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# Performance optimization of piecewise affine variable-gain controllers for linear motion systems $\stackrel{\mbox{\tiny{\%}}}{\to}$



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# ABSTRACT

To circumvent performance-limiting trade-offs encountered in the control of linear motion systems, we introduce a method for designing performance-optimal piecewise affine variable-gain feedback controllers. Variable-gain controllers are known to improve upon the performance trade-off between low-frequency tracking on the one hand and sensitivity to high-frequency disturbances on the other hand. However, the performance-based tuning of such variable-gain controllers is far from trivial. In this paper, we consider a class of variable gain controllers comprising a loop-shaped linear controller and a generic add-on piecewise affine variable gain element. This structure warrants both an intuitive design procedure of the linear part of the control design and a high level of versatility in the design of the nonlinear control part. The add-on piecewise affine controller by means of either a full model-based optimization approach of a certain  $\mathcal{L}_2$  performance indicator or by extending this approach with data-based elements. As a result, the controller design can be tuned for the disturbance situation at hand while optimizing performance. To illustrate the effectiveness of the approach, the proposed performance-based controller synthesis strategy is demonstrated on an industrial wafer scanner.

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# 1. Introduction

The control of industrial motion systems is mostly done using linear controllers of the proportional-integral-derivative (PID) type [1,2]. However, it is well-known that many linear control loops suffer from certain inherent performance trade-offs such as the waterbed effect [3,4]: an increase of low-frequency (below the bandwidth) disturbance suppression automatically yields an increase of noise amplification at high frequencies (above the bandwidth). Given this trade-off, linear motion controllers are designed to balance between low-frequency tracking and sensitivity to high-frequency disturbances. In the control design, this is often obtained by frequency-domain loop shaping [5].

To balance this trade-off in a more desirable manner, it has been shown that variable-gain control (also called N-PID control) can be effective [6-12]. In these references, it has been shown that the variable-gain controllers have the capability of outperforming linear controllers. This might seem contradictory to the well-known

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result that for a linear plant, *in an*  $H_{\infty}$ -*framework*, see [13], a linear controller is optimal and cannot be outperformed by nonlinear or time-varying controllers. However, this result relies on linearity and time-invariance of the plant, the disturbance characterization and the performance specification. In many industrial (motion control) applications, non-stationary disturbances are present and time-varying (and other than  $\mathcal{L}_2$ ) performance specifications are relevant. In these situations, nonlinear or time-varying controllers may yield superior performance over linear controllers.

In [8,10], the underlying linear controller part of a variable-gain controller can be designed based on well-known performancebased loop-shaping arguments, and stability of the nonlinear closed-loop system can be guaranteed by frequency-domain evaluation through the circle criterion [8,14]. Performance of the closed-loop system, however, very much depends on the design of the add-on variable-gain control part, which, in turn, is far from trivial. Typically, the design of this variable-gain control part is based on heuristic rules and depends on the specific application and (often unknown) disturbances at hand. Moreover, the type of nonlinearity is typically chosen *a priori*, e.g. a dead-zone characteristic [8,10] or a saturation characteristic.

This paper explicitly deals with the performance-based *design* of the variable-gain nonlinearity. This problem has, to a certain





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extent, also been addressed in [15], where a more constructive tuning method has been used to tune the parameters of an *a priori fixed* dead-zone characteristic. However, an a priori selection of a fixed nonlinear structure, e.g. based on a dead-zone characteristic, is not likely to induce the best time-domain performance in general as an ideal structure largely relates to the disturbance situation at hand.

In this paper, we develop a synthesis approach for a more general class of piecewise affine variable-gain controllers, as opposed to the tuning procedure for a particular variable-gain controller in [15]. This means that the controller design and tuning tailors the shape of the variable-gain element to the particular disturbance situation at hand. By increasing the number of segments of the piecewise affine structure, arbitrarily shaped characteristics can be constructed, thereby ensuring a high level of versatility of the performance-based nonlinear control design.

Using an  $\mathcal{L}_2$  performance criterion related to the tracking error. an iterative gradient-based quasi-Newton optimization scheme [16] is used for optimal performance-based controller synthesis. Two different approaches are presented in the synthesis of the piecewise affine variable-gain controllers: (1) a purely modelbased approach and (2) a data-based approach, both of which will be studied and compared in this paper. First, we present a computationally efficient model-based approach to synthesize the controllers, which is beneficial in a design-phase where no machine hardware is available yet, in situations where performing many experiments on a machine becomes prohibitive, or when performing (large-scale) parameter studies of the closed-loop system. Second, a *data-based* approach will be presented, which is especially suitable for situations where machine measurements are available and when accurate modeling of the disturbances acting on the system is challenging if not impossible. This paper extends the preliminary results in [17,18] by (1) comparing the model-based optimization approach and the data-based approach and applying both to an experimental motion control system, and (2) applying the data-based piecewise affine variable gain controller synthesis approach to an industrial wafer scanner.

The efficiency of the proposed model-based optimization approach stems from the computationally efficient Mixed-Time-Frequency algorithm [19], which can be used to compute the steady-state response of the closed-loop system with a piecewise affine variable gain controller. Typically, the computation of this steady-state response can be performed orders of magnitude faster than with regular forward integration of the closed-loop model. This makes the model-based method especially suitable for large-scale parameter studies, which generally require extensive simulations, or in a design-phase where no machine hardware is available yet.

The data-based method will adopt the iterative approach from [15], where measured closed-loop error-signals are used in combination with model knowledge in order to compute the gradients of the closed-loop error signals with respect to the variable-gain controller parameters, using a single experiment. The model knowledge, employed in this approach, consists of a controller model (which is exactly known) and a plant model of the system. A plant model is obtained by frequency response measurements. The effects of (unknown) machine-specific disturbances and perturbations are accounted for by the measured closed-loop error signals.

The paper has two main contributions. Firstly, two piecewise affine variable-gain controller synthesis methods are applied to and compared on an experimental motion system: an efficient model-based approach and a data-based machine-in-the-loop approach, which both allow for performance-optimal tuning of the nonlinear controller. This is achieved without making an a priori heuristic choice for the type of nonlinearity, allowing for the synthesis of controllers tailored to the disturbance situation at hand. Additionally, the data-based approach is used to synthesize a performance-optimal piecewise affine variable-gain controller on an industrial wafer scanner.

The remainder of the paper is organized as follows. In Section 2, we introduce the piecewise affine variable-gain control strategy and present conditions for guaranteed closed loop stability. These stability conditions will be used as constraints in the subsequent performance optimization. The model-based and data-based controller synthesis approach, will be presented in Section 3. The effectiveness of the two controller approaches will be assessed on an experimental motion control setup in Section 4. In Section 5, the data-based approach will be applied to one of the motion systems of an industrial wafer scanner. Conclusions are presented in Section 6.

# 2. Piecewise affine variable-gain control structure

Consider the variable-gain control structure depicted in Fig. 1. The nominal (linear) closed loop consists of the linear plant with transfer function  $\mathcal{P}(s), s \in \mathbb{C}$ , and linear feedback controller with transfer function  $\mathcal{C}(s)$ . The add-on variable-gain part of the controller consists of a linear shaping-filter  $\mathcal{F}(s)$  and a nonlinearity  $\varphi(e)$ , which is a continuous piecewise affine function on the tracking error e in the time domain.

Usually, the form of the nonlinearity  $\varphi$  is chosen heuristically based on a certain specific disturbance situation. For example, in the wafer scanning example considered in [8], a typical dead-zone characteristic is chosen for the nonlinearity  $\varphi(e)$ . As such, high-frequency small-amplitude disturbances during scanning are not amplified since they stay within the dead-zone length. Contrarily, low-frequency large-amplitude disturbances prior to scanning are additionally suppressed by the extra gain of the dead-zone nonlinearity for large errors.

It is known that other disturbance situations may require other shapes for the nonlinearity  $\varphi$ . To overcome this problem, we present an approach that avoids making such a heuristic a priori choice for the shape of the nonlinearity  $\varphi$  and enables to synthesize a nonlinearity that is tailor-made for the particular disturbance situation at hand. To facilitate such a general controller synthesis approach, we do not a priori specify a particular type of nonlinearity  $\varphi$ , but construct it on the basis of piecewise affine segments as depicted in Fig. 2. The odd continuous nonlinearity  $\varphi(e)$  with switching lengths  $\delta_i$  consists of *N* segments with slopes  $\alpha_i$ , which are defined as



Fig. 1. Closed-loop variable-gain control scheme.



