Cooperative Adaptive Cruise Control: Tradeoffs Between Control and Network Specifications

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Abstract-In this study, we consider a Cooperative Adaptive Cruise Control (CACC) system which regulates inter-vehicle distances in a vehicle string. Improved performance can be achieved by utilizing information exchange between vehicles through wireless communication besides local sensor measurements. However, wireless communication introduces networkinduced effects that may compromise the performance of the CACC system. Therefore, we approach the design of a CACC system from a Networked Control System (NCS) perspective. Network-induced imperfections in a NCS are mainly due to limited bandwidth of the network, multiple nodes sharing the same medium, and other limitations such as transmission delays and losses. Tradeoffs between CACC performance and network specifications need to be made for achieving desired performance under these network-induced constraints. In this paper, we present a NCS modelling framework that incorporates the effect of sample-and-hold and network delays that occur due to wireless communication. Moreover, we employ this model to study the so-called string stability performance of the string in which vehicles are interconnected by a vehicle following control law and a constant time headway spacing policy. Specifically, we study how string stability is affected by network-induced effects such as delays.

I. INTRODUCTION

The ever increasing demand for mobility in today's life brings additional burden on the existing ground transportation infrastructure for which a feasible solution in the near future lies in more efficient use of currently available means of transportation. For this purpose, development of Intelligent Transportation Systems (ITS) technologies that contribute to improved traffic flow stability, throughput and safety are needed. Cooperative Adaptive Cruise Control (CACC) extends the currently available Adaptive Cruise Control (ACC) with the addition of information exchange between vehicles through Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) wireless communication.

In today's traffic, limited human perception of traffic conditions and reaction characteristics constrain the lower limits of achievable safe inter-vehicle distances. Besides, erroneous human driving characteristics may cause traffic flow instabilities which result in so-called shockwaves. In dense traffic conditions, a single driver overreacting to a momentary disturbance (e.g. a slight deceleration of the predecessor) can trigger a chain of reactions in the rest of the follower vehicles. The amplification of such a disturbance can bring the traffic to a full stop kilometers away from the

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S. Öncü, N. van de Wouw and H. Nijmeijer are with the Department of Mechanical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands s.oncu@tue.nl; n.v.d.wouw@tue.nl; h.nijmeijer@tue.nl disturbance source and cause traffic jams for no apparent reason. ACC vehicles are also limited in their capability of sensing their environment, and therefore, require relatively high following distances for safe and comfortable operation. Information exchange between vehicles provides several advantages over ACC technology and manually driven cars, which can contribute significantly to improving the traffic flow, especially on highways.

One of the earliest studies towards regulating inter-vehicle distances to achieve improved traffic flow dates back to 60's in which the authors formulated the problem in an optimal control design framework [1]. In following years, many practical issues regarding a successful implementation were addressed especially in the scope of the California PATH program such as different inter-vehicle spacing strategies and information flow structures [2], heterogeneous traffic conditions [3], communication delays [4], and actuator limitations [5].

In recent years, developments in the fields of decentralized control of large-scale systems, formation control, decoupling of complex systems, and networked control systems have given rise to research on more systematic approaches from a system theoretical perspective. In [6], [7], [8] the problem is approached from a *spatially invariant systems* perspective. These studies mainly focus on characterizing the interconnection structure for obtaining scalable system properties. Stankovic and Siljak [9], [10] approached the problem from a large-scale systems perspective and used the inclusion principle to decompose the interconnected vehicle string into subsystems with overlapping states for which decentralized controllers were designed.

In this paper, we approach the problem of regulating intervehicle distances in a CACC system from a Networked Control Systems (NCS) perspective. In the fields of NCS, one considers the control of systems over a communication network [11], [12]. In the scope of this paper, control over a wireless communication network is the enabling technology that makes CACC realizable, but very few studies consider the imperfections that are introduced by the network [4], [13], [14]. This is mainly due to the fact that systematic NCS tools arose relatively recently. The main purpose of this paper is to emphasize the necessity for considering CACC in a NCS framework by studying the effects of wireless communication on the performance of an existing CACC controller. Moreover, we also demonstrate how these analyses can provide the designer with guidelines for making the tradeoffs between control and network specifications.

