Robust disturbance estimation for human–robotic comanipulation[‡]

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SUMMARY

External forces affect the dynamics of load-carrying robot devices. The knowledge of such disturbances is generally needed for control purposes. However, direct disturbance measurement using force sensors is not always possible. This paper introduces a force estimator for force-sensor-less robotic manipulators. The algorithm is based on the knowledge of the dynamics of the robotic device, whereas mass of the load is typically unknown. Using this algorithm, low-frequency external forces can be estimated robustly even for quasi-statically time-varying and uncertain loads. Experiments validate the proposed strategy in practice. Moreover, the applicability of the estimation algorithm is further illustrated by using it in a human–robot comanipulation setup in which the robot is providing additional coordinated forcing to alleviate human effort needed to manipulate the robot. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In this paper, we consider the problem of robust estimation of disturbance forces acting on loadcarrying robotic systems. This problem is encountered in the scope of both teleoperation and human–robotic comanipulation (think of a human and a robot jointly performing a load-carrying and positioning task). In both contexts, the robot interacts with the environment, and its dynamics are dependent on external forces induced by this interaction. These forces can be contact forces (interaction forces between environmental objects and the robot), or they can reflect an interaction between the robot and a human operator (as encountered in human–robotic comanipulation). It is well known that haptic robotic devices and teleoperation systems exploit information regarding the external forces (see [1] and [2], e.g., for haptic feedback).

In human–robotic comanipulation, knowledge on the unknown force applied by the human is typically needed to achieve coordinated comanipulation. One option for obtaining such disturbance information is to equip the robot with force sensors; for examples of such robotic devices, especially haptic devices, which use force sensors, the reader is referred to [1, 3]. However, in many cases, the most important external forces for multilink robots appear at the end effector. Note that force sensing at the end effector of the robot is often not feasible because the external forces will typically interact with the load directly (and not with the robot end effector). Besides, in some cases, the position at which the external forces are applied is a priori unknown and may be on a robot link as

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opposed to on the end effector. Moreover, the usage of force sensors can be expensive and increase the production costs of the robot.

For these reasons, a disturbance estimation scheme for force-sensor-less robots is needed. Disturbance observers (DOB) have been widely used in different motion control applications [4–6] for determining the disturbance forces, such as friction forces. However, the performance enhancement of these DOB strategies may lead to smaller stability margins for the motion control [7]; therefore, a robust design with respect to the environmental disturbances and model uncertainties is needed. Previous results on robustly stable DOB [8–11] are based on linear robust control techniques. Some nonlinear DOB have been developed for the estimation of harmonic disturbance signals in [12, 13].

Various strategies have also been considered for force-sensor-less control schemes estimating the external force. Eom *et al.* [14] proposed an adaptive disturbance observer scheme, and Ohishi *et al.* [15, 16] proposed an H^{∞} estimation algorithm. In [17], a control strategy called 'force observer' is introduced. This design uses an observer-type algorithm for the estimation of exogenous force. The drawback of these approaches is that they assume perfect knowledge of the model of the system.

In parallel with force estimation strategies, based on DOB, another approach using sensor fusion has been developed to diminish the noise levels of the force sensors. In [18], force and acceleration sensors are combined, whereas in [19], data from force sensors and position encoders are fused. Sensor fusion provides better qualitative results than those obtained by employing more expensive force sensors.

Two novel contributions are presented in the paper. Firstly, we present an estimation strategy for low-frequency external forces acting on a robotic manipulator with a load with unknown, possibly time-varying mass. Applications with time-varying mass can be encountered in the case of dispersing liquids from a container (e.g., painting or concrete pouring on building sites). This method extends a result presented in [20], which considered only the case with time-invariant mass. The proposed algorithm is robust for large uncertainties in the mass of the load. Secondly, this estimation strategy is used to solve a human–robot comanipulation problem. In recent years, the problem of cooperative motion control by a human and a robot has been tackled in [21] using 'interactive virtual impedance', whereas in [22], a set of tests for model identification is applied to tune the disturbance observer. In [23], a discussion on the state of the art in force-sensor-less power assist control is presented with an emphasis on the estimation of the human force using linear models for the robot with the load. The strategies discussed earlier assume more or less perfect knowledge of the dynamics of the robot with the load, that is, the mass of the load is considered known, or with very small uncertainty, and constant.

In the control scheme for the comanipulation problem proposed in this paper, the human is in charge of the position control, and the robot supplies an additional force, which is based on an estimation of the force that the human has applied to the load. As a consequence, the robot actions ensure that the task can be performed faster and with less human effort.

A preliminary version of the estimation approach proposed here has been published in [24], where it was limited to three-dimensional linear robots and constant load inertias and applied in the context of force-sensor-less bilateral teleoperation. Here, the estimation approach is extended to generic robotic applications and time-varying load inertias, and we propose to employ it in a strategy for human–robotic comanipulation.

The paper is structured as follows. Section 2 recalls some theoretical results that will be used in the sequel. In Section 3, we present the force estimation algorithm. In Section 4, we introduce a human–robot comanipulation control strategy that uses the estimation result. In Section 5, the effectiveness of the estimation algorithm is validated on an experimental one-degree-of-freedom (1-DOF) robot, and that of the comanipulation strategy is illustrated using an example of a 2-DOF robot. In the final section of the paper, the conclusions and some perspectives on future work are discussed.

2. PRELIMINARIES

In this section, we recall some definitions and results concerning the property of input-to-state stability as introduced by Sontag in [25], see also [26]. The input-to-state stability property of nonlinear systems is exploited in the proof of the main result of Section 3.

Consider the general nonlinear system

$$\dot{x}(t) = f(x(t), u(t)), \ x(0) = x_0,$$
(1)

with solutions $\varphi(t, x_0, u)$, where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is continuously differentiable. The set of all measurable locally bounded functions $u : \mathbb{R}^+ \to \mathbb{R}^m$, endowed with the supremum norm $\sup\{|u(t)|, t \ge 0\} < \infty$, is denoted as L_{∞}^m . A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is called a class \mathscr{K} -function, that is, $\gamma \in \mathscr{K}$, if it is continuous, strictly increasing, and $\gamma(0) = 0$. A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is called a class \mathscr{K}_{∞} -function if $\gamma \in \mathscr{K}$ and $\gamma(s) \to \infty$ as $s \to \infty$. A function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a class $\mathscr{K} \mathscr{L}$ -function if for each fixed $t \ge 0$, $\beta(\cdot, t) \in \mathscr{K}$, and for each fixed $s \ge 0$, $\beta(s, t)$ is decreasing to zero as $t \to \infty$.

Definition 1 ([25])

System (1) is input-to-state stable (ISS) if there exist a function $\beta \in \mathscr{KL}$ and a function $\gamma \in \mathscr{K_{\infty}}$ such that, for each input $u \in L_{\infty}^m$, all initial values x_0 and for any $t \ge 0$, the following inequality holds:

$$|\varphi(t, x_0, u)| \leq \beta(|x_0|, t) + \gamma(\sup_{0 \leq \tau \leq t} |u(\tau)|).$$

$$\tag{2}$$

Definition 2 ([25])

A smooth function $V : \mathbb{R}^n \to \mathbb{R}$ is called an ISS Lyapunov function for system (1) if there exist functions $\alpha_1, \alpha_2 \in \mathscr{K}_{\infty}, \alpha_3, \chi \in \mathscr{K}$ such that

$$\alpha_1(|x|) \leqslant V(x) \leqslant \alpha_2(|x|) \tag{3}$$

and

$$|x| \ge \chi(|u|) \Rightarrow \frac{\partial V}{\partial x} f(x, u) \le -\alpha_3(|x|)$$
(4)

hold for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$.

The quantitative aspects regarding the existence of an ISS Lyapunov function have been developed in [25] and [27]. These results are synthesized by the following theorem.

Theorem 1

If an ISS Lyapunov function exists for system (1), then system (1) is ISS with $\beta(\cdot, t) = \alpha_1^{-1} \circ \mu(\alpha_2(\cdot), t)$ and $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \chi$, where μ is the solution of the differential equation:

$$\frac{d}{dt}\mu(r,t) = -\alpha_3 \circ \alpha_2^{-1}(\mu(r,t)),\tag{5}$$

with the initial condition $\mu(r, 0) = r$.

Similar results have been developed in [28] concerning the input-to-state stability property of discrete-time nonlinear systems.

3. ROBUST DISTURBANCE ESTIMATOR

3.1. Problem statement

Consider the nonlinear system dynamics of a robot described by

$$M(q,t)\ddot{q} + D(q,\dot{q},t) = \tau + J^T(q)F_{\rm E},\tag{6}$$

where $q \in \mathbb{R}^n$ is the vector of generalized joint displacements, $\dot{q} \in \mathbb{R}^n$ is the generalized joint velocity vector, $\ddot{q} \in \mathbb{R}^n$ is the generalized joint acceleration vector, and $\tau \in \mathbb{R}^n$ is the robot torque vector. Moreover, $F_{\rm E} \in \mathbb{R}^d$ is the external force vector (*d* is the space dimension, d = 2 for

two-dimensional space or d = 3 for three-dimensional space) applied on a point mass rigidly attached to the end effector of the robot, $M \in \mathbb{R}^{n \times n}$ is the symmetric, positive definite inertia matrix, $D \in \mathbb{R}^n$ is the vector containing the sum of centripetal, Coriolis, friction, and gravitational forces/torques, and $J \in \mathbb{R}^{n \times d}$ is the Jacobian matrix relating the end-effector velocity $\dot{x} \in \mathbb{R}^d$ to the generalized joint velocity \dot{q} by $\dot{x} = J(q)\dot{q}$. We consider the case in which the robot carries an additional load of unknown and possibly quasi-statically time-varying mass m(t) at its end effector. In (6), the unknown load mass m(t) is incorporated in the inertia matrix M(q, t).