**Fig. 2.** Piecewise affine variable-gain element  $\varphi(e)$ .

$$\alpha_{i} = \frac{\partial \varphi}{\partial e} \quad \forall \delta_{i-1} < |e| < \delta_{i}, \tag{1}$$

with  $i \in \{1, 2, ..., N\}, \delta_0 = 0$  and  $\delta_N = \infty$ . The nonlinearity may be parameterized as follows:

$$\varphi(e) = \begin{cases}
\alpha_{N}(e + \delta_{N-1}) - \alpha_{1}\delta_{1} - \dots - \alpha_{N-1}(\delta_{N-1} - \delta_{N-2}) & \text{if } e \leq -\delta_{N-1} \\
\vdots \\
\alpha_{2}(e + \delta_{1}) - \alpha_{1}\delta_{1} & \text{if } -\delta_{2} \leq e \leq -\delta_{1} \\
\alpha_{1}e & \text{if } -\delta_{1} \leq e \leq \delta_{1} \\
\alpha_{2}(e - \delta_{1}) + \alpha_{1}\delta_{1} & \text{if } \delta_{1} \leq e \leq \delta_{2} \\
\vdots \\
\alpha_{N}(e - \delta_{N-1}) + \alpha_{1}\delta_{1} + \dots + \alpha_{N-1}(\delta_{N-1} - \delta_{N-2}) & \text{if } e \geq \delta_{N-1}.
\end{cases}$$
(2)

The maximum slope  $\alpha_{max}$  is defined as

$$\alpha_{\max} := \max_{i \in \{1, 2, \dots, N\}} \alpha_i. \tag{3}$$

Note that with this type of piecewise affine construction of the variable-gain element, by choosing *N* large enough, it is possible to generate (approximately) arbitrary continuous nonlinearities. Of course, part of the structure of the nonlinearity is still fixed by choosing the nonlinearity point-symmetric, an assumption that can easily be relaxed at the expense of having to tune additional parameters.

Let us now present conditions for the stability of the closedloop system as in Fig. 1 with piecewise affine variable gain elements as in (2) (see also Fig. 2). These stability conditions will ultimately be employed as constraints in the performance optimization strategies in Section 3. Input-to-state stability [20] of the closed-loop system, with respect to time-varying inputs *r* and *d*, see Fig. 1, can be assessed through circle-criterion arguments [14]. Hereto, the closed-loop dynamics in Fig. 1 can be written as a Lur'e-type system, see Fig. 3, of the following state-space form:

$$\dot{x} = Ax + Bu + B_w w \tag{4a}$$

$$e = Cx + D_w w \tag{4b}$$

$$u = -\varphi(e) \tag{4c}$$

with state  $x \in \mathbb{R}^{n_x}$  and external inputs  $w(t) \in \mathbb{R}^{n_w}$ , which typically consist of  $w(t) = [r(t), d(t)]^T$ , with reference input  $r(t) \in \mathbb{R}$  and force disturbance  $d(t) \in \mathbb{R}$ , see Fig. 1. The linear dynamics from input  $u \in \mathbb{R}$  to output  $e \in \mathbb{R}$  is characterized by the transfer function  $\mathcal{G}_{eu}(s)$ , which can be expressed as

$$\mathcal{G}_{eu}(s) = C(sI - A)^{-1}B = \frac{\mathcal{P}(s)\mathcal{C}(s)\mathcal{F}(s)}{1 + \mathcal{P}(s)\mathcal{C}(s)}.$$
(5)

For a linear system, stability, which can be assessed through the Nyquist criterion, guarantees a bounded state-response under bounded inputs acting on the system. For a nonlinear system, this property of a bounded state-response under bounded inputs, is not



Fig. 3. Lur'e-type description of piecewise affine variable-gain control system.

trivial, and is captured in the notion of input-to-state stability. The following theorem provides sufficient conditions under which system (4) is input-to-state stable (ISS) with respect to the disturbance input *w*.

Theorem 2.1. Consider system (4). Suppose

A1. The matrix A is Hurwitz;

A2. The continuous nonlinearity  $\varphi(e)$  satisfies the sector condition:

$$0 \leqslant \frac{\varphi(e)}{e} \leqslant \alpha_{max},\tag{6}$$

for all  $e \in \mathbb{R}, e \neq 0$ ;

A3 The transfer function  $\mathcal{G}_{eu}(s)$  given by (5) satisfies

$$\operatorname{Re}(\mathcal{G}_{eu}(j\omega)) > -\frac{1}{\alpha_{max}} \quad \forall \omega \in \mathbb{R}.$$
(7)

Then system (4) is ISS with respect to input w.

The proof follows from circle-criterion-type arguments [14,21,15].

**Remark 2.2.** Under a slightly stronger condition on the nonlinearity  $\varphi(e)$ , namely:

A2<sup>\*</sup> The nonlinearity  $\varphi(e)$  satisfies the *incremental* sector condition:

$$0 \leqslant \frac{\varphi(e_2) - \varphi(e_1)}{e_2 - e_1} \leqslant \alpha_{max},\tag{8}$$

for all errors  $e_1, e_2 \in \mathbb{R}, e_1 \neq e_2$ 

system (4) excited by a bounded piecewise continuous disturbance input *w*, will have a unique steady-state solution  $\bar{x}$  which is globally exponentially stable, and bounded for all  $t \in \mathbb{R}$  [10,22]. Systems with such a uniquely defined globally exponentially stable steady-state solution (for arbitrary bounded inputs *w*) are called exponentially convergent, see e.g. [23,24]. Moreover, if the disturbance input *w* is *T*-periodic, the unique steady-state solution  $\bar{x}$  will be *T*-periodic as well.

**Remark 2.3.** Considering the conditions in Theorem 2.1, the following remarks are in place:

- By design of a stabilizing feedback controller C(s) and stable shaping-filter  $\mathcal{F}(s)$ , which may be designed using loop-shaping arguments, the system matrix *A* in (4) will be Hurwitz, or equivalently the transfer function  $\mathcal{G}_{eu}(s)$  in (5) will have all poles in the open left-half complex plane.
- Note that the odd, continuous, piecewise affine nonlinearities  $\varphi$  that we consider in this paper, see Fig. 2 and (2), obey the sector condition (3) in A2 and also the (more strict) incremental sector condition (8) in A2\*.
- The frequency-domain circle-criterion condition (7) can be verified graphically using (e.g. measured) frequency response data. The shaping-filter  $\mathcal{F}(s)$  can be used to shape  $\mathcal{G}_{eu}(s)$  (see (5)) in order to satisfy this condition, which will be illustrated by examples in Sections 4 and 5.

Note that the stability result in Theorem 2.1 does not depend on the switching lengths  $\delta_i$  of the nonlinearity  $\varphi$ . Moreover, if the conditions of Theorem 2.1 are satisfied, the gains  $\alpha_i, i \in \{1, \ldots, N\}$ , can also be chosen freely in the range  $[0, \alpha_{max}]$ . Note, however, that these parameters are in fact very important for the performance of the closed-loop system. The freedom in the choice of the

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switching lengths  $\delta_i$  and additional gains  $\alpha_i$  will be used to design the *performance-optimal* piecewise affine variable-gain controller for the disturbance situation at hand. For this purpose, we propose an iterative  $\mathcal{L}_2$ -based optimization approach in Section 3.

#### 3. Controller synthesis for performance optimization

Typically, in motion control systems, the performance of the system relates to the tracking error e, see Fig. 1. Therefore, we propose the following type of tracking error based  $\mathcal{L}_2$  performance indicator:

$$J = \int_0^T e^2(t)dt,\tag{9}$$

where the interval [0, T] is an application-specific performance window. The goal of the controller synthesis method is to find, possibly in an iterative way, the optimal parameters of the piecewise affine characteristic  $\varphi(e)$ , see Fig. 2, in order to minimize the performance indicator *J* in (9). To this end, we constructively shape the nonlinearity  $\varphi(e)$  for the particular disturbances *w* at hand. Recall that the linear feedback controller C(s) and shaping-filter  $\mathcal{F}(s)$  may be designed using loop-shaping arguments.