In Section II, we introduce the general control objective,

the underlying longitudinal vehicle dynamics, and the control structure which together form the CACC vehicle model that will be used in the rest of the paper. In Section III, we use the CACC vehicle model to derive the interconnected vehicle string model. We use a lumped system model in order to represent the CACC control laws as state feedback controls that are suitable for the NCS analysis framework. We distinguish between two types of data employed in the CACC controller according to their way of being transferred as locally sensed and wirelessly communicated information. This allows us to consider the local sensor measurements as internal dynamics of the interconnected system and focus on the effects of network imperfections on the wirelessly communicated data. In Section IV, we present the CACC NCS model and derive the discrete-time interconnected system model. In Section V, we present the frequency response based string stability analysis and obtain maximum allowable time delays for different controller and network parameters. In Section VI, we demonstrate the results of the analysis with simulations by using a sampled-data NCS model of a two-vehicle string. The paper ends with discussion on future research directions.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

The general objective of a CACC system is to pack the driving vehicles together as tightly as possible in order to increase traffic flow while preventing amplification of disturbances throughout the string, which is known as string instability [2], [15]. These are two conflicting objectives when conventional methods are considered, since reducing inter-vehicle distances results in shockwaves which adversely affect the global traffic flow. Some other equally important requirements are related to safety, comfort, and fuel consumption, but are not in the scope of this work.

The vehicles forming the platoon are interconnected through the vehicle following objective. Each vehicle is requested to follow its predecessor while maintaining a desired, but not necessarily constant, distance. Here, we consider a constant time headway spacing policy where the desired spacing $(d_{r,i})$ between the front bumper of *i*-th vehicle to its predecessor's rear bumper is

$$d_{r,i} = r_i + h_{d,i} v_i, \tag{1}$$

where *i* is the vehicle index, r_i a constant term that forms the gap between consecutive vehicles at standstill, $h_{d,i}$ is the headway time constant representing the time that it will take the *i*-th vehicle to arrive at the same position as its predecessor when $r_i = 0$ and v_i is the vehicle velocity. For the sake of simplicity, $r_i = 0$ is taken in the rest of the paper since it does not affect the dynamics of the system in scope of this work. For similar reasons, the car length (L_i) will also be taken as zero. The actual distance between two consecutive vehicles (d_i) is

$$d_i = q_{i-1} - (q_i + L_i) = q_{i-1} - q_i,$$
(2)

where q_i is the absolute position of *i*-th vehicle in global coordinates. The local control objective, referred to as vehicle

following, can now be defined as regulating the error

$$e_i = d_i - d_{r,i},\tag{3}$$

to zero in the presence of disturbances. An additional requirement, so-called string stability, involves the global performance of the CACC vehicle string with regard to attenuation of disturbances along the vehicle string and will be defined in more detail at the end of this section.

A. Longitudinal Vehicle Dynamics Model

We use the following linearized third-order state-space representation of the longitudinal dynamics for each vehicle in the string:

$$\dot{q}_{i}(t) = v_{i}(t),
\dot{v}_{i}(t) = a_{i}(t),
\dot{a}_{i}(t) = -\eta_{i}^{-1}a_{i}(t) + \eta_{i}^{-1}u_{i}(t),$$
(4)

where $q_i(t), v_i(t), a_i(t)$ are respectively the absolute position, velocity, acceleration, η_i represents the internal dynamics and u_i is the control input of the *i*-th vehicle. This model is widely used in the literature as a basis of analysis [16]. Equivalently, by using Laplace transforms, $\mathscr{L}(q_i(t)) = Q_i(s)$ and $\mathscr{L}(u_i(t)) = U_i(s)$, the vehicle model can be represented by the following transfer function as in [17]:

$$G_i(s) = \frac{Q_i(s)}{U_i(s)} = \frac{1}{s^2(\eta_i s + 1)}, s \in \mathbb{C}.$$
 (5)

Note that the notational use of small letters for time-domain signals and capital letters for their frequency-domain counterparts will be retained throughout the rest of the paper.