For the sake of simplicity, we adopt the following assumption.

Assumption 1

The Jacobian matrix J is nonsingular at all times of operation.

Remark 1

The aforementioned assumption implies that we do not consider redundant robots, that is, d = n, and no kinematic singularities are encountered.

The objective of this control strategy is to determine an estimate $\hat{F}_{\rm E} \in \mathbb{R}^n$ of the external force. In the following section, we describe the force estimation algorithm.

3.2. Disturbance estimator design

We design an estimation controller strategy as schematically depicted in Figure 1. Herein, Σ_{Rm} represents the robot dynamics in (6), the controller C_{lin} compensates for the robot dynamics without the load, and the controller C estimates the external force \hat{F}_{E} , with $\tau = \tau_{\text{lin}} + \tilde{\tau}$, where τ_{lin} and $\tilde{\tau}$ are the outputs of the controllers C_{lin} and C, respectively. Assuming that the dynamics in (6) can be linearly parameterized with respect to the quasi-statically time-varying mass m(t) of the load (the inertial, gravitational, centripetal, Coriolis, and friction forces are typically linear with respect to the mass m(t)), then (6) can be written as

$$M_{\rm R}(q)\ddot{q} + D_{\rm R}(q,\dot{q}) + m(t)P_{\rm M}(\ddot{q},\dot{q},q) + \dot{m}(t)P_{\dot{\rm M}}(\dot{q},q) = \tau + J^{T}(q)F_{\rm E},\tag{7}$$



Figure 1. External force estimation controller.

where $M_{\rm R}(q)$ and $D_{\rm R}(q, \dot{q})$ contain the information concerning the robot dynamics without the end-effector load, m(t) (and $\dot{m}(t)$) is the function describing the evolution of the unknown mass of the load (and its time derivative), and $P_{\rm M}(\ddot{q}, \dot{q}, q)$, $P_{\rm M}(\dot{q}, q)$ represent the remaining terms that depend on the mass of the load. Now, we adopt the following assumption on the time dependency of the mass of the load.

Assumption 2

The load mass m(t) is assumed to be quasi-statically time-varying, that is, $\dot{m}(t) \approx 0$, and the term related to $\dot{m}(t)$ can be neglected in the dynamics of the robot.

Exploiting Assumption 2 in (7) yields

$$M_{\rm R}(q)\ddot{q} + D_{\rm R}(q,\dot{q}) + m(t)P_{\rm M}(\ddot{q},\dot{q},q) = \tau + J^T(q)F_{\rm E}.$$
(8)

The controller C_{lin} is designed on the basis of the idea of partial feedback linearization:

$$\tau_{\rm lin} = M_{\rm R}(q)\ddot{q} + D_{\rm R}(q,\dot{q}). \tag{9}$$

Remark 2

In building this controller, it has been assumed that an accurate model of the robot without the load is available. If this assumption is not met, then all the unmodeled dynamics will create an equivalent force at the end effector, which will be composed with the external force, and the resulting force will be estimated.

Introducing relation (9) and $\tau = \tau_{lin} + \tilde{\tau}$ in (6) leads to

$$m(t)P_{\rm M}(\ddot{q},\dot{q},q) = \tilde{\tau} + J^T(q)F_{\rm E},\tag{10}$$

where $P_{\rm M}$ and J are known and we have to design $\tilde{\tau}$, the output of controller C, such that, independent of the magnitude of the unknown mass of the load, estimation of the external force $F_{\rm E}$ can be achieved. Here, we assume that $m(t) \in [M_{\min}, M_{\max}], \forall t \in \mathbb{R}^+$, with $M_{\min} > 0$. By defining $\hat{F} := J^{-T}\tilde{\tau}$, (10) can be written as

$$m(t)J^{-T}(q)P_{\rm M}(\ddot{q},\dot{q},q) = \hat{F} + F_{\rm E}.$$
 (11)

If we define $\eta^{(p)} := J^{-T}(q)P_{\mathrm{M}}(\ddot{q},\dot{q},q)$, with $p \ge 1$ a constant integer and $\eta^{(p)}$ denoting the *p*th time derivative of η , then (11) is equivalent to the linear differential equation

$$m(t)\eta^{(p)} = \hat{F} + F_{\rm E}.$$
 (12)

If we consider the control strategy

$$\hat{F} = -\sum_{i=0}^{p} K_i \eta^{(i)},$$
(13)

with $K_i = \text{diag}(K_{i,1}, \ldots, K_{i,j}, \ldots, K_{i,n}) \in \mathbb{R}^{n \times n}$, $i = 0, \ldots, p$, then the output $\hat{F}_E = K_0 \eta$ represents the estimated human force. We care to stress that \hat{F} represents (part of) the control action applied, whereas \hat{F}_E represents an estimate for the external force F_E . The choice of parameters K_i should be made such that $K_i \ge 0$, for $i = 0, 1, \ldots, p - 1$, $mI_n + K_p > 0$, and $\{\lambda | m\lambda^p + \sum_{i=0}^p K_{i,j}\lambda^i = 0\} \cap \overline{\mathbb{C}^+} = \emptyset$ for all $m \in [M_{\min}, M_{\max}]$, where $I_n \in \mathbb{R}^{n \times n}$ is the $n \times n$ identity matrix. The control strategy is depicted in Figure 1 using a chain of p integrators. Owing to the diagonal structure of the matrices $K_i, i = 0, \ldots, p$, relation (12) can be written as a juxtaposition of equations:

$$m(t)\eta_j^{(p)} = -\sum_{i=0}^p K_{i,j}\eta_j^{(i)} + F_{\mathrm{E},j},$$
(14)

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where η_j and $F_{E,j}$, j = 1, ..., n, are the *j* th components of the vector η and the external force vector F_E , respectively. This means that system (12) can be seen as a decoupled system where the input $F_{E,j}$ only affects variable η_j and its time derivatives. From a physical point of view, this means that every actuator/encoder pair is used independently of the others after the separation of the η variables. For example, if the external force acts in two-dimensional space, even if the robot has more degrees of freedom available, only two DOFs are used for the force estimation. For each of decoupled differential equations (14), we can write the state-space representation of the single input-single output system as

$$\dot{x}^{j}(t) = \begin{pmatrix} -\frac{K_{p-1,j}}{m(t)+K_{p,j}} & \cdots & \cdots & -\frac{K_{0,j}}{m(t)+K_{p,j}} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix} x^{j}(t) + \begin{pmatrix} \frac{1}{m(t)+K_{p,j}} \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} u_{j}(t), \quad (15)$$
$$y_{j}(t) = K_{0,j} x_{p}^{j}(t), \qquad j = 1, 2, \dots, n,$$

where $u_j = F_{E,j}$ is the *j* th component of the external force vector, $x^j = \left(\eta_j^{(p-1)}, \ldots, \eta_j\right)^T$ is the state vector, and $y_j = \hat{F}_{E,j}$ is the *j* th component of the estimated force, the output of the system. Note that the desired behavior for system (15) is $y_j(t) \rightarrow u_j(t)$ as $t \rightarrow \infty$.

Let us now define the estimation error $e_j := u_j - y_j$, j = 1, 2, ..., n. We define the new state vector $\varepsilon^j := (\varepsilon_1^j, ..., \varepsilon_p^j) := (e_j, \dot{e}_j, ..., e_j^{(p-1)})$ containing the estimation error and its derivatives. Rewriting system (15) in terms of this new state variable ε^j leads to

$$\dot{\varepsilon}^j = A^j(t)\varepsilon^j + B^j(t)v_j(t), \quad j = 1,\dots,n,$$
(16)

where

$$A^{j}(t) := \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & 1 \\ -\frac{K_{0,j}}{m(t)+K_{p,j}} & \dots & \dots & -\frac{K_{p-1,j}}{m(t)+K_{p,j}} \end{pmatrix}$$
$$B^{j}(t) := \begin{pmatrix} 0 & \dots & \dots & 0 \\ \vdots & \dots & \dots & 0 \\ \vdots & \dots & \dots & \vdots \\ 0 & \dots & \dots & 0 \\ \frac{K_{1,j}}{m(t)+K_{p,j}} & \dots & \frac{K_{p-1,j}}{m(t)+K_{p,j}} & 1 \end{pmatrix},$$

and $v_j(t) := (\dot{u}_j, ..., u_j^{(p)})^T$.

The following technical result formulates the conditions under which error dynamics (16) are ISS with respect to $v_j(t)$, which also implies the global uniform asymptotic stability (GUAS) of $\varepsilon^j = 0$ for constant $u_j(t)$, that is, $v_j(t) = 0$, for $m(t) \in [M_{\min}, M_{\max}]$.