Let the to-be-optimized switching lengths  $\delta_i$  and gains  $\alpha_i$  of the nonlinearity  $\varphi$  at an iteration  $\kappa$  be collected in a vector  $\theta_{\kappa} \in \mathbb{R}^{2N-1}$  as

$$\boldsymbol{\theta}_{\kappa} = \left[\alpha_1, \dots, \alpha_N, \delta_1, \dots, \delta_{N-1}\right]^I, \tag{10}$$

for which we search the performance optimal values

$$\boldsymbol{\theta}_{opt} = \underset{\boldsymbol{\theta}_{\kappa}}{\operatorname{argmin}} \boldsymbol{J}.$$
 (11)

The optimization method we consider in this paper is a secondorder gradient-based Quasi-Newton algorithm, see Fig. 4, which is used to minimize the performance indicator J in (9). Depending on whether we employ the model-based or data-based approach, which will be discussed in Sections 3.1 and 3.2 respectively, we will either:

- Compute the error *e*(*t*) using the model (4) of the system and disturbances, with the advantage that this can be done in a computationally efficient manner,
- Perform an experiment to measure the error signal *e*(*t*), with the advantage that both the effect of model uncertainty and disturbances is directly incorporated in the measurements,

see step 1 in Fig. 4. With the obtained error signal e(t), the performance indicator *J* can be computed in step 2, see Fig. 4.

Only if  $J(\theta_{\kappa+1})$  is smaller than  $J(\theta_{\kappa})$  and  $\theta_{\kappa+1}$  lies within a region satisfying the constraints (e.g. induced by the stability conditions in Section 2), see step 3 in Fig. 4, the point  $\theta_{\kappa+1}$  is accepted as



Fig. 4. Schematic of the gradient-based Quasi-Newton optimization algorithm.

the new point, the iteration index  $\kappa$  is incrementally increased by 1, and we proceed to step 5. Otherwise, we proceed to step 4 and perform a line search (looping through steps 1–2–3) until the new point does satisfy these conditions. More detailed information on the constraints on  $\theta_{\kappa}$  (i.e. on the  $\alpha_i$ 's and the  $\delta_i$ 's) will be given in Section 3.3.

If a successful iteration is performed, the iteration index  $\kappa$  is raised by 1 and the new point  $\theta_{\kappa+1}$  is accepted, see step 5 in Fig. 4 (note that not each simulation/experiment in step 1 raises the iteration index, but only successful iterations raise the iteration index  $\kappa$  by 1). In step 6, The gradients  $\frac{\partial I}{\partial ta}(\theta_{\kappa})$  are either:

- Computed using finite-difference approximations in the modelbased approach.
- Computed using a model of the plant and controller and the measured tracking error data in the data-based approach. Details will be given in Section 3.2.

The Hessian estimate  $H_{\kappa}$  is obtained by using subsequent gradient information in a Broyden–Fletcher–Goldfarb–Shanno (BFGS) update:

$$H_{\kappa+1} = H_{\kappa} + \frac{q^T q}{q^T p} - \frac{H_{\kappa}^T p^T p H_{\kappa}}{p^T H_{\kappa} p}, \qquad (12)$$

where  $q = \partial J / \partial \theta(\theta_{\kappa+1}) - \partial J / \partial \theta(\theta_{\kappa}), p = \theta_{\kappa+1} - \theta_{\kappa}$ , and the initial Hessian estimate  $H_0$  is the identity matrix, see [16] for more details.

The following update is used in step 7 in the Quasi-Newton algorithm [16] to update the parameters  $\theta_{\kappa}$  of the piecewise affine nonlinearity:

$$\boldsymbol{\theta}_{\kappa+1} = \boldsymbol{\theta}_{\kappa} - \boldsymbol{H}_{\kappa}^{-1} \left( \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta}_{\kappa}) \right)^{\mathrm{T}},\tag{13}$$

where  $\theta_0$  is the initial parameter setting. Using the new parameters  $\theta_{\kappa+1}$  we return to step 1.

#### 3.1. Model-based controller synthesis

A model-based approach towards piecewise affine variable-gain controller synthesis can be valuable in a design-phase of a motion system, if no machine is yet available. Moreover, if (large-scale) parameter studies are to be conducted, an experimental approach can be prohibitive on an industrial machine, and, hence, a modelbased approach in such a case is more suitable.

When using a model-based approach, a model of the plant  $\mathcal{P}(s)$  and the external disturbances w(t) (e.g. a reference r(t) and/or a force disturbance d(t)) is needed, see Fig. 1. Typically, a sufficiently accurate (non-parametric) plant model can be obtained for motion systems using frequency response function measurements. The feedback controller  $\mathcal{C}(s)$  and shaping filter  $\mathcal{F}(s)$  are designed by the control engineer such that these are exactly known.

If we focus on *T*-periodic disturbances w(t) for the model-based approach, it is known that for a convergent system (4), see Remark 2.2, the steady-state output e(t) will also be *T*-periodic. Practically, *T*-periodic inputs will often occur due to the periodicity of the setpoints r(t), while external disturbances such as d(t) are often of a much higher frequency than the set-point such that these can be modeled (approximated) as being periodic. Given a *T*-periodic disturbance, and if conditions A1, A3 in Theorem 2.1 and condition A2\* in Remark 2.2 hold, the computationally efficient Mixed-Time–Frequency (MTF) algorithm [19] can be used to compute the *T*-periodic steady-state error  $e(\theta_{\kappa})$  at iteration  $\kappa$ , see step 1 in Fig. 4, and the corresponding performance  $J(\theta_{\kappa})$  in (9), see step 2 in Fig. 4. The main steps of the MTF algorithm can be summarized as follows (more details can be found in [19]):

- 1. Set MTF iteration index l = 0, set  $\epsilon_E > \epsilon_{reltol}$ , with  $\epsilon_{reltol}$  a termination parameter.
- 2. Compute the Fourier coefficients *W* of *w* using the Fast Fourier Transform (FFT).
- 3. Choose an arbitrary initial guess for the Fourier coefficients  $E_0$  of the steady-state solution  $e_0(t)$  (e.g. all zeroes).
- 4. Compute the time-domain signal  $e_0(t)$  from  $E_0$  using the Inverse Fast Fourier Transform (IFFT).
- 5. While  $\epsilon_E > \epsilon_{reltol}$ :
  - (a) Evaluate the nonlinearity  $u_{l+1}(t) = -\varphi(e_l(t))$  as in (4c) in time-domain.
  - (b) Compute the Fourier coefficients  $U_{l+1}$  of  $u_{l+1}(t)$  using the FFT.
  - (c) Evaluate the linear dynamics (4a) and (4b) in frequency domain:  $E_{l+1} = \mathcal{G}_{ew}W + \mathcal{G}_{eu}U_{l+1}$ .
  - (d) Compute  $e_{l+1}(t)$  from  $E_{l+1}$  using the IFFT.
  - (e) Check convergence of MTF algorithm, if

$$\epsilon_E = \frac{\|E_{l+1} - E_l\|}{\|E_l\|} > \epsilon_{reltol},\tag{14}$$

then continue, otherwise terminate the while loop and go to step 6.

(f) Raise MTF iteration index: *l* = *l* + 1, and go back to step 5a.
6. Set *e*(*θ<sub>κ</sub>*) = *e*<sub>*l*+1</sub>,

where  $\mathcal{G}_{ew} = C(sI - A)^{-1}B_w + D_w$  and  $\mathcal{G}_{eu}$  in (5) are the transfer functions from *w* to *e* and *u* to *e*, respectively, and  $\epsilon_{reltol}$  a parameter to determine sufficient convergence of the algorithm (e.g.  $\epsilon_{reltol} = 1e - 8$ ). The efficiency of the MTF algorithm hinges on the fact that the linear dynamics, see step (5c) and the upper part in Fig. 3, can be evaluated very efficiently in frequency domain, and the nonlinearity can be computed very efficiently in time-domain, see step (5a) and the bottom part in Fig. 3, hence the name Mixed-Time-Frequency algorithm. The algorithm converges to the unique steady-state solution for any initial guess and can be made as accurate as desired by increasing the number of Fourier coefficients considered.