B. Control Structure

Since our main focus is to investigate the network effects, the details of the controller design are omitted in this paper. We use an existing CACC controller design presented in [17], which has been successfully implemented in the scope of the Connect and Drive project and demonstrated with CACC equipped vehicles [18]. Here, we briefly summarize the main components.

The control structure for a single CACC equipped vehicle is as shown in Fig. 1. CACC operation is introduced as an addition to the underlying ACC in a feedforward fashion. The signal conditioning block, $H_i(s) = 1 + h_{d,i}s$ is used to implement the spacing policy given in (1). The feedback



Fig. 1. Control structure block diagram of a single CACC equipped vehicle.

controller $(C_{i,ACC}(s))$ that constitutes the ACC part is a PD-type controller that acts on locally sensed data (e.g. using radar) to perform the vehicle following objective and is given as:

$$U_{fb,i}(s) = C_{i,ACC}(s)E_i(s) = \omega_{k,i}(\omega_{k,i}+s)E_i(s),$$
 (6)

where $\omega_{k,i}$ is the bandwidth of the controller and is chosen such that $\omega_{k,i} << \omega_{g,i} = \frac{1}{\eta_i}$ holds in order to prevent actuator saturation. The time-domain equivalent setting of (6) is obtained by using (3) with (1) and (2) as follows:

$$u_{fb,i} = \omega_{k,i}^2 e_i + \omega_{k,i} \dot{e}_i, = \omega_{k,i}^2 (q_{i-1} - q_i - h_{d,i} v_i) + \omega_{k,i} (v_{i-1} - v_i - h_{d,i} a_i).$$
(7)

For notational convenience, $k_{p,i} = \omega_{k,i}^2$ and $k_{d,i} = \omega_{k,i}$ will be used to represent respectively the proportional and derivative gains of the ACC controller in the rest of the paper. This controller uses the relative distance and relative velocity between the host and the directly preceding vehicle, which are available as sensed measurements through a radar unit that is mounted in front of the vehicle.

Additional feedforward action is utilized to improve tracking performance and forms the CACC part of the controller $(C_{i,CACC}(s)$ in Fig. 1). It uses the acceleration of the directly preceding vehicle (a_{i-1}) . Following well-known design guidelines, the feedforward filter is given as follows:

$$U_{ff,i}(s) = C_{i,CACC}(s)A_{i-1}(s) = \frac{1}{H_i(s)G_i(s)s^2}A_{i-1}(s),$$
(8)

and achieves zero tracking error. Now, using $A_{i-1}(s) = s^2 Q_{i-1}(s) = s^2 G_{i-1}(s) U_{i-1}(s)$, the s^2 terms cancel and (8) can be rewritten as:

$$U_{ff,i}(s) = \frac{1}{H_i(s)G_i(s)}G_{i-1}(s)U_{i-1}(s).$$
 (9)

For a homogenuous vehicle string (i.e. identical vehicles and $G_i(s) = G_{i-1}(s)$), this reduces to

$$U_{ff,i}(s) = \frac{1}{H_i(s)} U_{i-1}(s) = \frac{1}{1 + h_{d,i}s} U_{i-1}(s).$$
(10)

Here, it can be seen that additional dynamics is introduced in the controller due to the velocity-dependent spacing policy, which gives the additional differential equation for the feedforward filter:

$$\dot{u}_{ff,i} = -h_{d,i}^{-1} u_{ff,i} + h_{d,i}^{-1} u_{i-1}, \qquad (11)$$

to be used in the state-space representation of CACC vehicle model.

