T) \end{aligned} \tag{2}$$

Here, we have assumed that the controller generates a zero command signal before receiving the first measurement. The S&H device challenges the MOR procedure as demonstrated by a motivating example below.

B. A motivating example

To shed light on the importance of the S&H effect in the scope of the model reduction of sampled-data control systems, we present a simple test-case of a two mass-spring-damper system, see Figure 2. The location of one mass (the output y) is controlled by applying force u to the other mass (the input), generated by a proportional controller, $u = Ky$. The performance output y_d is considered to be the same as the output y and the disturbance d is assumed to be zero.

Here, we design a stabilizing controller for the FOM, which is subsequently employed in both the continuous-time and sampled-data setting. Then, we reduce the open-loop plant in a standard manner (from input u to output y) by balanced residualization [5] and apply the same controller to the ROM. Then, we investigate how well the sampled-data ROM represents the original sampled-data control system. It should be noted that the stability of the FOM is checked by only simulation.

The result of the standard MOR approach is reported in Figure 3. The dimension of the FOM and the ROM is 4 and 3, respectively. Notably, the closed-loop response of the both FOM and ROM for a continuous-time implementation of the controller is stable and accurate; however, although the sampled-data implementation of the controller stabilizes the FOM, this implementation destabilizes the ROM. This result affirms that taking the sampled-data effect into account during reduction is crucial and MOR should take place in a sampled-data aware manner (by performing reduction with respect to additional input-output channels as advocated in Section III-B below).

In the next section, we will develop such a sampled-data aware reduction strategy that addresses the challenge illustrated by the above example.

III. METHODOLOGY

In this section, we propose an approach for sampled-data aware model reduction.

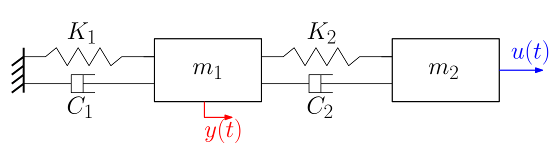

Fig. 2: Motivating example.

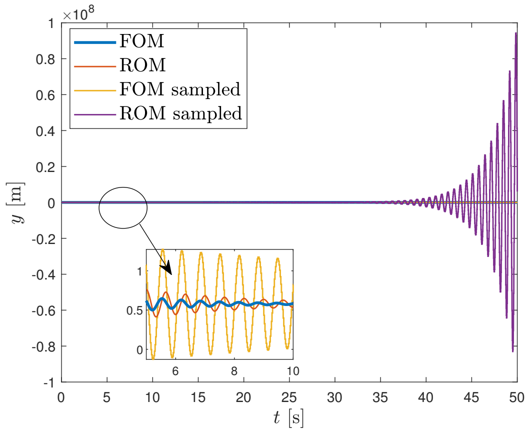

Fig. 3: Closed-loop system response of the FOM and the standard ROM of order 3. Continuous-time and sampled-data implementation of the controller with $K = -275$ and $T = 0.02$ s; controller designed for stabilizing the FOM.

A. Reformulation of model dynamics

To facilitate sampled-data aware model reduction, we rewrite the closed-loop system G_{cl} in (2) in another form [20], [18]:

$$\bar{G} : \begin{cases} \dot{x} = Ax + BKCx + Bw + B_d d, \\ z = KC\dot{x}, \\ y_d = C_d x, \end{cases} \tag{3}$$

$$w(t) = \Delta(z(t)) = \begin{cases} 0 & t \in [0, T), \\ -\int_{kT}^{t} z(s)\mathrm{d}s & t \in [kT, (k+1)T), \end{cases} \tag{4}$$

where w is introduced into the governing equations due to the presence of the sampling-and-hold effects (see the right side of Figure 1). In fact, w represents the difference between sampled-data control action and the ideal, continuous-time control action KCx. Note that we assume that the controller generates no command before receiving the first measurement.

We conjecture that taking into account the input-output pair w and z, associated to the sampling-and-hold effects, in the reduction improves the quality of the reduction in the sampled-data setting. This is further detailed in the next section.