In the model-based approach the gradients  $\partial J / \partial \theta$ , see step 6 in Fig. 4, can be efficiently and accurately obtained by finite-difference approximations. For example, the gradient of *J* with respect to parameter  $\delta_1$  can be simply obtained as

$$\frac{\partial J}{\partial \delta_1} \approx \frac{J(\delta_1 + \Delta) - J(\delta_1)}{\Delta},\tag{15}$$

for some finite difference  $\Delta$ , which in a model-based approach can be chosen small in order to obtain an accurate gradient estimate.

# 3.2. Data-based controller synthesis

In case machine measurements are available, the data-based machine-in-the-loop method discussed in this section can prove useful. This method in particular avoids the need to model disturbances w(t) acting on the system, which otherwise may be a challenging task in practice on an industrial machine, as for example on the industrial wafer scanner which will be considered in Section 5.

In order to arrive at a model formulation tailored for a databased machine-in-the-loop method, consider the following discrete-time representation of system (4) (with inputs w(t) consisting of a reference input r(t) and a force disturbance d(t), see Fig. 1):

$$\mathbf{x}(j+1) = \widehat{A}\mathbf{x}(j) + \widehat{B}\mathbf{u}(j) + \widehat{B}_r \mathbf{r}(j) + \widehat{B}_d d(j), \tag{16a}$$

$$e(j) = \widehat{C}x(j) + \widehat{D}_r r(j) + \widehat{D}_d d(j), \tag{16b}$$

$$u(j) = -\varphi(e(j)), \tag{16c}$$

where  $j \in \{1, ..., k\}$  denotes the discrete time-counter, i.e. e(j = 1) = e(t = 0) and e(j = k) = e(t = T) denotes the relation

between the sampled-data signal and continuous-time signal. Note that the system matrices in (16) depend on the discretization scheme and sampling rate  $t_s = T/(k-1)$  used. The linear part of (16) can be put in lifted form, see [18]:

$$\boldsymbol{e}_{\kappa} = \boldsymbol{G}_{eu}\boldsymbol{u}_{\kappa} + \boldsymbol{G}_{er}\boldsymbol{r} + \boldsymbol{G}_{ed}\boldsymbol{d}, \qquad (17a)$$

$$\boldsymbol{u}_{\kappa} = -\boldsymbol{\varphi}(\boldsymbol{e}_{\kappa}), \tag{17b}$$

where  $G_{eu}, G_{er}, G_{ed} \in \mathbb{R}^{k \times k}$  are Toeplitz matrices containing the impulse responses of the relevant transfer functions between u, r, d and error e respectively, see (5) for transfer function  $\mathcal{G}_{eu}(s)$ , and where  $\mathbf{r} = [r(1), \ldots, r(k)]^T$  and  $\mathbf{d} = [d(1), \ldots, d(k)]^T$ . At each iteration  $\kappa$  the sampled-data error signals are given by  $\mathbf{e}_{\kappa} \in \mathbb{R}^k$ , which can be measured on the machine with the current parameter setting  $\theta_{\kappa}$  and can straightforwardly be used to compute the sampled-data equivalent to the  $\mathcal{L}_2$  performance measure (9):

$$J = \boldsymbol{e}_{\kappa}^{T} \boldsymbol{e}_{\kappa}. \tag{18}$$

We will use (17) in the determination of the gradients  $\partial J/\partial \theta(\theta_{\kappa})$ . Particularly, we will determine the gradients using a combined model/data based approach using, on the one hand, a model of the motion system  $\mathcal{P}(s)$  (which is the same model as which is used in the model-based approach of Section 3.1),  $\mathcal{C}(s)$ ,  $\mathcal{F}(s)$  and, on the other hand, the measured error signals  $\boldsymbol{e}_{\kappa}$ , which account for the (unknown) external disturbances acting on the system. Note that non-parametric plant models of sufficient accuracy are often available from model fitting on measured FRF data, and that the measured error signals  $\boldsymbol{e}_{\kappa}$  are part of the iterative procedure, such that essentially no additional experiments are required for gradient estimation.

Note that the gradient of an  $\mathcal{L}_2$  performance measure as in (18) is given by

$$\frac{\partial J}{\partial \theta} = 2\boldsymbol{e}_{\kappa}^{T} \frac{\partial \boldsymbol{e}_{\kappa}}{\partial \theta}, \tag{19}$$

from which it follows that we need the gradients of the error signal  $e_{\kappa}$  with respect to the parameters  $\theta$ :

$$\frac{\partial \boldsymbol{e}_{\kappa}}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial \boldsymbol{e}_{\kappa}}{\partial \alpha_{1}}, & \dots, & \frac{\partial \boldsymbol{e}_{\kappa}}{\partial \alpha_{N}}, & \frac{\partial \boldsymbol{e}_{\kappa}}{\partial \delta_{1}}, & \dots, & \frac{\partial \boldsymbol{e}_{\kappa}}{\partial \delta_{N-1}} \end{bmatrix}.$$
(20)

Using (17), we can write the gradients with respect to the switching lengths  $\delta_i$  and  $\alpha_i$  as (see [18])

$$\frac{\partial \boldsymbol{e}_{\kappa}}{\partial \delta_{i}} = -\left(I + \boldsymbol{G}_{eu} \frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{e}_{\kappa}}(\boldsymbol{e}_{\kappa})\right)^{-1} \boldsymbol{G}_{eu} \frac{\partial \boldsymbol{\varphi}}{\partial \delta_{i}}(\boldsymbol{e}_{\kappa}), \tag{21a}$$

$$\frac{\partial \boldsymbol{e}_{\kappa}}{\partial \alpha_{i}} = -\left(I + \boldsymbol{G}_{eu} \frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{e}_{\kappa}}(\boldsymbol{e}_{\kappa})\right)^{-1} \boldsymbol{G}_{eu} \frac{\partial \boldsymbol{\varphi}}{\partial \alpha_{i}}(\boldsymbol{e}_{\kappa}), \tag{21b}$$

where  $\partial \varphi / \partial \delta_i(\boldsymbol{e}_k) = [\partial \varphi / \partial \delta_i(\boldsymbol{e}(1)), \dots, \partial \varphi / \partial \delta_i(\boldsymbol{e}(k))]^T \in \mathbb{R}^k$  and the diagonal matrix  $\partial \varphi / \partial \boldsymbol{e}_k(\boldsymbol{e}_k) \in \mathbb{R}^{k \times k}$  with diagonal entries  $[\partial \varphi / \partial \boldsymbol{e}(\boldsymbol{e}(1)), \dots, \partial \varphi / \partial \boldsymbol{e}(\boldsymbol{e}(k))]$ , and where the gradients  $\partial \varphi / \partial \boldsymbol{e}, \partial \varphi / \partial \delta_i$   $\forall i \in \{1, \dots, N-1\}$  and  $\partial \varphi / \partial \alpha_i \quad \forall i \in \{1, \dots, N\}$  are depicted in Fig. 5. The gradients as in Fig. 5 can easily be derived by considering the explicit formulas for the line segments of the piecewise affine function in (2).

The expressions for the gradients  $\partial e_{\kappa}/\partial \alpha_i$  and  $\partial e_{\kappa}/\partial \alpha_i$  in (21) are clearly both model and data based. The model-part comes from the Toeplitz matrix  $G_{eu}$ , based on the sampled-data model (16), which is obtained through a model of the plant  $\mathcal{P}(s)$  and the exactly known controller  $\mathcal{C}(s)$  and shaping filter  $\mathcal{F}(s)$ . The data-part comes from the measured error signals  $e_{\kappa}$  which are obtained from the experiments and accounts for the unknown external disturbances acting on the system.



**Fig. 5.** Piecewise affine nonlinearity and the gradients  $\frac{\partial \varphi}{\partial e}, \frac{\partial \varphi}{\partial \delta_i}$  and  $\frac{\partial \varphi}{\partial \alpha_i}$ .