C. CACC Vehicle Model

The general form of a CACC vehicle model with the given control structure explained in the preceding section is obtained by combining the vehicle longitudinal dynamics equations in (4) with the feedback and feedforward control laws given in (7) and (11):

$$\dot{x}_{i} = A_{i}x_{i} + \underbrace{B_{s,i}u_{i}}_{ACCpart} + \underbrace{B_{c,i}u_{i-1}^{*}}_{CACCpart},$$

$$A_{i} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\eta_{i}^{-1} & 0 \\ 0 & 0 & 0 & -h_{d,i}^{-1} \end{bmatrix},$$

$$B_{s,i} = \begin{bmatrix} 0 \\ 0 \\ \eta_{i}^{-1} \\ 0 \end{bmatrix}, \quad B_{c,i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ h_{d,i}^{-1} \end{bmatrix}, \quad (12)$$

with $x_i^T = [q_i \ v_i \ a_i \ u_{ff,i}]$. $B_{s,i}$ is the input vector corresponding to the input u_i which is generated by using locally available (sensed) data and $B_{c,i}$ is the input vector for the additional CACC input u_{i-1}^* , which is sent to the *i*-th vehicle through the wireless network and is therefore subject to network effects. The reason for this separation denoted with an asterisk (*) will become clear to the reader after we present the sampled-data model in Section IV. A time-domain representation of the feed-back/forward control input with the given spacing policy is

$$u_{i} = u_{fb,i} + u_{ff,i}, = K_{i,i-1}x_{i-1} + K_{i,i}x_{i},$$
(13)

$$K_{i,i-1} = \begin{bmatrix} k_{p,i} \\ k_{d,i} \\ 0 \\ 0 \end{bmatrix}^T, K_{i,i} = -\begin{bmatrix} k_{p,i} \\ k_{p,i}h_{d,i} + k_{d,i} \\ k_{d,i}h_{d,i} \\ -l \end{bmatrix}^T,$$

where l = 1 corresponds to an operational CACC, and l = 0 gives only ACC.

D. String Stability

An important requirement in a CACC system is to avoid amplification of disturbances throughout the string as the vehicle index increases. Hence, stability is not only studied in the time domain, but also in the spatial domain, such as the so-called mesh stability [19]. For 1-D systems this property is called *string stability* and can be quantified by the magnitude of the string stability transfer function [17]:

$$SS_{\Delta_i}(s) = \frac{\Delta_i(s)}{\Delta_{i-1}(s)}, \ i \ge 1,$$
(14)

where $\Delta_i(s) = \mathscr{L}(\delta_i)$ and $\delta_i \in \{q_i, v_i, a_i\}$ is the signal of interest. The string stability condition can then be obtained in the frequency domain $(s = j\omega)$ as follows:

$$|SS_{\Delta_i}(jw)| \le 1, \forall \omega, \ i \ge 1.$$
(15)

III. INTERCONNECTED VEHICLE STRING MODEL

A reference vehicle model (with state x_0) is introduced which may either represent the rest of the traffic as seen by the first vehicle in the string or the trajectory generated by the first vehicle in case there are no preceding vehicles.



Fig. 2. NCS model.

Also, the first CACC vehicle in the string requires special consideration. It is capable of transmitting information to its follower but receives no information from the preceding traffic through the network. By considering these two special cases for the reference and the first vehicles and using the CACC vehicle model in (12) for each operational CACC subsystem, we obtain the following equations for an *n*-vehicle string:

$$\dot{x}_{0} = A_{0}x_{0} + B_{s,0}u_{r},
\dot{x}_{1} = A_{1}x_{1} + B_{s,1}u_{1},
\dot{x}_{2} = A_{2}x_{2} + B_{s,2}u_{2} + B_{c,2}u_{1}^{*},
\vdots
\dot{x}_{n} = A_{n}x_{n} + B_{s,n}u_{n} + B_{c,n}u_{n-1}^{*}.$$
(16)