B. Model order reduction

The design of the controller, and/or its (simulation-based) performance analysis may be obstructed by the high dimension of the open-loop plant G. Therefore, typically MOR is needed to construct a model suitable for controller synthesis or for closed-loop performance analysis. Then, constructing an appropriate reduction in this context is challenged by the sampled-data effects (see the example in Section II-B).

As mentioned in (3) and in Figure 1, the sampled-data effect is associated to an additional input-output pair z, w. When employing standard MOR, the effect of this extra input-output pair is ignored while this may lead to instability. Therefore, we propose to take this extra input-output pair explicitly into account during reduction (sampled-data aware MOR).

To perform reduction with the new input-output terms, we rewrite (3)-(4) as follows:

$$\begin{aligned} &\bar{G}^* : \begin{cases} \dot{x} = Ax + Bu + Bw + B_d d, \\ z = KC\dot{x}, \\ y = Cx, \\ y_d = C_d x, \end{cases} \\ &w = \Delta(z), \\ &u = Ky \end{aligned} \tag{5}$$

with Δ as in (4). This closed-loop system is shown in Figure 4. This reformulation suggests that reducing the system while taking only the input-output pairs $[u \quad d]^T$ and $[y \quad y_d]^T$ into account might not suffice (standard MOR) in the sampled-data context. Instead, we propose to reduce the system while taking into account the inputs $[u \quad d \quad w]^T$ and outputs $[y \quad y_d \quad z]^T$ (sampled-data aware MOR).

With the above vision on the model reduction of sampled-data systems in mind, we propose to reduce $\bar{G}^*$ in (5) with these extended input-output pairs in mind. In the next section, we address the challenge of guaranteeing the stability of the sampled-data ROM and also its accuracy given a stabilizing controller designed for the FOM.

C. Preservation of stability, accuracy and performance in sampled-data aware reduction

Since the Δ-operator in (4) is L_2-gain bounded with gain $\frac{2T}{\pi}$ [7], i.e., $\|\Delta\| \leq \frac{2T}{\pi}$, the stability analysis of closed-loop FOM in the transformed version (3)-(4) can be performed by the following small-gain condition [19] (see right side of Figure 1):

$$\|\bar{G}\|_\infty \frac{2T}{\pi} < 1, \tag{6}$$

where $\bar{G}$ is governed by (3) and $\|\bar{G}\|_\infty$ is the H_∞-norm of $\bar{G}$ from input w to output z. Based on this inequality, we can decide either on the maximum allowable sampling time T or the requirement on H_∞-norm of the transformed system $\bar{G}$, which can be tuned by designing the controller K. Here, we assume that the controller K is already designed and we aim at performing many simulations to study the effect of the disturbance d on the system. In the following, we will extend this result for the closed-loop system (5) and its reduced-order version. Before going into details, we adopt the following assumption.

Assumption 1: There exists an asymptotically stabilizing static output-feedback controller $u = Ky$ for the sampled-data

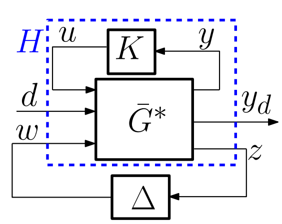

Fig. 4: Closed-loop system reformulation.

FOM as in (2). This controller renders the matrix $A+BKC$ Hurwitz.

We aim to derive conditions under which the closed-loop sampled-data ROM is also asymptotically stable with the sampled-data controller designed for the FOM. To this end, the controller is incorporated within the linear FOM to construct the system H, as shown in Figure 4, which shows the closed-loop system if the controller was implemented in a continuous-time manner, i.e., with $w=0$. This system is governed by the following equation:

$$u=Ky \xrightarrow{(5)} H: \begin{cases} \dot{x} = (A+BKC)x + Bw + B_d d, \\ z = KC\dot{x} \\ y_d = C_d x. \end{cases} \tag{7}$$