Theorem 2

Consider systems (16). If there exist matrices $P_j = P_j^T > 0$ and scalars $\rho_j > 0$, j = 1, ..., n, such that the following LMIs are satisfied:

$$P_j A_i^j + \left(A_i^j\right)^T P_j \le -\rho_j P_j, \ i \in \{1, 2\} \text{ and } j \in \{1, \dots, n\},$$
 (17)

with

$$A_1^j = \begin{pmatrix} 0 & I \\ -K_{0,j}\underline{\alpha^j} & \dots & -K_{p-1,j}\underline{\alpha^j} \end{pmatrix},$$
(18)

$$A_2^j = \begin{pmatrix} 0 & I \\ -K_{0,j}\overline{\alpha^j} & \dots & -K_{p-1,j}\overline{\alpha^j} \end{pmatrix},$$
(19)

and $\underline{\alpha}^{j} = \frac{1}{M_{\max} + K_{p,j}}$, $\overline{\alpha}^{j} = \frac{1}{M_{\min} + K_{p,j}}$, then systems (16) are ISS with respect to the input $v_{j}(t)$ for each $j \in \{1, ..., n\}$. In particular, functions β^{j} and γ^{j} (Theorem 1) are respectively given by

$$\beta^{j}(r,t) = \sqrt{\frac{\lambda_{\max}(P_j)}{\lambda_{\min}(P_j)}} r e^{-\frac{\lambda_{\min}(P_j)}{\lambda_{\max}(P_j)}\frac{\rho_j}{4}t}$$
(20)

and

$$\gamma^{j}(r) = \frac{4}{\rho_{j}} \frac{\lambda_{\max}(P_{j})}{\lambda_{\min}(P_{j})} r,$$
(21)

where $\lambda_{\min}(P_j) = \min(\operatorname{eig}(P_j))$ and $\lambda_{\max}(P_j) = \max(\operatorname{eig}(P_j))$.

Proof

Appendix I.

Corollary 1

Consider systems (15) with a constant input $u_j(t) = U_j$. Under the conditions of Theorem 2, the equilibrium point $x^j = \left(0, \ldots, 0, \frac{U_j}{K_{0,j}}\right)^T$ is globally uniformly asymptotically stable (GUAS).

Using Corollary 1, we prove that $y(t) \rightarrow u(t)$ when $t \rightarrow \infty$ (i.e., the estimated force $\hat{F}_{\rm E}(t)$ converges to the external force $F_{\rm E}(t)$), for a constant input signal u(t) (i.e., a constant external force) and quasi-statically time-varying parameter $m(t) \in [M_{\min}, M_{\max}]$. In other words, the estimation algorithm provides exact estimation of a constant unknown external force. Theorem 2 shows that for a nonconstant external force, the estimation error remains bounded and that this bound can be related to the time derivatives of the external force via a linear ISS gain relation (see (21)). Moreover, the function $\beta^{j}(r, t)$ as in (20) reflects a bound on the transient convergence rate of the estimation algorithm.

Remark 3

In this section, we have chosen to estimate the external force, but it is also possible, using the same approach, to estimate the equivalent torque applied by the environmental forces $\tau_{\rm E} = J^T(q)F_{\rm E}$. In this case, (10) becomes

$$m(t)P_{\rm M}(\ddot{q},\dot{q},q) = \tilde{\tau} + \tau_{\rm E},\tag{22}$$

which means that we can directly substitute $\eta^{(p)} := P_{\mathrm{M}}(\ddot{q}, \dot{q}, q)$ and obtain a relation similar to (12) and proceed with the algorithm as for estimation of the external force. In this second case, Assumption 1 is no longer needed because the Jacobian J(q) does not need to be invertible to determine the new variable η .

Remark 4

In practice, for both pseudo-linearizing controller (9) and feedback controller (13), knowledge about the acceleration of the joints is needed. Such an acceleration signal can be obtained by numerical differentiation (and additional filtering) of the joint velocity signal, by employing acceleration observers (see e.g., [29, 30]), or by using an accelerometer.

Remark 5

This disturbance estimator is not suitable for estimating reaction forces induced by (hard) unilateral constraints. Because these forces typically exhibit the same spectrum as the actuator input of the robotic device, the algorithm proposed here is not fast enough to track these forces.

4. HUMAN-ROBOTIC COMANIPULATION STRATEGY

4.1. Setup description

In this section, we exploit the estimation algorithm proposed in the previous section in the scope of robot-assisted load carrying by human operators. The main goal of the robot in achieving comanipulation is to scale up the force that the human operator applies to the load. In that way, the human will 'feel' a load with lower mass but will still be in charge of the position control of the load. When designing a robot control scheme for this purpose, we face the following problems:

- The mass of the load is unknown and possibly (quasi-statically) time varying.
- The force that the human operator applies is unknown because there are no force sensors on the load; the human operator is in direct contact with the load to be transported. The only measurements available are the position coordinates of the robot links.

In Figure 2, we present the problem setup in more detail. The human operator has a desired trajectory x_d (in Cartesian coordinates of the load) in mind and establishes a position control strategy H so that, using the (visual) feedback loop (x), he can achieve the positioning goal. Using this strategy, the human operator will apply the force F_H to the load with the mass m(t). The problem is that in many applications, the mass is too heavy for the human to transport or the speed achieved is too low. The assisting robotic device with the load m(t) is represented by the dynamic block Σ_{Rm} . The controller C estimates the human force, F_H , by \hat{F}_H , using the measurements of the motor encoders from the joints of the robot. Hereto, we employ the disturbance estimation strategy of Section 3 (refer back to Figure 1 for further details about Σ_{Rm} and C). We note that we assume that the human force on the load is directly transferred into an equal force (no torque) on the end effector.

This estimated force is amplified by a factor Φ , and the resulting force is applied to the load, thereby amplifying the human operator power. The block *FK* represents a forward kinematics block from the joint coordinates q to Cartesian coordinates x. One can observe the positive feedback loop in Figure 2. We note once more that the human operator is in charge of the path planning and position control.

4.2. Controller design

The unknown variables in the problem discussed in the previous section are the mass of the load and the human operator's force. The only measurements available are the joint coordinates of the robotic device. Using this partial information, we have to estimate the human force, and the robot should



Figure 2. Setup for human-robotic comanipulation.



Figure 3. Temporal division of the control strategy.

apply an additive force that scales the human force. As the available measurements do not allow a direct control strategy because of unknown parameters and signals (a force control is dependent on the mass of the load), we propose to tackle the problem in the following two temporal steps:

- (1) Estimate the human operator force.
- (2) Apply the scaled force.

The question that arises is how to obtain this temporal division in the algorithm (Figure 3). In this respect, it is important to note the difference between the frequencies with which a human operator and the robot can perform their tasks. Studies [31] and [32] have shown that a human can perform a task with a frequency of up to 6 Hz, which is much slower than the typical sampling frequency used in a robotic control scheme. This means that if the frequency with which the two steps of our procedure are implemented is significantly higher than 6 Hz, then the robotic device can correctly track the force of the human operator and apply the scaled force to achieve its goal.

The force generated by the robot is a signal similar to a pulse width modulation signal (Figure 3). Such an input signal generates a series of accelerations and decelerations with a frequency of $\frac{1}{T}$, with T the length of a cycle. This frequency should be set above the maximal frequency that a human can perceive to avoid that the operator feels a possibly disturbing vibration induced by the algorithm. In [33], it has been shown that a human subject can feel a vibrating object with frequencies up to 500 Hz. Unfortunately, no research has been carried out for the perception of signals other than periodical ones in position (i.e., it is not known what is the human perception for periodic signals in acceleration). Moreover, human perception greatly depends on the amplitude of the vibration because for higher amplitudes, the perception limit is 500 Hz, whereas for lower amplitudes, the sensitivity limit decreases to 40 Hz. This information should also be taken into consideration when choosing the cycle period T.

The second issue of this design is to determine the real amplification coefficient, Ψ (Figure 3). Because the desired amplification coefficient is Φ , which is applied during the entire period of the cycle T, we must determine a new scaling coefficient Ψ because the amplification period lasts only for a time interval with length $T - T_0$. Assuming that the effect of the robot action should be the same in both cases, that is, the average robot force is the same during one cycle period, one can determine the scaling factor Ψ (Section 4.4).

The algorithm for the estimation of external force (in this case, the external force is the human force) was discussed in Section 3, whereas the algorithm effectuating the amplification of the human force is presented in Section 4.4. The stability of the overall comanipulation strategy is discussed in Section 4.3.

4.3. Stability of the estimation algorithm over the two phases

Because of the model switching introduced in the comanipulation algorithm, we have to study the stability of the estimation error dynamics over the entire cycle of the comanipulation algorithm as in Figure 3. For this reason, we are using a discrete-time modeling approach, and we first study the behavior of the estimation error dynamics by sampling the continuous-time estimation error signal $\varepsilon^{j}(t)$ at the beginning of each cycle, and next, we also study the intersample behavior.

4.3.1. Stability of the discrete-time system. To simplify the notation, we consider each input–output decoupled system (between the external force, which in the comanipulation setup is the human force, and the estimated force) separately.