With the estimated gradient  $\partial e_{\kappa}/\partial \theta$ , the gradient  $\partial J/\partial \theta$  in (19) can be computed, which can be used in the Quasi-Newton optimization scheme, see step 6 in Fig. 4. As such, a machine-in-the-loop optimization approach is developed that optimizes for the best performance in terms of (18).

**Remark 3.1.** Of course, also other optimization methods can be used to find the optimal parameters minimizing the performance indicator *J*. In particular, any gradient-based optimization routine, such as e.g. Gauss–Newton [15], can directly be employed in combination with the estimated gradients in (19).

**Remark 3.2.** Although no rigorous results are posed on (local) convexity of the optimization problem, from simulations and experiments it follows that this is the case. In this paper, 3 parameters are being optimized, which does not allow for the visualization of the objective function as a function of these parameters. In [19], 2 parameters of a fixed dead-zone structure are being optimized, which can be visualized, and where it is clear that the optimization problem considered there is locally convex. On the basis of the results in this paper, we expect such property to remain valid in case of higher dimensions as considered in this paper.

# 3.3. Optimization constraints

The parameter vector  $\theta_{\kappa}$  of the piecewise affine variable-gain controller that will be optimized contains the switching lengths  $\delta_i$  and the gains  $\alpha_i$ . The frequency-domain condition A3 in Theorem 2.1 gives an upper-bound for the maximum gain  $\alpha_{max}$  that can be used such that input-to-state stability of the closed-loop system can be guaranteed. Hence, the parameters  $\alpha_i$  should be constrained to

$$0 \leqslant \alpha_i \leqslant \alpha_{max} \quad \forall i \in \{1, \dots, N\}.$$
(22)

Also for the  $\delta_i$  parameters, we formulate certain constraints (although not needed from a stability point of view). In order to explain the rationale behind these constraints, consider the case that we have N = 2 segments, see Fig. 2, such that we have only one switching length, namely  $\delta_1$ , and two gains  $\alpha_1$  and  $\alpha_2$ . If  $\delta_1 = 0$ , the gain  $\alpha_1$  is completely irrelevant, and hence, the gradient  $\partial J/\partial \alpha_1 = 0$ . As a consequence,  $\alpha_1$  will not be altered, most likely preventing the optimization from finding the optimal piecewise affine variable-gain controller. Similarly, if the measured error signal

does not exceed  $\delta_1$ , i.e.  $\delta_1 \ge \max_{j \in \{1,...,k\}} |e(j)|$ , the gain  $\alpha_2$  will be completely irrelevant, again preventing  $\alpha_2$  from being adapted. To circumvent these difficulties, the following constraints are formulated for the optimization:

$$\delta_1 \ge \epsilon_1,\tag{23a}$$

$$\delta_1 \leqslant \epsilon_2 \max_{j \in \{1, \dots, k\}} |e(j)|, \tag{23b}$$

for some parameters  $\epsilon_1 > 0$  and  $0 \le \epsilon_2 \le 1$ , e.g.  $\epsilon_2 = 0.9$ . Under these constraints,  $\alpha_1$  and  $\alpha_2$  will always influence the response, which will increase the possibility of finding the optimal variable-gain controller parameters. An explicit illustration of the positive effect of including these constraints can be found in [17].

In the general case of N - 1 different  $\delta_i$  parameters, the constraints can be formulated as

$$\delta_i - \delta_{i-1} \ge \epsilon_1 \quad \forall i \in \{1, \dots, N-1\},$$
(24a)

$$\delta_{N-1} \leqslant \epsilon_{2\max_{j \in \{1,\dots,k\}}} |e(j)|. \tag{24b}$$

**Remark 3.3.** Since the performance optimization approach is a gradient-based one, and the optimization problem is in general not convex, there will of course be no straightforward guarantees that a global optimum of the performance map will be found, as with any gradient-based optimization scheme. Starting the optimization for different initial parameter sets, as we will do in Sections 4 and 5, can increase the likelihood of finding the global optimum. Additionally, as mentioned, the constraints (22) and (24) further support finding the global optimum, see also [17]. Furthermore, if the problem exhibits an increasingly large number of local optima, it can be worthwhile to investigate optimization strategies which are specifically tailored towards finding global optima, such as for example simulated annealing or ideas from the artificial intelligence community such as particle swarm optimization.

#### 4. Comparative analysis for a motor-load motion system

In this section, we apply and compare both the model-based and the data-based piecewise affine variable-gain controller synthesis approach to a 4th-order motion (motor-load) system consisting of two rotating inertias interconnected by a flexible shaft, see Fig. 6. We consider non-collocated actuation, i.e. the case in which the measurement by the encoder (at the left side) will be separated from the actuation by the motor (at the right side).

# 4.1. Modeling and controller design of the motor-load system

Consider the measured frequency response function of the plant  $\mathcal{P}(j\omega)$  in Fig. 7, obtained by closed-measurements with a sample frequency of 4 kHz. From this frequency response data, a 4th-order plant model  $\mathcal{P}(s)$  is estimated:

$$\mathcal{P}(s) = \frac{5.173e8}{s^4 + 5.484s^3 + 1361e5s^2}.$$
(25)



Fig. 6. Experimental motor-load setup consisting of two interconnected rotating inertias.



Fig. 7. Measured frequency response function and 4th-order model fit.

This plant model is used in the simulations of the model-based approach, see Section 3.1, and is used in the determination of the gradients in the data-based approach, see Section 3.2.

A nominal linear low-gain controller C(s) (corresponding to  $\varphi(e) = 0$ , see Fig. 1) has been designed, using loop-shaping arguments [5], consisting of a lead-filter, notch-filter, integrator, and a 2nd-order low-pass filter:

$$C(s) = \frac{1.216e - 7s^4 + 3.942e - 6s^3}{8.510e - 15s^6 + 2.727e - 11s^5} \\ \cdots \frac{+1.674s^2 + 0.4551s + 2.199}{+4.045e - 8s^4 + 2.951e - 5s^3 + 9.602e - 3s^2 + s}.$$
 (26)

Stability of the linear system can easily be checked by verifying the Nyquist criterion. (Input-to-state) stability of the nonlinear system, however, needs to be assessed through the conditions of Theorem 2.1. Condition A3 of Theorem 2.1 is important for the design of the shaping filter  $\mathcal{F}(s)$ , see Fig. 1. Tuning of  $\mathcal{F}(s)$  aims at adding a significant amount of allowable additional gain  $\alpha_{max}$  while satisfying the circle-criterion condition (7). Consider Fig. 8, where  $\mathcal{G}_{eu}(j\omega)$ , see (5), is plotted for the case without shaping filter  $\mathcal{F}(s)$  (i.e.  $\mathcal{F}(s) = 1$ ). If no shaping filter  $\mathcal{F}(s)$  is used, the maximum additional gain that can be put on the system is  $\alpha_{max} = -1/-0.75 = 1.3$ . By using a notch filter



**Fig. 8.** Circle criterion condition  $\operatorname{Re}(\mathcal{G}_{eu}(j\omega)) > -1/\alpha_{max} \ \forall \omega \in \mathbb{R}$  with  $\alpha_{max} = 3$ .

 $\mathcal{F}(s) = (\omega_p/\omega_z)^2 (s^2 + 2\beta_z \omega_z s + \omega_z^2) / (s^2 + 2\beta_p \omega_p s + \omega_p^2), \quad \text{where}$   $\omega_p = \omega_z = 17 \cdot 2\pi \text{ rad/s}, \ \beta_p = 2, \ \text{and} \ \beta_z = 0.4, \ \text{a higher additional}$ gain  $\alpha_{max} = 3$  is allowed as indicated by the dashed vertical line in the circle criterion plot of Fig. 8. Considering piecewise affine nonlinearities as in Fig. 2 with  $\alpha_i \leq \alpha_{max} \quad \forall i \in \{1, \dots, N\}$ , see also Remark 2.3, guarantees that condition A2 (and also A2\*, see Remark 2.2) is satisfied. Lastly, since  $\mathcal{C}(s)$  has been designed to be a stabilizing controller, and  $\mathcal{F}(s)$  is a stable filter, condition A1 of Theorem 2.1 is satisfied, see also Remark 2.3. As a result, conditions A1–A3 are satisfied, which guarantees the closed-loop system with piecewise affine variable-gain controller is inputto-state stable.

