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BOOK REVIEWS

UNIFORM OUTPUT REGULATION OF NONLINEAR SYSTEMS: A CONVERGENT DYNAMICS APPROACH. Alexey Pavlov, Nathan van de Wouw and Henk Nijmeijer, Birkhäuser, Basel, 2006. No. of pages: IX+172. ISBN 978-0-8176-4445-1

The book deals with the problem of output regulation for nonlinear systems, namely the problem of offsetting, in nonlinear dynamics, the presence of exogenous signals, possibly representing references to be tracked and/or disturbances to be rejected, generated as solutions of an autonomous system usually referred to as 'exosystem'.

The book presents in a coordinated, self-contained and clear way the main outcomes of a 4-year research project conducted at the Eindhoven University of Technology and mainly developed within the PhD work of the first author.

The distinctive feature of the approach followed in the book is to adopt, as underlying stabilization tool, the concept of convergent dynamics as originally proposed by B. P. Demidovich in [1], revived and further developed by the authors to make it effective to the problem of output regulation (see [2]). In words, a dynamical system driven by inputs is said to be convergent if it 'forgets' the effect of initial conditions so that any trajectory converges toward a unique steady-state solution defined over the whole time axis and only dependent on the inputs. In this context, the main intuition developed in the book is to design a controller so that the closed-loop system, with input the exogenous signal, is convergent and its consequent unique steady state solution is shaped so that the regulation objective is fulfilled.

The book is divided into eight chapters and one appendix reporting the technical proofs of the main results.

In Chapter 1 the authors introduce and discuss the problem of output regulation and the concept of convergent system by framing the actual literature and proposed solutions. This section is very well-written and makes the reader aware of past developments as well as of recent results and trends. As far as the problem of output regulation is concerned, the authors emphasize how the actual literature of *global* output regulation lacks a generally accepted problem formulation and setting especially in terms of underlying assumptions, and how their convergence approach may contribute to provide a neat framework. As far as the definition of convergent systems is concerned, the authors present and discuss alternative properties and definitions proposed so far in the literature (such as contraction property, incremental stability, incremental ISS, etc.), by emphasizing how the proposed definition of convergence relies upon a topological property of the system having the advantage of being coordinate independent and not relying upon an operator description of the system.

In Chapter 2 convergent systems are precisely defined and characterized in terms of properties and sufficient conditions. Besides presenting the basic definition of convergence due to Demidovich, the authors discuss a number of extensions by taking into account the effect of particular classes of inputs and introducing a notion of Input-to-State convergence. Basic properties of convergent systems are introduced and discussed as well. Besides others, specific emphasis is given to the remarkable property that any convergent system driven by a periodic input has a periodic steady state with the same period of the forcing term. This property, more than others, reveals the closeness between the class of nonlinear convergent systems and the one of linear Hurwitz systems, as also remarked by the authors in several parts of the book. Finally, in the second part of the chapter, sufficient conditions for convergence and Input-to-State convergence are detailed also in the presence of dynamics, which are not continuously differentiable everywhere. The given conditions are indeed quite compact and neat even though, as expected, quite restrictive. As meaningful classes of systems which lend themselves to be systematically analyzed by means of the proposed conditions, the authors develop the case of piecewise affine systems and of the so-called Lur'e systems.

In general, the chapter is enjoyable to read and well-organized, with a notation that is never cumbersome.

Chapter 3 is entirely dedicated to present the framework of uniform output regulation, both in the global and in the local settings, by introducing notations, terminology and main formulations. With respect to the conventional frameworks, the novelty of the proposed formulation relies in the fact that the internal stability requirement is given in terms of uniform (with respect to time) convergence of the closed-loop system. As a particular case of the problem of output regulation, also problems of observer design and controlled synchronization are framed. This is a nice and original choice as it highlights how the theory of output regulation can be successful also in unconventional settings, different from the 'usual' tracking and disturbance rejection problems.