Recall the definitions of the estimation error $e_j := u_j - y_j$ as the difference between the human force and the estimation of the human force and the state vector $\varepsilon^j := (\varepsilon_1^j, \ldots, \varepsilon_p^j) := (e_j, \dot{e}_j, \ldots, e_j^{(p-1)})$ containing the estimation error and its time derivatives. The error dynamics in

the first phase of the cycle $(kT \le t < kT + T_0, \forall k \in \mathbb{N})$ are given by relation (16).

During the second phase of the algorithm $(kT + T_0 \le t < (k + 1)T, \forall k \in \mathbb{N})$, the estimation error dynamics are described by the following system of equations:

$$\dot{\varepsilon}^{j}(t) = I_{p}v^{j}(t), \tag{23}$$

where I_p is the $p \times p$ identity matrix. Herein, we used the aforementioned definitions of ε^j and v^j and the fact that the time derivatives $\dot{y}, y^{(2)}, \ldots, y^{(p-1)}$ (where y is the estimate of human force) are all zero, because we keep the estimation of the human force constant in the amplification phase. We will now exploit an exact discretization of system (16), (23). The solution of system (16) at time $t = kT + T_0$, with $k \in \mathbb{N}$, is

$$\varepsilon^{j}(kT + T_{0}) = e^{Q_{j}(kT + T_{0})}\varepsilon^{j}(kT) + \int_{kT}^{kT + T_{0}} e^{Q_{j}(kT + T_{0} - s)}B^{j}(s)v^{j}(s)ds,$$
(24)

where $Q_j(t) = \int_{kT}^t A^j(s) ds = \int_0^{t-kT} A^j(kT + \tau) d\tau$. Define $w_k^j := \int_0^{T_0} e^{Q_j(T_0 - \tau)} B^j(kT + \tau) v^j(kT + \tau) d\tau$. Then, (24) can be written as $\varepsilon^j(kT + T_0) = e^{Q_j(kT + T_0)} \varepsilon^j(kT) + w_k^j$. Now, consider the second phase of the algorithm. The solution of system (23) is

$$\varepsilon^{j}((k+1)T) = \varepsilon^{j}(kT + T_{0}) + \int_{kT + T_{0}}^{(k+1)T} v^{j}(s) \mathrm{d}s.$$
(25)

Let $\mu_k^j := \int_0^{T-T_0} v^j (kT + T_0 + \tau) d\tau$; then using relation (24) and (25), one obtains $\varepsilon^j ((k+1)T) = e^{Q_j (kT+T_0)} \varepsilon(kT) + w_k^j + \mu_k^j$. Let us now define $\varepsilon_k^j := \varepsilon^j (kT)$, the estimation error vector sampled at the beginning of the cycle, and $\omega_k^j := w_k^j + \mu_k^j$, the input of the discretized system. The resulting discrete-time system can then be formulated as follows:

$$\varepsilon_{k+1}^{j} = e^{\mathcal{Q}_{j}(kT+T_{0})}\varepsilon_{k}^{j} + \omega_{k}^{j}, \qquad (26)$$

where

$$Q_{j}(kT+T_{0}) = \begin{pmatrix} 0 & T_{0} & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & T_{0} \\ -K_{0}^{j}\sigma_{k}^{j} & \cdots & \cdots & -K_{p-1,j}\sigma_{k}^{j} \end{pmatrix},$$
(27)

with $\sigma_k^j := \int_{kT}^{kT+T_0} \frac{1}{m(t)+K_p^j} dt$. Here, (26) is a nonlinear parameter-varying system with unknown parameter σ_k^j . Because $m(t) \in [M_{\min}, M_{\max}], \forall t$, is bounded, the unknown parameter σ_k^j , is also bounded by $\sigma_k^j \in \left[\frac{T_0}{M_{\max}+K_p^j}, \frac{T_0}{M_{\min}+K_p^j}\right]$ for all k. The challenge that remains is to study the inputto-state stability of discrete-time nonlinear system (26) with respect to the input ω_k^j . But before we proceed to this step, we need to evaluate the exponential of the matrix $Q_j(kT + T_0)$ in more detail. Namely, input-to-state stability of (26) implies, firstly, the global uniform asymptotic stability of $\varepsilon^j = 0$ when $\omega^j = 0$ (i.e., when $v^j(t) = 0$: constant human force) and the boundedness of the error ε^{j} for bounded $v^{j}(t)$ (i.e., time-varying human forcing with bounded time derivatives).[¶] For the sake of transparency of notation, denote $Q_{j}(kT + T_{0}) =: Q_{j,k}$.

To compute the matrix exponential $e^{Q_{j,k}}$, we are using a procedure similar to the one introduced in [34] and [35], which employs the Cayley–Hamilton theorem. Herein, it is exploited that if $p(\lambda) = \det(\lambda I_n - A)$ is the characteristic polynomial of a matrix $A \in \mathbb{R}^{n \times n}$, then p(A) = 0.

This means that given the matrix $Q_{j,k}$, for any $i \ge p$, there exists a set of coefficients $c_{i,r}^j \in \mathbb{R}$ such that the *i*th power of $Q_{j,k}$ can be expressed in terms of its first p-1 powers:

$$Q_{j,k}^{i} = c_{i,0}^{j} I_{p} + c_{i,1}^{j} Q_{j,k} + \dots + c_{i,p-1}^{j} Q_{j,k}^{p-1}.$$
(28)

We now exploit (28) to determine the exponential of the matrix $Q_{j,k}$:

$$e^{\mathcal{Q}_{j,k}} = \sum_{i=0}^{\infty} \frac{\mathcal{Q}_{j,k}^{i}}{i!} = \sum_{i=0}^{\infty} \frac{1}{i!} \sum_{r=0}^{p-1} c_{i,r}^{j} \mathcal{Q}_{j,k}^{r},$$
(29)

or

$$e^{\mathcal{Q}_{j,k}} = \sum_{r=0}^{p-1} \mathcal{Q}_{j,k}^r \sum_{i=0}^{\infty} \frac{c_{i,r}^j}{i!}.$$
(30)

Let $q_r^j := \sum_{i=0}^{\infty} \frac{c_{j,r}^j}{i!}$, which means that $e^{Q_{j,k}} = \sum_{r=0}^{p-1} Q_{j,k}^r q_r^j$. Using (27), we can decompose $Q_{j,k}$ as follows: $Q_{j,k} = U + \sigma_k^j L_j$, where

$$U = \begin{pmatrix} 0 & T_0 & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & T_0 \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \text{ and } L_j = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ -K_0^j & \cdots & -K_{p-1,j} \end{pmatrix}.$$
 (31)

Because the matrices U and L_j are noncommutative, we have that

$$\left(U + \sigma_k^j L_j\right)^r = \sum_{s=0}^r \left(\sigma_k^j\right)^s \sum_{t=1}^{C_r^s} \Pi_t \left(U^{r-s}, L_j^s\right),$$
(32)

where $\Pi_t \left(U^{r-s}, L_j^s \right)$ is the *t*-th noncommutative product of r-s matrices U and s matrices L_j and C_r^s is the *r*-combinations from a set with s elements $\left(C_r^s = \frac{s!}{r!(s-r)!} \right)$. But as $\sum_{t=1}^{C_r^s} \Pi_t \left(U^{r-s}, L_j^s \right)$ is independent of the unknown parameter σ_k^j , denote $S_{r,s}^j := \sum_{t=1}^{C_r^s} \Pi_t \left(U^{r-s}, L_j^s \right)$, which allows to write $Q_{j,k} = \left(U + \sigma_k^j L_j \right)^r = \sum_{s=0}^r \left(\sigma_k^j \right)^s S_{r,s}^j$. Consequently, the expression for the exponential matrix becomes

$$e^{Q_{j,k}} = \sum_{r=0}^{p-1} \sum_{s=0}^{r} \left(\sigma_k^j\right)^s q_r^j S_{r,s}^j.$$
(33)

For each scalar product $\left(\sigma_k^j\right)^s q_r^j$, we define

$$\overline{a}_{r,s}^{j} := \max_{\substack{\sigma_{k}^{j} \in \left[\frac{T_{0}}{M_{\max} + K_{p}^{j}}; \frac{T_{0}}{M_{\min} + K_{p}^{j}}\right]}} \left\{ \left(\sigma_{k}^{j}\right)^{s} q_{r}^{j} \right\}$$
(34)

[¶]In essence, we aim to formulate an extension of Theorem 2 in the scope of the cyclic comanipulation algorithm as in Figure 3.

and

$$\underline{a}_{r,s}^{j} := \min_{\substack{\sigma_{k}^{j} \in \left[\frac{T_{0}}{M_{\max} + K_{p}^{j}}; \frac{T_{0}}{M_{\min} + K_{p}^{j}}\right]}} \left\{ \left(\sigma_{k}^{j}\right)^{s} q_{r}^{j} \right\}.$$
(35)

Then, there always exists a $\xi_{r,s}^j \in [0, 1]$ such that

$$\left(\sigma_k^j\right)^s q_r^j = \xi_{r,s}^j \overline{a}_{r,s}^j + \left(1 - \xi_{r,s}^j\right) \underline{a}_{r,s}^j.$$

$$(36)$$

Introducing relation (36) in expression (33) leads to

$$e^{Q_{j,k}} = \sum_{r=0}^{p-1} \sum_{s=0}^{r} \left[\xi_{r,s}^{j} \overline{a}_{r,s}^{j} + \left(1 - \xi_{r,s}^{j}\right) \underline{a}_{r,s}^{j} \right] S_{r,s}^{j}, \tag{37}$$

for some $\xi_{r,s}^j \in [0, 1]$.