By defining a new state vector $\Sigma_n = [x_0^T \ x_1^T \ x_2^T \ \cdots \ x_n^T]^T$, we lump *n* subsystems together with the reference model (x_0) and use the input of the reference vehicle model $(u_0 = u_r)$ as the exogenous input to the lumped system. We also substitute the locally available control inputs, according to $(13), u_i = [K_{i,i-1} \ K_{i,i}][x_{i-1}^T \ x_i^T]^T$ to obtain the following vehicle string model that is suitable for the upcoming NCS model analysis:

$$\dot{\Sigma}_n = A_{\Sigma_n} \Sigma_n + B_{\Sigma_n} \widehat{u}^* + B_r u_r, \qquad (17)$$

with

$$A_{\Sigma_n} = \begin{bmatrix} A_0 & 0 & 0 & \cdots & 0\\ \Lambda_{1,0} & \Lambda_{1,1} & 0 & \cdots & 0\\ 0 & \Lambda_{2,1} & \Lambda_{2,2} & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & 0 & \Lambda_{n,n-1} & \Lambda_{n,n} \end{bmatrix},$$

$$B_{\Sigma_n} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ B_{c,2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_{c,n} \end{bmatrix}, B_r = \begin{bmatrix} B_{s,0} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$
$$\Lambda_{j,j-1} = B_{s,j}K_{j,j-1}, \qquad \Lambda_{j,j} = A_j + B_{s,j}K_{j,j},$$

where $j \in \{1, 2, ..., n\}$.

IV. CACC NCS MODEL

The CACC NCS model schematics is shown in Fig. 2. In the model, the constant, though uncertain, network-induced delay denoted by τ is given by

$$\tau = \tau^* + (l-1)h, \quad l \in \mathbb{Z}, \quad \tau^* \in [0,h],$$
 (18)

where h is the constant sampling interval. The data is sent over the network at sampling instants $s_k = kh$. The controller responds instantaneously to newly arrived data. Using the CACC model given in the previous section, continuoustime CACC NCS model for a *n*-vehicle string becomes

$$\dot{\Sigma}_n = A_{\Sigma_n} \Sigma_n + B_{\Sigma_n} \hat{u}^* + B_r u_r, \hat{u}^*(t) = \hat{u}_{k-l+1}, \quad t \in [s_k + \tau^*, s_{k+1} + \tau^*].$$
 (19)

Note that $\hat{u}^*(t)$ is piecewise constant due to the zero order hold (ZOH) used to translate the discrete-time control commands (sent over the wireless network) to the continuoustime input $\hat{u}^*(t)$. The discrete-time NCS model description is based on exact discretization of (19) at the sampling instants $s_k = kh$ by using $\sum_{n,k} = \sum_n (s_k), k \in \mathbb{N}$:

$$\Sigma_{n,k+1} = e^{A_{\Sigma_n}h}\Sigma_k + \int_0^{h-\tau^*} e^{A_{\Sigma_n}s} ds B_{\Sigma_n}\widehat{u}_{k-l+1} + \int_{h-\tau^*}^h e^{A_{\Sigma_n}s} ds B_{\Sigma_n}\widehat{u}_{k-l} + \int_0^h e^{A_{\Sigma_n}s} ds B_r u_{r,k}.$$
(20)

Next, we write the model in state-space notation using the augmented state vector $\xi_k = [\Sigma_{n,k}^T \ \hat{u}_{k-1}^T \ \hat{u}_{k-2}^T \ \dots \ \hat{u}_{k-l}^T]^T$ as in [20]. Then, the discrete-time NCS model is given by

$$\xi_{k+1} = A_{\xi}(\tau, h)\xi_k + B_{\xi}(\tau, h)\widehat{u}_k + \Gamma_r(h)u_{r,k}, \qquad (21)$$