The ROM obtained from applying balanced truncation or balanced residualization [5] [1] to H, which gives the ROM $\hat{H}$, is governed by the following system of equations:

$$\hat{H}: \begin{cases} \dot{\hat{x}} = (\hat{A}+\hat{B}K\hat{C})\hat{x} + \hat{B}\hat{w} + \hat{B}_d d, \\ \hat{z} = K\hat{C}\dot{\hat{x}} \\ \hat{y}_d = \hat{C}_d \hat{x}, \end{cases} \tag{8}$$

where it will be closed by $\hat{w}=\Delta(\hat{z})$ with Δ as in (4). Here, $\hat{(\cdot)}$ denotes the reduced-order matrices and variables. Here, $\hat{x} \in \mathbb{R}^{\hat{n}}$ with $\hat{n} \ll n$, where $\hat{n}$ is the dimension of the reduced-order model. To analyze the stability of $\hat{H}$ with $w=\Delta(\hat{z})$, i.e., the reduced-order sampled-data control system, we will use the following lemma.

Lemma 1: The L_2-gain from input $\hat{w}$ to output $\hat{z}$ for the ROM $\hat{H}$, denoted by $\hat{\gamma}_{zw}$, abides by the following inequality

$$\hat{\gamma}_{zw} \leq \gamma_{zw} + \epsilon_{zw}, \tag{9}$$

where γ_{zw} is the L_2-gain of H from input w to output z and ϵ_{zw} is the error bound due to the reduction of H to $\hat{H}$ from input w to output z.

Remark 1: An error bound is readily available when using balancing-based methods for the LTI system H, i.e., $\epsilon_{zw} = \sum_{i=\hat{n}+1}^{n} \sigma_i$, where σ_i's are the discarded Hankel singular values. If balanced residualization is used to obtain the ROM, a feedthrough term appears in (8), which can be handled in a straightforward manner.

To ensure the stability of the closed-loop sampled-data ROM, we use the following theorem.

Theorem 1: Consider Assumption 1. If the inequality

$$\frac{2T}{\pi}(\gamma_{zw}+\epsilon_{zw}) < 1 \tag{10}$$

is satisfied, then the closed-loop sampled-data ROM is asymptotically stable.

Proof: A direct application of small-gain theorem to (8) closed with $\hat{w}=\Delta(\hat{z})$ enforces $\|\Delta\|\hat{\gamma}_{zw} < 1$ for stability. Replacing $\|\Delta\|$ with its L_2-gain bound, $\frac{2T}{\pi}$ and using inequality (9) to replace $\hat{\gamma}_{zw}$ yields (10). ■

Remark 2: Equation (10) suggests that for stabilizing the ROM with the sampled-data controller designed for the FOM, 1) the sampling interval should be low enough, and/or 2) the ROM $\hat{H}$ should be close enough to the FOM H (i.e., ϵ_{zw} small enough).

Suppose a sampled-data controller $u=Ky$ is given. The following theorem gives an error bound on the performance output error induced by reduction. In the following theorem, γ_{ij} denotes the L_2-gain of system H from input j to output i. Moreover, ϵ_{ij} is the reduction error bound from H to $\hat{H}$ from input j to output i.

Theorem 2: Adopt Assumption 1 and suppose inequality (10) holds. Then, the following error bound on the performance output holds, $\|y_d - \hat{y}_d\| \leq \epsilon \|d\|$ with

$$\begin{aligned} \epsilon = & \epsilon_{y_d d} + \epsilon_{y_d w} \frac{\frac{2T}{\pi}\gamma_{zd}}{1-\frac{2T}{\pi}\gamma_{zw}} \\ & + \frac{(\gamma_{y_d w}+\epsilon_{y_d w})}{1-\frac{2T}{\pi}(\gamma_{zw}+\epsilon_{zw})}\left(\epsilon_{zd} + \epsilon_{zw}\frac{\frac{2T}{\pi}\gamma_{zd}}{1-\frac{2T}{\pi}\gamma_{zw}}\right). \end{aligned} \tag{11}$$