**Remark 4.1.** A possible extension to the piecewise affine variablegain controller synthesis could be to optimize the parameters of the filter  $\mathcal{F}(s)$  for performance as well. However, through the circle-criterion condition (7) this would also directly influence stability conditions. Whereas now, stability guarantees and performance optimization are separated, such an extension would destroy such separation and render the optimization problem more complex (even more so since more parameters would have to be optimized for).

The nominal linear low-gain controller, see Fig. 1 with  $\varphi(e) = 0$ , is given by C(s). The linear high-gain controller, see Fig. 1 with  $\varphi(e) = \alpha_{max}e$ , is given by  $C(s)(1 + \alpha_{max}\mathcal{F}(s))$ . The high-gain controller induces better low-frequency tracking properties, but due to the waterbed effect, this also induces a (high) frequency region in which the tracking properties degrade. This waterbed effect is clearly visible in the sensitivity characteristics of the closed-loop system,

$$S(j\omega) = \frac{1}{1 + \mathcal{P}(j\omega)\mathcal{C}(j\omega)},\tag{27}$$

$$\mathcal{S}(j\omega) = \frac{1}{1 + \mathcal{P}(j\omega)\mathcal{C}(j\omega)(1 + \alpha_{max}\mathcal{F}(j\omega))},$$
(28)

for the low-gain and high-gain setting, respectively, see Fig. 9.

## 4.2. Disturbance specification

In this section, we will specify two illustrative disturbance situations. We will consider these particular disturbance situations in order to clearly illustrate that the optimal piecewise affine nonlinearity  $\varphi$  depends on the disturbance situation at hand, and that this optimal  $\varphi$  can be designed using the model-based and databased optimization approaches. We consider two *T*-periodic reference signals r(t), with T = 2, that should be tracked (and set d(t) = 0 here, see Fig. 1), both of which are shown in Fig. 10. Both non-stationary disturbances have low-frequency (below the bandwidth) contents of 5 Hz at the start, and high-frequency (above the



**Fig. 9.** Sensitivity characteristics  $S(j\omega) = 1/(1 + P(j\omega)C(j\omega))$  and  $S(j\omega) = 1/(1 + P(j\omega)C(j\omega))(1 + \alpha_{max}F(j\omega)))$  of the experimental setup for the low-gain and high-gain controller setting, respectively. The dashed vertical lines indicate the two frequencies used in the reference signal r(t) in Fig. 10.



**Fig. 10.** One period of r(t) for disturbance situation 1 and 2 for the motor-load system.

bandwidth) contents of 15 Hz at the end, see Figs. 9 and 10. In Section 5, an industrial wafer scanner will be considered in which the disturbances acting on the system are unknown (and typically more complex).

#### 4.3. Performance measure

The performance measure we use to quantify the performance of the nonlinear controllers is of the integral squared error type (9). However, in order to quantify the effect of low-frequency (i.e. below the bandwidth) performance improvement and highfrequency (i.e. above the bandwidth) performance degradation when increasing the controller gain, we propose to use the following  $\mathcal{L}_2$  steady-state performance indicator:

$$J = c_{lf}J_{lf} + c_{hf}J_{hf}, (29)$$

where

$$J_{lf} = \int_0^T e_{lf}^2(t) dt, \quad J_{hf} = \int_0^T e_{hf}^2(t) dt,$$
(30)

where T = 2 s is the period of the reference r(t) in Fig. 10, and  $e(t) = e_{lf}(t) + e_{hf}(t)$ . The low-frequency part of the steady-state error  $e_{lf}(t)$  is obtained by low-pass filtering<sup>1</sup> the error signal e(t) with a 2nd-order low-pass filter  $C_{lp}(s) = \omega_{lp}^2 / (s^2 + 2\beta_{lp}\omega_{lp}s + \omega_{lp}^2)$ , where  $\omega_{lp} = 10 \cdot 2\pi$  rad/s (around the bandwidth), and  $\beta_{lp} = 0.7$ . The high-frequency part is given by  $e_{hf}(t) = e(t) - e_{lf}(t)$ . The coefficients  $c_{lf} \ge 0$  and  $c_{hf} \ge 0$  in (29) allow to balance the importance of both the low-frequency and high-frequency part of the error. In this section, the nominal linear low-gain controller is normalized to yield J = 1. This is achieved by selecting  $c_{lf}$  and  $c_{hf}$  in such a way that

 $c_{if}J_{if} = c_{hf}J_{hf} = 0.5$ , i.e. the low-frequency and high-frequency part of the error signal are considered equally important.

**Remark 4.2.** Note that since the low-pass filtering operation is linear, the gradients  $\partial \mathbf{e}_{lf}/\partial \theta$  can simply be obtained by computing the low-pass filtered version of  $\partial \mathbf{e}/\partial \theta$  in (20). The high-frequency part simply follows from  $\partial \mathbf{e}_{hf}/\partial \theta = \partial \mathbf{e}/\partial \theta - \partial \mathbf{e}_{lf}/\partial \theta$ .

The piecewise affine nonlinearity  $\varphi(e)$  we will consider in the following subsections consists of N = 2 segments, see Fig. 2. In general, it is recommended to start with a small value of N in order to keep the computational burden of the optimization problem low. Especially in situations where two distinct controllers can be preferred at different times, it is expected that a piecewise affine variable-gain controller with N = 2 can already yield improved performance. For more complex disturbance situations, nonlinearities with N > 2can possibly yield improved performance, but at the expense of computational complexity of the optimization problem.

The constraint in (22) is used for the gains  $\alpha_1$  and  $\alpha_2$  and the constraints (23) are used for the parameter  $\delta_1$  in both the modelbased and data-based approach. We use  $\epsilon_2 = 0.8$  to prevent  $\delta_1$  from exceeding the maximum absolute error and  $\epsilon_1 = 0.05$  rad for disturbance situation 1, and  $\epsilon_1 = 0.02$  for disturbance situation 2, to prevent  $\delta_1$  from becoming zero, see Section 3.3. First we will discuss the experimental results with the data-based approach in Section 4.4 after which we will compare the result with the model-based approach in Section 4.5.

#### 4.4. Data-based results

The results of the tuning of the piecewise affine variable-gain controllers, using data-based optimization, see Section 3.2, are shown in Fig. 11. The optimizations are done for 5 different sets of initial parameters  $\theta_0 = [\alpha_{1,0}, \alpha_{2,0}, \delta_{1,0}]^T$ . The upper-plots show the error-response for the linear low-gain controller  $(C(s), \varphi(e) = 0)$ , the linear high-gain controller  $(C(s)(1 + \alpha \mathcal{F}(s)))$ ,  $\varphi(e) = \alpha_{max}e)$ , and the optimal piecewise affine variable-gain controller synthesized for the disturbance situation at hand. The other plots show the iteration history as a function of the iteration index  $\kappa$  and the optimal nonlinearity  $\varphi(e)$  found for the disturbance situation at hand. A fixed number of 25 experiments was used in the optimizations (note that only successful iterations, see Fig. 4, lead to a raise by 1 of the iteration index  $\kappa$ ).

For disturbance situation 1, see the left part in Fig. 11, the optimal piecewise affine variable-gain controller synthesized (for all different starting points) is a dead-zone variable-gain controller with  $\alpha_1 = 0$ ,  $\alpha_2 = 3$  and  $\delta_1 = 0.19$  rad with corresponding performance indicator J = 0.72. The variable-gain controller outperforms the linear low-gain controller with J = 1 and high-gain controller with J = 2.42. The reason for the fact that the dead-zone for  $\varphi$  is now optimal is as follows. When comparing to the low- and high-gain controller in Fig. 11, the variable-gain controller does not induce any additional gain if a high-frequency small-amplitude disturbance is present, thereby performing equally well as the low-gain controller. However, when only a low-frequency large-amplitude disturbance is present, additional gain is induced such that the low-frequency disturbance suppression is improved compared to the case of low-gain linear control.