with

$$\begin{split} A_{\xi}(\tau,h) &= \begin{bmatrix} e^{A_{\Sigma_n}h} & M_{l-1} & M_{l-2} & \cdots & M_0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & 0 \\ \vdots & & \ddots & & \cdots \\ 0 & \cdots & 0 & I & 0 \end{bmatrix}, \\ B_{\xi}(\tau,h) &= \begin{bmatrix} M_l^T & I & 0 & \cdots & 0 \end{bmatrix}^T, \\ \Gamma_r(h) &= \int_0^h e^{A_{\Sigma_n}s} ds B_r, \\ M_j(\tau,h) &= \begin{cases} \int_{h-t_{j+1}}^{h-t_j} e^{A_{\Sigma_n}s} ds B_{\Sigma_n} & \text{if } 0 \le j \le 1, \\ 0 & & \text{if } 1 < j \le l, \end{cases} \end{split}$$

where $t_0 := 0$, $t_1 = \tau^*$ and $t_2 := h$. The CACC control inputs $\hat{u}_k = [u_{1,k} \dots u_{n-1,k}]$ are sent through the wireless network. By substituting the control law given in (13) for (n-1) vehicles we obtain

$$\widehat{u}_k = \widehat{K} \Sigma_{n-1,k},$$

with

$$\widehat{K} = \begin{bmatrix} K_{1,0} & K_{1,1} & 0 & \cdots & 0 \\ 0 & K_{2,1} & K_{2,2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & K_{n-1,n-2} & K_{n-1,n-1} \end{bmatrix},$$



(b) Control structure block diagram.

Fig. 3. 2-vehicle string (a) Schematic representation, (b) Control structure block diagram.

which can equivalently be represented as a full state-feedback control law by using the augmented state vector ξ_k as follows:

$$\widehat{u}_k = \begin{bmatrix} \widehat{K} & 0_{(n-1)\times(l+\rho)} \end{bmatrix} \xi_k = K_\xi \xi_k, \qquad (22)$$

where $0_{(n-1)\times(l+\rho)} \in \mathcal{R}^{(n-1)\times(l+\rho)}$ and ρ is such that $x_i \in \mathcal{R}^{\rho}$ (i.e. $\rho = 4$ for the given CACC vehicle model in (12)). Now, we substitute (22) into (21) to obtain the closed loop CACC NCS model:

$$\xi_{k+1} = \overline{A_{\xi}}(\tau, h)\xi_k + \Gamma_r(h)u_{r,k}, \qquad (23)$$

with $\overline{A}_{\xi}(\tau, h) = A_{\xi}(\tau, h) + B_{\xi}(\tau, h)K_{\xi}$. We will use this model in the next section to perform a string stability analysis.

V. DISCRETE-TIME FREQUENCY RESPONSE ANALYSIS

String stability of the discrete-time CACC NCS model is analyzed by using a discrete-time frequency response approach. Similar to the continuous-time frequency domain condition given in Section II, string stability is quantified by the magnitude of the discrete-time string stability transfer function $(SS_{\Delta_i}(z))$, where $z = e^{j\omega}$ is the \mathbb{Z} -transform variable and $\Delta_i(z) = \mathbb{Z}\{\Delta_i(k)\}$. Discrete-time frequency response condition for string stability is then given as

$$|SS_{\Delta_i}(z)| = \left|\frac{\Delta_i(z)}{\Delta_{i-1}(z)}\right| \le 1, \forall \omega, i = 1, ..., n,$$
(24)

where $\Delta_i(z) = \mathcal{Z}(\delta_i)$ and $\delta_i \in \{q_i, v_i, a_i\}$ is the signal whose propagation along the string is of interest. To compute SS_{Δ_i} in (24) we note that

$$\frac{\Delta_{i}(z)}{\Delta_{i-1}(z)} = \frac{\Delta_{i}(z)}{u_{r}(z)} \left(\frac{\Delta_{i-1}(z)}{u_{r}(z)}\right)^{-1},
= \Psi_{\Delta_{i,r}}(z)(\Psi_{\Delta_{i-1,r}}(z))^{-1},$$
(25)

where the discrete-time transfer functions $(\Psi_{\Delta_{i,r}}(z))$ are extracted from (23) by using

$$\Psi_{\Delta_i,r}(z) = C_{\Delta_i}(zI - \overline{A}_{\xi}(\tau, h))^{-1}\Gamma_r(h), \ i = \{1, 2, ..., n\},$$
(26)



Fig. 4. Maximum allowable ratio $(\frac{\tau}{h})$ of time delays (τ) to sampling interval (h) for given vehicle following controllers $(K_1(s) = K_2(s) = \omega_k(\omega_k + s))$ with different bandwidths (ω_k) .

where C_{Δ_i} is chosen accordingly to yield $\delta_i = C_{\Delta_i} \xi_k$.