Let us define $\Gamma_l^j := \frac{p(p+1)}{2} \overline{a}_{r,s}^j S_{r,s}^j$ and $\delta_l^j := \xi_{r,s}^j \frac{2}{p(p+1)}$, with $l = \frac{r(r+1)}{2} + s + 1$, for $l \in \{1, \dots, \frac{p(p+1)}{2}\}, r \in \{0, \dots, p-1\}, s \in \{0, \dots, r\}.$ Similarly, we define $\Gamma_l^j = \frac{p(p+1)}{2} \underline{a}_{r,s}^j S_{r,s}^j$ and $\delta_l^j = \left(1 - \xi_{r,s}^j\right) \frac{2}{p(p+1)}$, with $l = \frac{p(p+1)}{2} + \frac{r(r+1)}{2} + s + 1$, for $l \in \left\{\frac{p(p+1)}{2} + 1, \dots, p(p+1)\right\}$, $r \in \{0, \dots, p-1\}$, $s \in \{0, \dots, r\}$. This means that expression (37) is equivalent to

$$e^{Q_{j,k}} = \sum_{l=1}^{p(p+1)} \delta_l^j \Gamma_l^j,$$
(38)

with $\sum_{l=1}^{p(p+1)} \delta_l^j = 1$. Thus, we have now found the generators for a convex set that overapproximates the exponential of matrix $e^{Q_{j,k}}$, with the uncertain parameter σ_k^j . Notice that $q_r^j = \sum_{i=0}^{\infty} \frac{c_{i,r}^j}{i!}$ is an infinite sum and will in practice be approximated by a finite sum of length N. Next, we provide an explicit upper bound on the 2-norm of the approximation error induced by such truncation.

Theorem 3

Consider an integer $N \in \mathbb{N}$ and a real positive scalar ϑ_j such that

•
$$\zeta_{j} = \sqrt{\frac{\Lambda_{j}}{\vartheta_{j}}} < 1$$
, where

$$\Lambda_{j} = \max_{\substack{\sigma_{k}^{j} \in \left[\frac{T_{0}}{M_{\max} + K_{p}^{j}}; \frac{T_{0}}{M_{\min} + K_{p}^{j}}\right]} \left\{ \operatorname{eig}\left(\mathcal{Q}_{j,k}^{T} \mathcal{Q}_{j,k}\right) \right\}.$$
(39)
• $\forall i \ge N, \sqrt{\vartheta_{j}^{i}} < i!.$

Then,

$$\left\|\sum_{i=N}^{\infty} \frac{Q_{j,k}^{i}}{i!}\right\|_{2} \leqslant \frac{\zeta_{j}^{N}}{1-\zeta_{j}} \text{ for all } k.$$

$$\tag{40}$$

Proof Appendix II.

Using Theorem 3, we can choose N such that the approximation error is small (even as low as the machine accuracy), and we can correctly evaluate matrices Γ_I^J , which are the generators for the polytopic overapproximation of $e^{Q_{j,k}}$.

Theorem 4

Consider discrete-time systems (26). If there exist matrices $\Omega_j = \Omega_j^T > 0$ and scalars $\zeta_j > 0$, such that the following LMIs are satisfied:

$$\Gamma_l^{jT}\Omega_j\Gamma_l^j - \Omega_j \leqslant -\varsigma_j\Omega_j, l \in \{1, \dots, p(p+1)\},\tag{41}$$

where Γ_l^j are defined earlier, then systems (26) are ISS with respect to the inputs ω_k^j for each $j \in \{1, ..., n\}$.

Proof

Appendix III.

LMIs (41) are defined for the nontruncated Γ_l^j , but in practice, we evaluate the vertex matrices by using a truncation after N iterations as provided by Theorem 3. The errors can be as low as the machine accuracy, just as the errors obtained from the numerical solver of the LMIs. Moreover, we can gain some robustness for these evaluation errors if the scalar ς_i is chosen greater than 0.

4.3.2. Intersample behavior. According to Theorem 2, the error dynamics are ISS for $t \in (kT, kT+T_0]$, which implies that for bounded $\varepsilon^j(kT)$, $\varepsilon^j(t)$ will be bounded for $t \in [kT, kT+T_0)$. Using Theorem 4, we can prove that the error dynamics are ISS on the sampling instance t = kT, with $k \in \mathbb{N}$. To prove the stability of the overall continuous-time system, we need to show that the error dynamics are also bounded for $t \in [kT + T_0, (k + 1)T]$.

The continuous-time error dynamics for $t \in (kT + T_0, (k + 1)T)$ are given by

$$\dot{\varepsilon}^{j}(t) = I_{p} v^{j}(t). \tag{42}$$

The solution of (42) for time $t \in (kT + T_0, (k + 1)T)$ is given by

$$\varepsilon^{j}(t) = \varepsilon^{j}(kT + T_{0}) + \int_{kT + T_{0}}^{t} \upsilon^{j}(s) \mathrm{d}s$$
(43)

or

$$\varepsilon^{j}(t) = \varepsilon^{j}(kT + T_{0}) + \begin{pmatrix} u_{j}(t) - u_{j}(kT + T_{0}) \\ \vdots \\ u_{j}^{(p-1)}(t) - u_{j}^{(p-1)}(kT + T_{0}) \end{pmatrix}.$$
 (44)

Given the fact that u_j is the human force and therefore a bounded signal with bounded time derivatives and using Theorem 2, we know that $\varepsilon^j (kT + T_0)$ is also bounded. Hence, we can conclude that for any $t \in (kT + T_0, (k + 1)T), \varepsilon^j(t)$ is also bounded.

4.4. Scaling the human force

With the maximal human operation frequency, we can determine the period T of one cycle of the algorithm, which includes the estimation and amplification stages, see Figure 3. Using the analysis in Theorem 2, one can obtain an upper bound for the settling time for the estimator (from the expression of the function β^j in (20)). If we consider that the output has settled if the error has dropped below 5% of the initial value, then the settling time for function β^j as in (20) is given by $T_s^j \ge -\frac{4}{\rho_j} \frac{\lambda_{\max}(P_j)}{\lambda_{\min}(P_j)} \ln(0.05) = -\frac{4}{\rho_j} \frac{\lambda_{\max}(P_j)}{\lambda_{\min}(P_j)} (-2.9957) \simeq 3\frac{4}{\rho_j} \frac{\lambda_{\max}(P_j)}{\lambda_{\min}(P_j)}$. The duration of the estimation phase T_0 is chosen to be longer than the maximum settling time for each input–output channels, that is, $\max_{j=1,...,n} \left\{ 3\frac{4}{\rho_j} \frac{\lambda_{\max}(P_j)}{\lambda_{\min}(P_j)} \right\} < T_0 < T$. This means that the scaled force is applied for a time interval of length $T - T_0$ during one cycle, the scaled force is applied for only a fraction of time $(T - T_0)$. Therefore, we have to determine the new scaling factor Ψ , which leads to an

overall scaling factor Φ . The human applies the average force $\frac{1}{T} \int_{kT}^{(k+1)T} F_{\rm H} dt$, over the cycle k. We consider the following assumption:

Assumption 3

 $F_{\rm H}$ is constant during each cycle, that is, $F_{\rm H}(t) = F_{\rm H}(kT), \forall t \in [kT, (k+1)T).$

Because $\frac{1}{T}$ is chosen to be significantly larger than the maximum frequency of human operator, this is a reasonable assumption. Under this assumption, the human applies the force $F_{\rm H}(kT)$, and the robot should apply the force $\Phi F_{\rm H}(kT)$, presumably with $\Phi > 0$. The robot applies the force $F_{\rm R}$, with

$$F_{\rm R}(t) = \begin{cases} \hat{F}(t), \, kT \le t < kT + T_0 \\ \Psi \hat{F}_{\rm H}(kT + T_0), \, kT + T_0 \le t < (k+1)T \end{cases}$$
(45)

Under Assumption 3, systems (15) reach the equilibrium point $\left(\eta_{j}^{(p-1)}, \ldots, \eta_{j}\right)^{T} = \left(0, \ldots, 0, \frac{F_{\mathrm{E},j}}{K_{0,j}}\right)^{T}$. This means that $\hat{F} = -\sum_{i=0}^{p} K_{i} \eta^{(i)} = K_{0} \eta = -\hat{F}_{\mathrm{H}}$. Consequently, the average force supplied by the robot is approximately given by

$$F_{\rm R} = \frac{1}{T} \left(\int_{kT}^{kT+T_0} (-1)\hat{F}_{\rm H}(t) \mathrm{d}t + \int_{kT+T_0}^{(k+1)T} \Psi \hat{F}_{\rm H}(kT+T_0) \mathrm{d}t \right), \tag{46}$$

where Ψ is the scaling factor we have to determine and we have ignored the torque corresponding to the controller C_{lin} because the human force is supposed to move only the load and not the robot links. Let us suppose that $\hat{F}_{\text{H}}(kT + T_0) = F_{\text{H}}(kT + T_0)$, that is, the estimation is working; then by also using Assumption 3, the right-hand side of relation (46) is equivalent to

$$\frac{1}{T} \left(\Psi(T - T_0) F_{\rm H}(kT) - \int_{kT}^{kT + T_0} \hat{F}_{\rm H}(t) \mathrm{d}t \right),\tag{47}$$

where the second term is approximately equal to $\frac{T_0}{T}F_{\rm H}(kT)$. Note that the lower the settling time for the estimation procedure, the better the approximation. Using this approximation and the requirement that $\frac{1}{T}\int_{kT}^{(k+1)T}F_{\rm R}dt = \Phi F_{\rm H}(kT)$, we obtain the following equation from which we can determine the scaling factor Ψ :

$$\Phi F_{\rm H}(kT) = \frac{1}{T} (\Psi(T - T_0) F_{\rm H}(kT) - T_0 F_{\rm H}(kT))$$
(48)

or

$$\Psi = \frac{\Phi T + T_0}{T - T_0}.\tag{49}$$

The estimation/force scaling algorithm is now fully defined, and the design goals have been reached. The human operator now has a supplementary force at his disposal that can enhance his performance by manipulating a larger variety of unknown loads.