**Remark 4.3.** For the considered disturbance situations, an increase to N = 3 segments, see Fig. 2, still yields for optimal nonlinearities a dead-zone characteristic and a saturation characteristic. This confirms that for the illustrative disturbance situations considered, N = 2 is sufficient in order to improve the performance of the system. More complex disturbance situations however may benefit from nonlinearities with N > 2.

<sup>&</sup>lt;sup>1</sup> Since we do this off-line in-between simulations/experiments, the filtering can be applied in both forward and recursive direction, to avoid phase distortion [25].



**Fig. 11.** Experimental data-based results of the iteration history of piecewise affine variable-gain controller synthesis for N = 2 segments and five different initial parameter sets. Results for disturbance situation 1 (left) leading to an optimal dead-zone characteristic and results for disturbance situation 2 (right) leading to an optimal saturation characteristic. The measured error signals are used to compute the performance *J*.

The optimal piecewise affine variable-gain controller for disturbance situation 2, see the right part in Fig. 11, consists of a saturation nonlinearity with  $\alpha_1 = 3$ ,  $\alpha_2 = 0$  and  $\delta_1 = 0.028$  rad with corresponding performance indicator J = 0.756. The variable-gain controller outperforms the linear low-gain controller with J = 1 and high-gain controller with J = 1.55. The additional gain within the saturation band achieves equal low-frequency disturbance suppression to the high-gain controller. However, by limiting the amount of additional gain for  $|e| > \delta_1$ , the high-frequency disturbance amplification is kept to a minimum, being almost equal to that of the low-gain controller.

Note that, depending on the disturbance situation that is present for the experimental setup, the data-based controller synthesis method automatically finds the best suitable piecewise affine characteristics. Moreover, the shapes of the nonlinearities  $\varphi(e)$ being optimized (dead-zone and saturation) correspond well to our intuition, considering the two different disturbance situations. This makes the data based method a valuable tool for automated tuning of the piecewise affine variable gain controller. In Section 4.5, we compare the results to the model-based approach from Section 3.1.

#### 4.5. Model-based vs. data-based

In order to compare the data-based results to the model-based optimization approach from Section 3.1 (without repeating

iteration history plots as in Fig. 11), consider the comparison in Fig. 12 for disturbance situation 1. Here, we plot the iteration history of both optimization approaches (with experiments for the data-based method, and simulations using the MTF algorithm for the model-based method) for the same initial parameter set  $\theta_0$  used to initialize both optimizations (for the model-based method, a stopping criteria related to the norm of the difference between two subsequent evaluations of *J* has been used). In the lower-plots the optimization history is shown as a function of the iterations  $\kappa$ . Clearly, both methods converge to the same type of optimal variable-gain controller, namely a dead-zone characteristic for  $\varphi$ , with  $\alpha_1 = 0, \alpha_2 = 3$  and  $\delta_1 \approx 0.2$  rad. The time-domain response is shown in the upper-plot of Fig. 12, from which it can be concluded that the match between simulation and experiment is fairly good.

To further compare the model-based approach with the databased approach, consider the results in Table 1, which show the parameters of the optimal variable-gain controllers found and the related optimal performance indicators for both performance situations. Clearly, the model-based approach gives similar results for the optimal variable-gain controller parameters and associated optimal performance indicator *J*, which is also visualized in Fig. 12. Of course this match between experiments and simulations hinges on the fact that the references r(t) that are supplied to the system are well-known in the situation considered in this section. Nevertheless, the model-based optimizations are computationally very



Fig. 12. Comparison of the data-based optimization approach (measured error signals) and model-based optimization approach (simulated error signals) for disturbance situation 1.

efficient with the MTF algorithm (matter of seconds vs. several minutes for the experiments) which makes it a good method for (large-scale) parameter studies or situations where no machine is available yet.

In this section, the model-based and data-based controller synthesis method have been applied to a motor-load motion system in order to synthesize performance-optimal piecewise affine variable-gain controllers tuned for illustrative disturbance situations. Both methods have been successfully applied, in order to automatically shape the nonlinearity  $\varphi$  depending on the disturbance situation at hand.

A prerequisite for using the model-based method is that a reasonable model of the disturbances acting on the system is available, since these are crucial for the model-based performance evaluation of the closed-loop system. If such a model of the disturbances is not readily available, but a machine is available for measurements, one can still utilize the data-based machine-in-the-loop method, see Section 3.2. This will be the case in the next section where we will consider an industrial wafer scanner as being representative for a class of nowadays nano-positioning motion systems.

#### Table 1

Comparison of optimal piecewise affine variable-gain controllers synthesized using model-based approach (using simulations) and data-based approach (using experiments).

	Disturbance 1		Disturbance 2	
	Model-based	Data-based	Model-based	Data-based
α <sub>1</sub>	0	0	3	3
α2	3	3	0	0
$\delta_1$	0.179 rad	0.19 rad	0.025 rad	0.028 rad
J	0.698	0.720	0.729	0.756

## 5. Application to an industrial wafer scanner

The piecewise affine variable-gain control strategy and databased performance optimization strategy will be applied in this section to a high-precision wafer scanner [26]. A wafer scanner is a system used to produce integrated circuits (IC's), see Fig. 13. Light, emitted from a laser, falls on a reticle (mounted on a reticle stage), which contains an image of the chip to be processed. The light is projected onto a wafer (mounted on a wafer stage) by passing through a lens system. The effect of this illumination, in combination with a photo-resist process results in the desired IC



**Fig. 13.** Schematic representation of the scanning process in a wafer scanner with the reticle stage (top) containing the pattern and wafer stage (bottom) with wafer containing the IC's.

pattern being produced. The reticle-stage and wafer-stage both perform high-speed scanning motions in order to efficiently process the wafers.

The smallest features that can be manufactured on IC's nowadays are in the order of 10 nm [26]. The positioning of the wafer scanner needs to be accurate enough in order to produce such small features; therefore, the allowable tracking error during exposure of the wafer is only a few nanometers. In this section, we will focus on the wafer stage, see Fig. 13, in particular on the *y*-direction of the wafer stage. Typically, for a wafer stage application, as stated in [26], "the lithographic tools design needs to be robust, avoiding elaborate manual adjustments on a machine-to-machine basis". Therefore, to facilitate an automated performance-based tuning of the piecewise affine variable gain controller, we will use the data-based optimization in this section.

## 5.1. Stability conditions

Stability of the nonlinear variable-gain controlled wafer scanner can be guaranteed by applying Theorem 2.1. A stabilizing nominal linear low-gain controller C(s) for the *y*-direction is present (by default) on the machine, see Fig. 14 for the resulting (measured) low-gain open-loop frequency response function  $\mathcal{P}(j\omega)C(j\omega)$  and high-gain open-loop frequency response function  $\mathcal{P}(j\omega)C(j\omega)$  $(1 + \alpha_{max}\mathcal{F}(j\omega))$ . The feedback controller C(s), together with the design of a stable shaping filter  $\mathcal{F}(s)$ , guarantees that condition A1 of Theorem 2.1 is satisfied.

The shaping filter  $\mathcal{F}(s)$  is again designed using the graphical frequency-domain circle-criterion condition (7) as shown in Fig. 15 (based on experimental data), in order to obtain a significant amount of allowable additional gain  $\alpha_{max}$ . The current design of  $\mathcal{F}(s)$  uses a notch filter and a low-pass filter, allowing a maximum gain of  $\alpha_{max} = 4$  to be used in the variable-gain controller, see Fig. 15. As a consequence, conditions A2 and A3 of Theorem 2.1 are also satisfied, hence, the closed-loop system is guaranteed to be input-to-state stable. Note that by restricting  $\alpha_{max}$  to even smaller values, robustness can be added to the nonlinear control design.

#### 5.2. Performance specification

The illumination process on the wafer scanner takes place during the constant velocity part of the wafer stage setpoint during the time-interval  $t \in [0.01, 0.04]$  s. An acceleration part precedes this constant velocity part of the setpoint during the time-interval  $t \in [0, 0.01]$  s. The aim of the piecewise affine variable-gain control-



**Fig. 14.** Open-loop characteristic  $\mathcal{P}(j\omega)\mathcal{C}(j\omega)$  of the wafer scanner for the low-gain controller and high-gain controller.