Here, we demonstrate the string stability analysis approach of the interconnected vehicle string for which the schematic representation and the control structure block diagram are shown in Fig. 3. For the sake of brevity, we use the CACC NCS model presented in the previous section with n = 2vehicles, which is the smallest number of vehicles where string (in)-stability behavior can be observed. The discretetime CACC NCS model was obtained as explained in Section IV for $0 \le \tau \le 6h$ (i.e. l = 6 in (18)). Discrete-time transfer functions are extracted by using (26) with $\delta_i = v_i$ in order to inspect the response of the CACC system to a velocity disturbance. In Fig. 4, the maximum allowable ratio $\left(\frac{\tau}{h}\right)$ of constant time delays (τ) to sampling intervals (h) are shown for string stable operation of the CACC vehicle string. The results are obtained for different bandwidths (ω_k) of the underlying vehicle following controllers denoted by K_1 and K_2 in Fig. 3.(b) for which the control law was given in (6). Maximum allowable ratio of the time delay to sampling interval $\left(\frac{\tau}{b}\right)$ where string stability condition (24) is satisfied is depicted with a color code for different sampling interval (h), headway distance (h_d) pairs.

The analyses show that a high sampling frequency is desired to achieve string stability with relatively low intervehicle distances (h_d) while tolerating large delays. However, from a practical point of view, increasing the sampling frequency limits the number of nodes that can operate reliably in the same network, hence also limiting the number of vehicles in a string. Another observation is related to the selection of the vehicle following controller bandwidths. Although high bandwidth controllers are desirable for better asymptotic tracking, it can be seen that from a string stability point of view, this choice impairs the robustness of the controller against communication delays. Therefore, the design of a CACC system involves making tradeoffs between the vehicle following controller, network performance and string stability performance criteria in the face of network-induced



Fig. 5. Four representative points chosen for comparison.



delays. The presented analyses can be used as guidelines for the designer in making these tradeoffs.

VI. SIMULATION RESULTS

For validation of the results, simulations were performed using the sampled-data NCS model in (19) with n = 2. Frequency domain analysis results given in Fig. 5 were compared with simulation results. In Fig. 6, velocity plots corresponding to the representative (h, h_d) pairs marked in Fig. 5 are given. In the time plots, string stability is evaluated by inspecting amplification of the response to a velocity disturbance. Hence, the peak of the velocity response for the preceding vehicle constitutes the string stability boundary which is marked with the green line in Fig. 6. CACC under ideal conditions (without network effects) is also included for comparison. Simulation results are consistent with the analysis results and demonstrate how string stability performance can be affected by the network.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we presented a Networked Control System (NCS) framework for analyzing the effects of networkinduced impairments on Cooperative Adaptive Cruise Control (CACC) string stability performance. The effect of sampling frequency, zero-order-hold and constant network delays was inspected by modeling the CACC as a NCS model. String stability was studied by using discrete-time frequency response plots. Herewith, the maximum allowable ratio of delays was obtained for various network, vehicle following controller, and inter-vehicle spacing parameters. We demonstrated the validity of the results by performing simulations with the sampled-data NCS model of a 2-vehicle string.

There are many other imperfections and constraints in a NCS, such as variable sampling/transmission intervals, packet dropouts, variable transmission delays, and communication constraints [12]. Future studies will focus on extending the NCS modeling and analysis framework presented in this paper to incorporate these NCS imperfections.

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