Proof: For this proof, we introduce the notation $y_d(w,d)$ denoting the performance output of the FOM for inputs w and d. Similar signals are defined for different inputs. To derive the error bound (11), we write:

$$\begin{aligned} & \|y_d(w,d) - \hat{y}_d(\hat{w},d)\| = \\ & \|y_d(w,d) - \hat{y}_d(w,d) + \hat{y}_d(w,d) - \hat{y}_d(\hat{w},d)\| \leq \\ & \|y_d(w,d) - \hat{y}_d(w,d)\| + \|\hat{y}_d(w,d) - \hat{y}_d(\hat{w},d)\| \leq \\ & \epsilon_{y_d w}\|w\| + \epsilon_{y_d d}\|d\| + \hat{\gamma}_{y_d w}\|w-\hat{w}\|. \end{aligned} \tag{12}$$

Here, $\hat{\gamma}_{y_d w}$ can be replaced by $\gamma_{y_d w}+\epsilon_{y_d w}$ similar to the inequality in (9). To bound $\|w\|$, we exploit the Lipschitz boundedness of operator Δ as follows:

$$\begin{aligned} & \|w\| \leq \frac{2T}{\pi}\|z\| \leq \frac{2T}{\pi}(\gamma_{zw}\|w\| + \gamma_{zd}\|d\|) \\ & \Rightarrow \|w\| \leq \frac{\frac{2T}{\pi}\gamma_{zd}}{1-\frac{2T}{\pi}\gamma_{zw}}\|d\|. \end{aligned} \tag{13}$$

To compute a bound for the rightmost term in the right-hand side of (12), we use the gain of the operator Δ again, i.e.,

$$\|w-\hat{w}\| \leq \frac{2T}{\pi}\|z(w,d) - \hat{z}(\hat{w},d)\|, \tag{14}$$

where $z(w,d)$ is defined similarly to $y_d(z,w)$ above. Following similar steps, we define a bound on $\|z(w,d) - \hat{z}(\hat{w},d)\|$ as follows:

$$\begin{aligned} & \|z(w,d) - \hat{z}(\hat{w},d)\| = \\ & \|z(w,d) - \hat{z}(w,d) + \hat{z}(w,d) - \hat{z}(\hat{w},d)\| \leq \\ & \|z(w,d) - \hat{z}(w,d)\| + \|\hat{z}(w,d) - \hat{z}(\hat{w},d)\| \leq \\ & \epsilon_{zw}\|w\| + \epsilon_{zd}\|d\| + \hat{\gamma}_{zw}\|w-\hat{w}\|. \end{aligned} \tag{15}$$

Using (14) in (15) and then inserting (12) yields:

$$\|w-\hat{w}\| \leq \frac{\epsilon_{zw}\frac{\frac{2T}{\pi}\gamma_{zd}}{1-\frac{2T}{\pi}\gamma_{zw}} + \epsilon_{zd}}{1-\frac{2T}{\pi}\hat{\gamma}_{zw}}\|d\|. \tag{16}$$

Here, $\hat{\gamma}_{zw}$ can be replaced by $\gamma_{zw}+\epsilon_{zw}$ as in (9). Inserting (16) and (13) into (12) yields the desired result. ■

Remark 3: Since balancing methods do not provide a priori reduction error bounds for individual input-output pairs, we need to use the overall error bound for the entire input-output

[1] For simplicity, the feed-through term coming from balanced residualization is not considered here.

pairs instead of all ϵ_{ij}'s in (11) to obtain an a priori error bound for the sampled-data ROM.

Remark 4: A less-conservative but a posteriori error bound can be derived similar to (11) by replacing $(\gamma_{y_d w} + \epsilon_{y_d w})$ and $(\gamma_{zw} + \epsilon_{zw})$ with $\hat{\gamma}_{y_d w}$ and $\hat{\gamma}_{zw}$, respectively.

We can also use the error bound (11) to obtain a bound on the L_2-gain of the closed-loop sampled-data ROM.