**Fig. 15.** Circle criterion condition  $\text{Re}(\mathcal{G}_{eu}(j\omega)) > -1/\alpha_{max} \quad \forall \omega \in \mathbb{R} \text{ with } \alpha_{max} = 4 \text{ for the wafer scanner.}$ 

ler is to obtain smaller errors prior to scanning (during acceleration) without deteriorating the performance during scanning, since this allows for an improvement in the speed of the scanning process, which, in turn, yields more throughput (i.e. higher machine-productivity). Therefore, we will use the following specific non-stationary performance indicator for the wafer scanner application:

$$J = \gamma \left( c_1 \int_0^{0.01} e(t)^2 dt + c_2 \int_{0.01}^{0.04} e(t)^2 dt \right), \tag{31}$$

and use  $c_1 = 1$  and  $c_2 = 4$  to emphasize the importance of the error during the scanning process. Since the measured error signals are of nm order of magnitude, we use  $\gamma = 1e19$  to scale *J* up to values with unitary order of magnitude. For optimizing the variable-gain element with N = 2 segments, we use constraints (22) with  $\alpha_{max} = 4$  and constraints (23) with  $\epsilon_1 = 1$  nm and  $\epsilon_2 = 0.8$ . We choose N = 2, in order to limit the computational burden of the optimization problem, and investigate whether this provides enough freedom in the design of the nonlinearity to improve performance. From practical experience with the control of wafer stages it is known that two distinctive non-stationary disturbance effects are visible in closed-loop error measurements, as we will see in the following section, which further motivates our choice for N = 2 segments.

# 5.3. Data-based optimization results

The *y*-direction of the wafer stage is only a small part of the complete machine in which many different movements and sources of disturbances can be present that, due to cross-talk for example, disturb the *y*-direction. Therefore, we will rely in this section on the data-based optimization approach in order to optimize the piecewise affine variable gain controller. This machine-in-the loop approach will incorporate the effects of the disturbances acting on the system through the measured tracking-error responses, see Section 3.2, that will serve as input for deriving the gradients, see (21).

Three optimizations of the piecewise affine function  $\varphi$  have been carried out using the data-based approach for three different initial parameter sets  $\theta_0 = [\alpha_{1,0}, \alpha_{2,0}, \delta_{1,0}]^T$ . The measured timedomain tracking-error responses of the low-gain, high-gain and the optimal piecewise affine variable-gain controller synthesized (from measurement set 3) are shown in Fig. 16. We care to stress here that a well-tuned feed-forward has been designed for the system, resulting in the nm-scale tracking errors as depicted in the figure. However, the feed-forward design is not perfect; here, the remaining performance improvement is achieved by means of (nonlinear) feedback. In this scope, we also note that the tracking errors shown in Fig. 16 show limited recurrence (since this part is already compensated for by the feed-forward controller), but feature the following similar characteristics from experiment to experiment: during the acceleration phase from  $t \in [0, 0.01]$  s, a low-frequency contribution in the error is present which can be well suppressed by additional gain of the controller. In the scanning phase, however, from  $t \in [0.01, 0.04]$  s, the error signal is mostly dominated by high-frequency contents, which is amplified under additional controller gain. This is clearly visible from the measured linear low-gain and high-gain controller responses. The optimal piecewise affine variable-gain controller induces some additional gain ( $\alpha_1 = 1.2$ ) for small errors  $|e| \le \delta_1 = 3.3$  nm and large additional gain  $\alpha_2 = 3.9$  for errors exceeding  $\delta_1$ , see Fig. 18 (for measurement set 3).

The optimization history obtained is shown in Fig. 17. The optimizations were terminated manually, in case improvement in the performance I was no longer observed (with 9, 11 and 12 experiments respectively for set 1.2 and 3). The fact that only 3 successful iterations were needed for set 1, is due to the fact that the optimization parameters (apparently) have starting values close to the optimal values. Note once more that the amount of successful iterations  $\kappa$  (resulting in a lower *I*), see Fig. 4, can be lower than the amount of experiments performed. All three optimizations converged to the same qualitative shape of the nonlinearity, see Fig. 18, where small additional gain is used for small errors, but large additional gain is used for large errors exceeding  $\delta_1$ . The quantitative discrepancy between the three optimal nonlinearities may be caused by the fact that real measurements on an industrial machine are performed, where the disturbances acting on the system differ slightly from trial to trial, as was the case for example with the experiments in Section 4. However, the three controllers obtained for the three different series of optimizations do yield very similar performance in terms of the performance indicator J, namely J = 2.93, J = 3 and J = 2.92, for measurement sets 1, 2, and 3, respectively (the optimal response in set 3 is shown in Fig. 16). The nonlinear controllers clearly outperform the linear controller limits for the low-gain controller with I = 5.04 and high-gain controller with I = 10.23. From a robustness point of view, it is beneficial that the three quantitatively different piecewise affine variable gain controllers give similar performance, since this indicates that the performance is not highly sensitive to the optimization parameters near the optimum (albeit for a slightly different disturbance situation).



**Fig. 16.** Measured time-domain error response of low-gain controller, high-gain controller and optimal piecewise affine variable-gain controller (from measurement set 3 in Fig. 17).



**Fig. 17.** Experimental results of iteration history for three different initial parameter sets (measurement set 1, 2, 3) as a function of the iterations  $\kappa$ .



**Fig. 18.** Optimal piecewise affine nonlinearities  $\varphi(e)$  for the three different measurement sets.

Although the disturbance situation changes from experiment to experiment for the wafer stage application, the error-response has similar (non-repetitive) characteristics for every experiment (a low-frequency contribution during the acceleration phase of the reference, and a high-frequency contribution afterwards). If the disturbance situation changes significantly over time, then employing a single variable-gain controller setting will likely be sub-optimal. In such a case, a re-tuning of the parameters, or an optimization strategy that stays operational during the operation of the machine, can be two viable options to achieve increased performance.

**Remark 5.1.** In case stochastic components are present in the measured error profiles, as in the wafer stage application for example, a possible extension to the presented deterministic optimization approach can be to apply optimization methods that take into account such stochasticity. In general, this will require additional measurements that can be used to filter out the

stochastic components, to give unbiased estimates of the performance measure and gradients. Iterative feedback tuning concepts, see e.g. [27–30], can possibly be used in this context, although it should be investigated whether these concepts apply to the *nonlinear* piecewise affine variable-gain control feedback configuration as considered in this paper.

In this section, the data-based approach has been applied successfully to the performance-based tuning of a piecewise affine variable gain controller for a wafer stage of an industrial wafer scanner. The disturbance modeling in such a nano-positioning industrial machine can be very challenging. The data-based method therefore uses measured error-signals each iteration in order to account for the disturbances acting on the system. By the use of readily available models of the plant and (exactly known) controller, no additional experiments are needed to determine the gradients needed in the optimization, keeping the number of experiments needed for the optimization small. The results show that an automated performance-based synthesis of the piecewise-affine structure of the variable-gain is possible, and adapts to the disturbance situation at hand. This is crucial for performance-based machine-specific tuning of the industrial machines in the field.

#### 6. Conclusions

In this paper, we have proposed a generic piecewise affine nonlinearity in a variable-gain motion control context with the aim to improve the performance compared to linear motion controllers. By not fixing the shape of the nonlinearity a priori, we developed a strategy to synthesize a variable-gain controller that optimizes the performance for the particular disturbance situation at hand. We proposed a computationally efficient model-based approach and a data-based machine-in-the loop approach, giving a machine-dedicated calibration procedure. We have illustrated the controller synthesis approach on an experimental setup, where a good match between the model-based and the data-based method was obtained. The data-based method was also applied to an industrial wafer scanner, where improved scanning stage performance is demonstrated using piecewise affine variable-gain control, compared to the linear controllers. Two piecewise affine segments appeared to be sufficient to improve the performance in the considered applications. The piecewise affine description of the variable-gain element allows for a generic class of nonlinearities to be synthesized, paving the way to general performancebased nonlinear controller design. In this respect, we consider an extension in the line of general linear-parameter-varying controllers for linear motion systems an interesting research topic for future work.

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