5. AN APPLICATION

In Section 5.1, we will first illustrate the effectiveness of the disturbance estimator proposed in Section 3 by means of experiments. In Section 5.2, we present an application of the human–robotic comanipulation strategy proposed in Section 4 to a 2-DOF robotic manipulator.

5.1. Disturbance estimation

5.1.1. Experimental results. The force estimation control scheme presented in Section 3 has been implemented on an experimental 1-DOF haptic device for estimating the human force applied to the device. The mechanical setup has been designed in the Dynamics and Control Laboratory of the Eindhoven University of Technology, the Netherlands, for an eye surgery robotics project (Figure 4). The details of the mechanical design have been presented in [36, 37]. The dynamics of this model are represented in Figure 5 in the upper gray rectangle, where $F_{\rm H}$ is the human force applied to the lever, F is the equivalent force applied by the motor translated to a force acting at the tip of the lever, F_m is the force measured by the force sensor (which is only used for validation purposes), x is the position of the lever tip measured using the motor encoder, m is the system inertia, and F_f is friction force.

The estimation strategy is applied to determine the human force, knowing that the system inertia (composed of the robot mechanical parts and the human hand) ranges between $m \in [0.25, 0.75]$ kg. In Figure 5, the control blocks are presented. The blocks $\frac{s}{\alpha(s)}$ and $\frac{s^2}{\alpha(s)}$ provide estimates for the velocity and the acceleration, respectively, where $\frac{1}{\alpha(s)}$ is a second-order low-pass filter with poles above the human frequency range of 6 Hz, that is, $\alpha(s) = 3.4483s^2 + 3103s + 931035$ to avoid



Figure 4. 1-DOF haptic device for eye surgery [36].



Figure 5. Estimation of the human force on the 1-DOF device.

amplification of high-frequency measurement noise. The parameters K_0 and K_1 are chosen as follows: $K_0 = 27 \cdot 10^6$ and $K_1 = -0.2$. The block F_{fid} provides an estimation of the friction force using the friction model identified in [38]. The results of the estimation procedure are compared with those measured by the force sensor available on the 1-DOF haptic device for the robot without load (Figure 6). Even though the measured force is subject to noise, this figure clearly shows that the estimation algorithm is providing a consistent tracking of the human force. Of course, the results of the estimation method are affected by the accuracy of the model used.

Next, we added an additional load of 0.11 kg to the end effector and applied the same estimation algorithm to estimate the human force in a second experiment. We note that no knowledge on the additional load is used in the estimator and this experiment clearly illustrates (Figure 7) that the estimator is robust against relatively large uncertainties in the load, which is a property not present in conventional DOB (see the next section for additional comparative results).

5.1.2. Comparison with a nonlinear disturbance observer. In this section, we will compare in simulation the force estimation algorithm proposed in the previous section with the nonlinear disturbance observer (NDOB) introduced in [39] and extended in [40]. Both algorithms are applied on a two-link robot model in the horizontal plane, Figure 8. We assume that the links are rigid and the joints



Figure 6. Results of the estimation algorithm on the 1-DOF device (solid line, estimated force; dotted line, measured force).



Figure 7. Results of the estimation algorithm on the 1-DOF device with extra load (solid line, estimated force; dotted line, measured force).



Figure 8. A two-link robot.

are frictionless. The dynamics of the robot can be described by

$$M_{\rm R}(q)\ddot{q} + D_{\rm R}(q,\dot{q}) + m(t)P_{\rm M}(\ddot{q},\dot{q},q) = \tau + J^T(q)F_{\rm H},$$
(50)

where $\tau_{\rm H} = J^T(q)F_{\rm H}$ is the torque applied by the human operator, $\tau = (\tau_1 \quad \tau_2)^T$ represents the actuator torques, and

$$M_{\rm R} = \begin{pmatrix} J_1 + \frac{m_1 l_1^2}{4} + m_2 l_1^2 & \frac{m_2 l_1 l_2}{2} \cos(\theta_2 - \theta_1) \\ \frac{m_2 l_1 l_2}{2} \cos(\theta_2 - \theta_1) & J_2 + \frac{m_2 l_2^2}{4} \end{pmatrix}, \quad D_{\rm R} = \begin{pmatrix} -\frac{m_2 l_1 l_2}{2} \dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \\ \frac{m_2 l_1 l_2}{2} \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) \end{pmatrix},$$
$$P_{\rm M} = \begin{pmatrix} l_1^2 \ddot{\theta}_1 + l_1 l_2 \ddot{\theta}_2 \cos(\theta_2 - \theta_1) - l_1 l_2 \dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \\ l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + l_2^2 \ddot{\theta}_2 + l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) \end{pmatrix}, \quad J = \begin{pmatrix} -l_1 \sin \theta_1 & -l_2 \sin \theta_2 \\ l_1 \cos \theta_1 & l_2 \cos \theta_2 \end{pmatrix}.$$

Herein, l_i , m_i , and J_i are the length, mass, and moment of inertia about the center of mass of link i, i = 1, 2, respectively. Moreover, m(t) represents the mass of the load.

For simulation purposes, we consider the following parametric settings: $l_1 = l_2 = 0.6$ m, $m_1 = m_2 = 2$ kg, and $J_1 = J_2 = \frac{m_1 l_1^2}{12} = 0.06$ kgm² for the robot links. We assume that the mass of the load varies between $M_{\min} = 10$ kg and $M_{\max} = 50$ kg by the law $m(t) = 40e^{-\frac{t}{2}} + 10$ (exponential shape), $t \ge 0$ (we did not choose a quasi-static function for this parameter to show that our algorithm can cope with more uncertainties in our system).

For the estimation phase, we have chosen only one integrator (p = 1) per input–output channel with the gains in (13) given by

$$K_1 = \begin{pmatrix} -5 & 0\\ 0 & -5 \end{pmatrix},\tag{51}$$

$$K_0 = \begin{pmatrix} 10^5 & 0\\ 0 & 10^5 \end{pmatrix}.$$
 (52)

For the NDOB introduced in [39], the defining parameter is the matrix $L = 500I_2$, where I_2 is the 2×2 identity matrix. Moreover, because, on the one hand, this algorithm requires knowledge of the model of the robot with the load and, on the other hand, the mass of the load m(t) is only known to satisfy $m(t) \in [M_{\min}, M_{\max}] = [10, 50]$ kg, $\forall t$, we have chosen a nominal value of 30 kg to be used in the NDOB.

For both simulations, we have considered that position and velocity measurements in the robot joints are available; the acceleration information is obtained by numerical differentiation of the velocity signal. On the model of the robot, we apply an external force $F = \begin{pmatrix} F_x & F_y \end{pmatrix}$ with $F_x = 10\cos(2\pi t), F_y = 10\sin(2\pi t)$, the force in x-direction and y-direction, respectively.



Figure 9. Robustness of the force estimation algorithm for time-varying load mass $m(t) = 40e^{-\frac{t}{2}} + 10$.



Figure 10. Robustness of the force estimation algorithm for time-varying load mass $m(t) = 20 \left(2 \left(\frac{t}{0.5} - \left\lfloor \frac{t}{0.5} - 0.5 \right\rfloor\right) + 1\right) + 10.$

The results for these simulations are presented in Figure 9, where the dashed line is the external (human) force, the dotted line is the estimated force using the algorithm introduced in [39, 40], and the solid line is the force estimated using the algorithm introduced in this paper. For both cases, we have considered null initial conditions for the estimated forces. We have chosen a fast convergence rate for the two estimators because the emphasis of this comparison is on stationary error for the two methods. One can see that our algorithm tracks perfectly the exogenous force, whereas the other algorithm is not robust to the variation of mass of the load. The same algorithm was also applied for a different time-varying load mass $(m(t) = 20 (2 (\frac{t}{0.5} - \lfloor \frac{t}{0.5} - 0.5 \rfloor) + 1) + 10$, which expresses a sawtooth shape), and the results can be seen in Figure 10. One can observe in this figure that the algorithm introduced in this paper is robust against such (unknown) time-dependent variations in the load mass, whereas the algorithm in [39, 40] is not.