Corollary 1: Consider a sampled-data controller $u = Ky$ that asymptotically stabilizes the FOM (5) satisfying the condition in (10). The L_2-gain of the closed-loop sampled-data ROM is bounded by $(\kappa + \epsilon)$, i.e.:

$$\|\hat{y}_d\| \leq (\kappa + \epsilon) \|d\| \,, \tag{17}$$

where κ is the L_2-gain of the FOM from d to y_d and ϵ is given in (11).

Proof: This can be proved by using the triangular inequality. ∎

In the next section, we show the results of applying the samples-data aware reduction method and analysis results to the motivating example in Section II-B and also to a more extensive illustrative test case.

IV. Results

In this section, we first apply our methods to the motivating example in Figure 2 and then introduce an illustrative test case.

A. Mass-spring-damper test-case

Performing the sampled-data aware MOR to the example in Section II-B yields the result in Figure 5. In this scenario, a stabilizing sampled-data controller is given for the FOM and we desire to construct a stable, accurate sampled-data ROM. Standard MOR did not provide such result as shown in Figure 3. Figure 5 shows that using the proposed sampled-data aware MOR approach, stability is preserved for the ROM and the ROM accurately described the response of the FOM. For this test case, we did not even satisfy (10), and as can be seen, stability is preserved. This is related to the conservativeness of (10). To satisfy (10), i.e., guaranteeing having a stable sampled-data ROM with $K = -275$, the sampling time should be less than 0.014 s.

To compare the small-gain conditions for the FOM as in [20] and for the ROM as in (10), stability regions for the closed-loop sampled-data FOM and closed-loop sampled-data ROM is reported in Figure 6. This result confirms that the stability regions of the FOM and ROM are very similar to each other, and, therefore, the ROM-based simulation analysis of the controller can lead to significant speedup.

B. An illustrative test case

The test-case considered here is based on [22]. It is an Euler-Bernoulli beam with the location of the inputs, measure outputs, disturbance and performance measure shown in Figure 7. The slender beam has the following dimensions: length×height×width = 1.3 m×3 mm×0.1 m. Moreover, the beam material properties are as follows: a mass density of 7746 kg/m^3 and Young's modulus of 200 GPa. Moreover, the beam is subject to a disturbance $d = 0.1\sin(10t)$ N representing an external force, which causes the beam to vibrate in the vertical plane. To attenuate the effect of this disturbance, an actuation force u can be applied by a static sampled-data controller, which acts on a measurement y of the vertical deflection at some point of the beam, at a different location than the performance measure. The dynamics of the beam is modelled using Euler-Bernoulli beam elements, leading to a linear time-invariant dynamical system of the form

$$M\ddot{q} + D\dot{q} + K_{\text{beam}}q = b_u u + b_d d, \tag{18}$$

with nodal coordinates q containing the deflection and rotation of each node and M, D, and K_{beam} representing the mass matrix, damping matrix, and stiffness matrix, respectively. Moreover, b_u and b_d are vectors with appropriate dimension, which take into account the effect of the respective forces at the right locations. Equation (18) can easily be written in the form of (1).

Considering 20 elements, the model written in form of (1) contains $n = 80$ states. Reducing from $n = 80$ to $\hat{n} = 20$ by balanced residualization yields $\epsilon_{zw} = 36.8$ (this value can be used for all ϵ_{ij}'s in (11)). We synthesize the feedback gain for the FOM: $K = -1200$. Using (10), we restrict $T = 0.0081$ s to obtain a stable closed-loop ROM. Figure 8 compares $\hat{y}_d$ for the closed-loop sampled-data ROM with y_d from the closed-loop sampled-data FOM. This figure confirms that the 1) ROM with the additional input-output channel is an accurate representation of the original model, 2) stability of the closed-loop ROM is preserved by the sampled-data aware reduction method.

V. Conclusions

Sampled-data control systems are ubiquitous in industry. Model order reduction of such systems requires special atten-

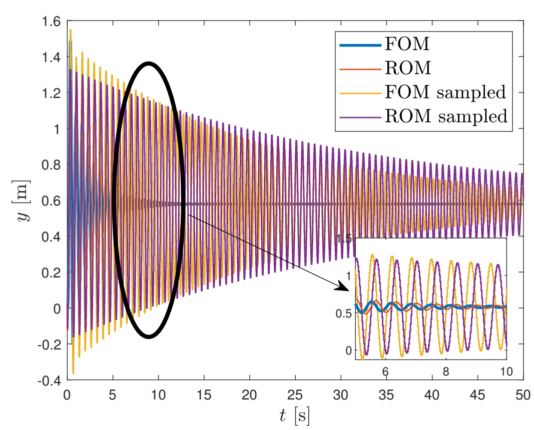

Fig. 5: Closed-loop system response of the FOM and the sampled-data aware ROM of order 3. Continuous-time and sampled-data implementation of the controller with $K = -275$ and $T = 0.02$ s; controller designed for stabilizing the FOM even without satisfying (10) and just by enriching the input-output channels.

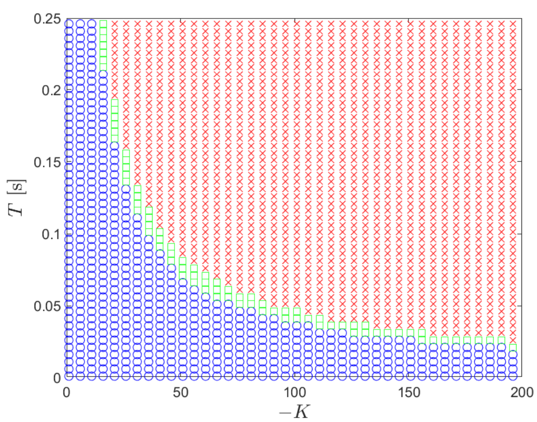

Fig. 6: Comparison of stability regions; sampling time T vs controller gain $-K$; blue circle: both closed-loop sampled-data FOM and ROM are stable; green rectangle: stable closed-loop sampled-data FOM; red cross: stability of FOM and ROM cannot be deduced from [20] and (10).

tion due to the sampling and hold effect, since it can destroy 1) the quality of the reduction and 2) stability through reduction. In this paper, first, a new perspective for model order reduction of sampled-data systems is introduced (sampled-data aware model order reduction). Second a condition is formulated under which stability is preserved, where the easy-to-check condition depends on the controller gain, the error bound on the reduction of the linear time-invariant continuous-time (reduced-order) plant and the sampling interval. Moreover, an error bound for the sampled-data reduced-order system is provided. The illustrative examples show the effectiveness of the proposed approach.

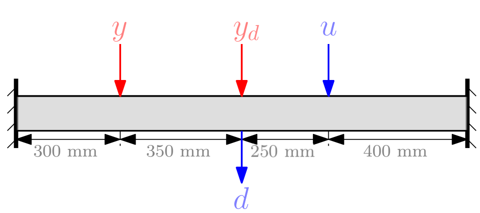

Fig. 7: The Euler-Bernoulli beam.

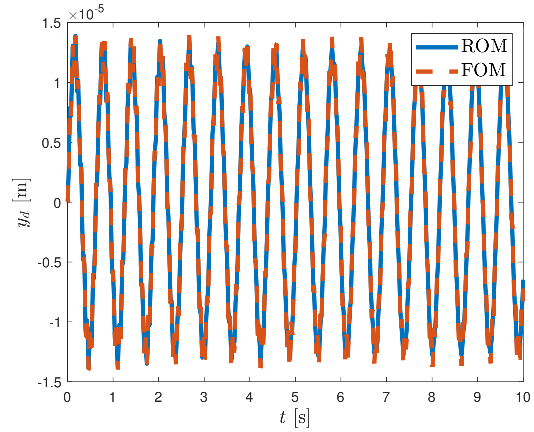

Fig. 8: Evolution of the performance output for the closed-loop FOM and ROM.

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