5.2. Human-robot comanipulation

In this section, we will apply the estimation and amplification strategy proposed in Section 4 to the two-link robot model in the horizontal plane, see Figure 8, with the time-varying load mass as $m(t) = 40e^{-\frac{t}{2}} + 10$, $t \ge 0$. We assume now a robotic setup under more realistic conditions including friction in the joints and quantization errors for the encoders and for the gyroscopes used to obtain position and velocity measurements, respectively. The (set-valued) friction model (including both Coulomb and viscous friction) that we have introduced for every joint of the two-link robot is defined by

$$F_f(\dot{q}_i) \in a\dot{q}_i + b\operatorname{Sign}\left(\dot{q}_i\right),\tag{53}$$

with $i \in \{1, 2\}$, where Sign(·) represents the set-value-sign function

$$\operatorname{Sign}(x) = \begin{cases} -1, x < 0\\ [-1, 1], x = 0\\ 1, x > 0 \end{cases}$$
(54)

and a = 5 Nm/s and b = 2 Nm. To emphasize the robustness of our algorithm, we do not take into account the friction model in the design of the controller. We have considered the following sensor specifications: encoders with 10⁴ pulses/rad resolution and 16-bit gyroscopes. The acceleration information is again obtained by numerical differentiation of the velocity measurement. The sampling and the control update rates are set to 10 kHz.

Knowing that a human operator can not generate signals with a frequency greater than 6 Hz, the cycle period for our design is T = 0.01 s $(\frac{1}{T} = 100 \gg 6)$. For the estimation phase, we have chosen, as in the previous simulation, only one integrator (p = 1) per input–output channel with parameters given in (51) and (52).

We have solved LMIs (17) for $\rho_j = 4000$ yielding $P_j = 1$ ($\lambda_{\max}(P_j) = \lambda_{\min}(P_j) = 1$), j = 1, 2. Consequently, estimation error dynamics (16) is ISS with respect to $v_j(t)$, j = 1, 2. Now, the ISS result in Theorem 2 provides an ultimate bound on the estimation error of $\frac{4}{\rho_j} \sup(\overline{v}_j(t)) = 0.001 \sup(\dot{u}_j(t))$, j = 1, 2.

The ISS property also provides some important insights for the design of the global controller because it allows to determine the period of the estimation cycle T_0 . The function β^j , j = 1, 2, as in (20), is providing a bound on the convergence rate for the system: $\beta(r,t) = re^{-1000t}$, and the settling time is $T_s = \max_{j=1,2} \left\{ 3\frac{4}{\rho_j} \right\} = 0.003$ s. As a consequence, we have chosen $T_0 = 0.005$ s > T_s .

We assume that the desired value for the scaling parameter Φ is 3, that is, the robot adds a force equivalent to three times the human force. Hence, according to relation (49), $\Psi = 7$.

Regarding the stability of the estimation algorithm over both phases, it is easy to check that $Q^{j,k} = -\frac{K_0^j T_0}{m(t)+K_1^j} \in [-100; -\frac{100}{9}]$. Therefore, we can consider matrices $\Gamma_1^j = e^{-100}$ and $\Gamma_2^j = e^{-\frac{100}{9}}$, $\varsigma_j = 0.5$, and $\Omega_j = 1$ to satisfy LMIs (41). Consequently, the conditions of Theorem 4 are met and, therefore, the discrete-time system is ISS. Concerning the intersample behavior, the argument presented in Section 4.3.2 holds in this particular case as well.

The simulation setup from Figure 2 contains also the human operator. We have emulated the human behavior by a proportional-derivative controller on each input-output channel with a first-order low-pass filter for frequencies higher than 6 Hz and saturation bounds on the human force level. The 'human' controller on each Cartesian direction has been emulated by a linear transfer function:

$$H(s) = \frac{K_d(T_d s + 1)}{T_{PL}s + 1} = \frac{250(1+s)}{\frac{1}{2\pi}s + 1}, \ s \in \mathbb{C}$$
(55)

with saturation at ± 250 N. We have chosen this 'human'-like behavior to emulate the fact that the human response is typically below a 6-Hz bandwidth [32, 33] and that according to common labor legislation, the human worker is not allowed to carry a load with a mass higher than 25 kg,

see [41,42]. The human acts as a motion controller, see Figure 2, aiming to stabilize the set point in end-effector task space given by (x, y) = (0.5, 0.5).

Let us now present the simulation results where the initial condition $(q_1, q_2, \dot{q}_1, \dot{q}_2) = (\pi/4, 0, 0, 0)$ is used. The simulation of the estimation algorithm is presented in Figure 11 for a short time frame (the dashed-dotted line is the human force, and the solid line is the estimated force). Because of the friction in the joint for which there is no compensation mechanism, the estimated force will converge to the sum of the friction force in the joint and the external force. In practice, a model for the friction in the joints will be available, and the residues that will remain because of the inconsistencies between the friction model and the real friction will be small with respect to the external force. Here, we opted for the introduction of relatively large friction force without compensation to illustrate the robustness of the strategy for such model uncertainties.

In Figures 12 and 14, the results obtained by the estimation and amplification control algorithm introduced in this article (solid line) are compared with the use of human force alone (dotted line) and the use of human force in conjunction with the compensation for the dynamics of the robot links (dashed-dotted line), that is, the partial feedback linearization controller τ_{lin} from (9).

This simulation focusses on two important issues of this comparison: the end-effector displacement (Figures 12 and 13) and the applied human force (Figure 14). Figures 12 and 13 show clear differences between the end-effector displacements resulting from the approach proposed in this paper and the case of human actuation (with compensation of the link dynamics). Clearly, the



Figure 11. Comparison human force estimation with applied human force.



Figure 12. Load displacement.



Figure 13. Convergence of the load displacement (lines of 'human' and 'human and links compensation' visually overlap).



Figure 14. Human force (lines of 'human' and 'human and links compensation' visually overlap).

transients in case of the 'estimation and amplification' approach proposed in this paper are more high-frequent and slighter weaker damped. This can be understood from the fact that the amplification strategy amplifies the human action and that the human acts as a motion controller to achieve set point control. Hence, the gain of the 'human motion controller' is effectively increased, causing these changes in the transient behavior. The benefits of the approach of estimating and amplifying the human actuation proposed here are twofold. First, the most important benefit is the fact that the human is supported by the force amplification strategy. In Figure 14, we only selected the first 4 s of the simulation when higher human force input is required. One can see that even though there is a saturation of the human force, our algorithm manages to provide extra force such that the human force will desaturate sooner. Moreover, by using this approach, the set point (in end-effector task space given by (x, y) = (0.5, 0.5) is approached with much higher accuracy, see Figures 12 and 13, than with the other approaches (where the steady-state errors are due to frictional effects incorporated in the simulation, see (53)). The latter effect is again due to the amplification support for the human motion controller. This means that the extra power delivered by the robotic manipulator using the control strategy introduced in this paper ensures a more accurate control for the human over the load.

6. CONCLUSIONS

A new disturbance force estimator algorithm for a force-sensor-less robotic setup has been introduced. The estimation strategy is robust to large uncertainties on the mass of the load that the robotic device is manipulating at the end effector. Moreover, this unknown mass may also quasi-statically vary over time. We have validated the effectiveness of the estimation approach in experiments.

The applications for this force estimation algorithm range from haptic devices to the estimation of compliant contact forces. In this paper, we have applied the proposed estimation algorithm to support a human-robotic comanipulation strategy. Herein, the robotic device enhances the force that a human operator applies to the load. In this application, the force estimation algorithm is used for determining the force applied by the human, in the first step of the strategy and, then, this force is amplified, in the second phase of the strategy.

The perspectives of this study are the extension of the concept to *n*-DOF haptic devices and teleoperation setups.

APPENDIX

I. Proof of Theorem 2

Let $\alpha^{j}(t) = \frac{1}{m(t) + K_{p,j}} \in [\underline{\alpha}^{j}, \overline{\alpha^{j}}], j = 1, ..., n$. Then, $\alpha^{j}(t) = \lambda^{j}(t)\underline{\alpha}^{j} + (1 - \lambda^{j}(t))\overline{\alpha^{j}}, j = 1, ..., n$, with $0 \leq \lambda^{j}(t) \leq 1, \forall t \geq 0$. Hence, the time-varying matrix $A^{j}(t)$ can be written as a convex combination of two matrices A_1^j and A_2^j :

$$A^{j}(t) = \lambda^{j}(t)A_{1}^{j} + (1 - \lambda^{j}(t))A_{2}^{j}, \forall t \in \mathbb{R}^{+}, j = 1, \dots, n,$$
(56)

with A_1^j and A_2^j as in (18) and (19), respectively, and $\lambda^j(t) \in [0, 1], \forall t$. Because there exist $P_j = P_j^T > 0$ and $\rho_j > 0$ such that (17) is satisfied, it holds that

$$P_{j}A^{j}(t) + (A^{j})^{T}(t)P_{j} = P_{j}\left(\lambda^{j}(t)A_{1}^{j} + (1-\lambda^{j}(t))A_{2}^{j}\right) + \left(\lambda^{j}(t)\left(A_{1}^{j}\right)^{T} + (1-\lambda^{j}(t))\left(A_{2}^{j}\right)^{T}\right)P_{j} = \lambda^{j}(t)\left(P_{j}A_{1}^{j} + \left(A_{1}^{j}\right)^{T}P_{j}\right) + (1-\lambda^{j}(t))\left(P_{j}A_{2}^{j} + \left(A_{2}^{j}\right)^{T}P_{j}\right) \leq -\lambda^{j}(t)\rho_{j}P_{j} - (1-\lambda^{j}(t))\rho_{j}P_{j} = -\rho_{j}P_{j}.$$
(57)

Let us define the *P*-norm $|x|_P := \sqrt{x^T P x}$ and consider the candidate ISS Lyapunov functions $V_j = \frac{1}{2} |\varepsilon^j|_{P_j}^2$. The time derivative of V_j along the solutions of (16) satisfies

$$\dot{V}_{j} = \frac{1}{2} \left(\varepsilon^{j} \right)^{T} \left(P_{j} A^{j}(t) + \left(A^{j}(t) \right)^{T} P_{j} \right) \varepsilon^{j} + \left(\varepsilon^{j} \right)^{T} P_{j} B^{j}(t) v_{j}(t).$$
(58)

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Let $\overline{v}_j(t) := B^j(t)v_j(t)$. Then using (57), relation (58) can be written as

$$\dot{V}_j \leq -\frac{1}{2}\rho_j |\varepsilon^j|_{P_j}^2 + |\varepsilon^j|_{P_j} \sup_{t \in \mathbb{R}^+} |\overline{v}_j(t)|_{P_j}$$
(59)

$$\Rightarrow \dot{V}_{j} \leq -\frac{1}{4}\rho_{j}|\varepsilon^{j}|_{P_{j}}^{2} + |\varepsilon^{j}|_{P_{j}} \left(-\frac{1}{4}\rho_{j}|\varepsilon^{j}|_{P_{j}} + \sup_{t\in\mathbb{R}^{+}}|\overline{v}_{j}(t)|_{P_{j}}\right),\tag{60}$$

which means that

$$|\varepsilon^{j}|_{P_{j}} \ge \frac{4}{\rho_{j}} \sup_{t \in \mathbb{R}^{+}} |\overline{v}_{j}(t)|_{P_{j}} \Rightarrow \dot{V}_{j} \le -\frac{1}{4}\rho_{j}|\varepsilon^{j}|_{P_{j}}^{2}.$$
(61)

After straightforward computations, we ultimately arrive at the following implications:

$$\|\varepsilon^{j}\| \ge \frac{4}{\rho_{j}} \sqrt{\frac{\lambda_{\max}(P_{j})}{\lambda_{\min}(P_{j})}} \sup_{t \in \mathbb{R}} (\overline{v}_{j}(t)) \Rightarrow \dot{V}_{j} \le -\frac{\rho_{j}}{4} \lambda_{\min}(P_{j}) \|\varepsilon^{j}\|^{2}.$$
(62)

Define the functions

$$\alpha_1^j(r) := \frac{\lambda_{\min}(P_j)}{2} r^2, \quad \alpha_2^j(r) := \frac{\lambda_{\max}(P_j)}{2} r^2, \quad \alpha_3^j(r) := \frac{\rho_j}{4} \lambda_{\min}(P_j) r^2, \quad \chi^j(r) := \frac{4}{\rho_j} \sqrt{\frac{\lambda_{\max}(P_j)}{\lambda_{\min}(P_j)}} r.$$
(63)

Then, the solution of differential equation (5) is $\mu(r,t) = re^{-\frac{\lambda_{\min}(r_j)}{\lambda_{\max}(P_j)}\frac{\rho_j}{2}t}$. Using these definitions. Definition 2 and Theorem 1 we can conclude that

Using these definitions, Definition 2, and Theorem 1, we can conclude that system (16) is ISS with the functions β^{j} and γ^{j} defined as in (20) and (21), respectively.

II. Proof of Theorem 3

$$\left\|\sum_{i=N}^{\infty} \frac{\mathcal{Q}_{j,k}^{i}}{i!}\right\|_{2} \leq \sum_{i=N}^{\infty} \left\|\frac{\mathcal{Q}_{j,k}^{i}}{i!}\right\|_{2} \leq \sum_{i=N}^{\infty} \frac{\left\|\mathcal{Q}_{j,k}^{i}\right\|_{2}}{i!} \leq \sum_{i=N}^{\infty} \frac{\sqrt{\left(\Lambda_{j}\right)^{i}}}{i!},\tag{64}$$

where the inequality $||A^i||_2^2 \leq ||A||_2^2 \times \ldots \times ||A||_2^2 = \max(\operatorname{eig}((A^T A))^i)$ has been used. Using the property that $\forall a \in \mathbb{R}^+, \exists N \in \mathbb{N}$ such that $\forall i \geq N, \sqrt{a^i} < i!$, inequality (64) becomes

$$\left\|\sum_{i=N}^{\infty} \frac{Q_{j,k}^{i}}{i!}\right\|_{2} \leqslant \sum_{i=N}^{\infty} \frac{\sqrt{\left(\Lambda_{j}\right)^{i}}}{i!} \leqslant \sum_{i=N}^{\infty} \zeta_{j}^{i}.$$
(65)

Let us now employ the known result of convergence of geometric series, which states that $\forall a \in [0, 1]$, $\lim_{n \to \infty} \sum_{i=0}^{n} a^{i} = \lim_{n \to \infty} \frac{1-a^{n+1}}{1-a} = \frac{1}{1-a}$, to obtain

$$\left\|\sum_{i=N}^{\infty} \frac{Q_{j,k}^{i}}{i!}\right\|_{2} \leqslant \frac{\zeta_{j}^{N}}{1-\zeta_{j}} \text{ for all } k.$$
(66)

III. Proof of Theorem 4

By using the Schur complement, relations (41) can be written as

$$\begin{pmatrix} -\Omega_j & \Gamma_l^{j^T} \Omega_j \\ \Omega_j \Gamma_l^j & -\Omega_j \end{pmatrix} \leqslant -\varsigma_j \Omega_j.$$
 (67)

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Multiplying every inequality (67) with δ_I^j and summing them up, we obtain

$$\begin{pmatrix} -\Omega_j \sum_{l=1}^{p(p-1)} \delta_l^j & \sum_{l=1}^{p(p-1)} \delta_l^j \Gamma_l^{j^T} \Omega_j \\ \Omega_j \sum_{l=1}^{p(p-1)} \delta_l^j \Gamma_l^j & -\Omega_j \sum_{l=1}^{p(p-1)} \delta_l^j \end{pmatrix} \leqslant -\varsigma_j \Omega_j \sum_{l=1}^{p(p-1)} \delta_l^j,$$
(68)

which according to (38) is

$$\begin{pmatrix} -\Omega_{j} & e^{\mathcal{Q}_{j,k}T}\Omega_{j} \\ \Omega_{j}e^{\mathcal{Q}_{j,k}} & -\Omega_{j} \end{pmatrix} \leqslant -\varsigma_{j}\Omega_{j}$$

$$(69)$$

or

$$e^{\mathcal{Q}_{j,k}T}\Omega_j e^{\mathcal{Q}_{j,k}} - \Omega_j \leqslant -\varsigma_j \Omega_j.$$

$$\tag{70}$$

Let the candidate Lyapunov function be $V_k^j = (\varepsilon_k^j)^T \Omega_j \varepsilon_k^j$. We compute $\Delta V^j = V_{k+1}^j - V_k^j$:

$$\Delta V_{j} = \left(\varepsilon_{k}^{j}\right)^{T} \left(e^{\mathcal{Q}_{j,k}}\right)^{T} \Omega_{j} e^{\mathcal{Q}_{j,k}} \varepsilon_{k}^{j} - \left(\varepsilon_{k}^{j}\right)^{T} \Omega_{j} \varepsilon_{k}^{j} + 2\left(\varepsilon_{k}^{j}\right)^{T} \left(e^{\mathcal{Q}_{j,k}}\right)^{T} \Omega_{j} \omega_{k}^{j} + \left(\omega_{k}^{j}\right)^{T} \Omega_{j} \omega_{k}^{j},$$
(71)

which according to (70) gives

$$\Delta V_{j} \leq -\varsigma_{j} \left(\varepsilon_{k}^{j}\right)^{T} \Omega_{j} \varepsilon_{k}^{j} + 2 \left(\varepsilon_{k}^{j}\right)^{T} \left(e^{\mathcal{Q}_{j,k}}\right)^{T} \Omega_{j} \omega_{k}^{j} + \left(\omega_{k}^{j}\right)^{T} \Omega_{j} \omega_{k}^{j}.$$
(72)

After some straightforward computations, we can show that

$$\left\|\varepsilon^{j}\right\|_{2} \geq \frac{2}{\varsigma_{j}} \sqrt{\frac{\lambda_{\max}(\Omega_{j})}{\lambda_{\min}(\Omega_{j})}} \sup_{k \in \mathbb{N}} \left(\omega_{k}^{j}\right) \Rightarrow \Delta V_{j} \leq -\frac{\varsigma_{j}}{2} \|\varepsilon^{j}\|_{2}^{2}, \tag{73}$$

where $\lambda_{\max}(\Omega_j)$ and $\lambda_{\min}(\Omega_j)$ are the largest and the smallest eigenvalues of matrix Ω_j , respectively.

Equation (73) implies that system (26) is ISS with respect to the input ω_k^j ; see [28] for sufficient conditions for the ISS of discrete-time systems, in which a discrete-time equivalent of Theorem 1 is presented.

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