Chapters 4 and 5 are the real core of the book. Chapter 4 addresses the problem of uniform output regulation in terms of necessary and sufficient conditions, while Chapter 4 discusses the constructive part of the regulator design.

From a theoretical viewpoint, the backbone of Chapter 4 is a 'global' invariant manifold theorem, which aims to extend the well-known (local) center manifold theorem under convergence assumptions. With this result at hand, the authors present necessary and sufficient conditions for the problem. In one of the possible variants presented in the book, the conditions are given in terms of the so-called 'generalized internal model property' (that is the ability of the regulator to reproduce the steady state control input able to enforce an identically zero-regulated error) and convergence property required to any regulator solving the problem at hand. The problem of robust uniform output regulation in presence of uncertainties is also addressed. The convergence framework allows the authors to formulate very simple and neat conditions that are organized in a easy-to-follow way. Noteworthy is the final part of Chapter 3 that addresses possible applications of the global invariant manifold theorem. In particular, by taking advantage of the property that convergent systems excited by periodic inputs have periodic steady-states, a nonlinear frequency response function associated with a convergent system is introduced as nonlinear equivalent of the frequency response function for a linear system and its graphical representation is investigated in order to obtain an original variant of the Bode plot for this class of nonlinear systems. Regretfully, only a few preliminary results and intuitions are presented in this part that, in the intention of the authors, just aim to intrigue the reader and promote a new research area to be definitely developed.

As classically proposed in the literature on output regulation, the regulator structure discussed in Chapter 5 is then composed by two units, at first 'internal model unit', which is designed so that the regulator has the generalized internal model property, and a 'stabilization unit' designed to make the closed-loop system convergent. Most of the chapter is focused on different aspects of the latter and specifically how to make the system Input-to-State convergent. In this respect, the authors introduce design procedures based on backstepping, quadratic stability and H_∞ optimization methods by giving a lot of emphasis on how the proposed methods apply straightforwardly to the class of the piecewise affine systems and of the Lur'e systems.

In order to overcome a few intrinsic limitations in the applicability of the theory in a global setting, in Chapter 6 the authors develop further the local and approximated version of the theory, which turns out to be more appealing for practical purposes. More specifically, the Chapter first addresses the problem of estimating the region of attraction of the closed-loop system in the case where the Demidovich condition, on which the design of the stabilization unit is based, is satisfied only locally. The analysis in this part is very detailed and attempts to obtain a nonconservative estimation by means of Lyapunov arguments. Then, in the second part of the chapter, the problem of approximated output regulation is dealt with. In this case the problem is to estimate a bound on the asymptotic regulation error in the case where the solution of the regulator equations, on which the design of the internal model unit is based, is known only approximately. The theoretical aspects of both the parts are exemplified by using the translational oscillator with a rotational actuator (TORA) system as benchmark.

The TORA system is also the benchmark used in Chapter 8 to illustrate an intensive experimental activity conducted at the Eindhoven University of Technology. This chapter presents a few issues related to the implementation of the regulator and the performances obtained in presence of nonidealities typical of experimental setups. Remarkably, the authors use the benchmark also to present an objective analysis of their theory and to identify limits to be overcome in future researches. Indeed, Chapters 6 and 7 are more directed toward readers interested to practical aspects of the theory and to figure out its applicability in real scenarios.

Finally, Chapter 8 concludes with an overview on the presented arguments, final remarks and future research directions opened by the presented theory.

In summary, the book is well-written and presents a fresh perspective to the problem of output regulation. The book is not intended to be a general treatment on nonlinear output regulation, but rather aims to present a different and original way of approaching the problem by means of the convergence tool. In this respect, the book is a valuable addition to the theory of nonlinear output regulation and definitely recommended to researchers working in this field. Unfortunately the class of nonlinear systems that can be successfully dealt with the proposed tool is limited. As also remarked by the authors, there is a need for new design and analysis tools for convergent systems to wide the cases in which the theory can be successfully applied. However, whenever applicable, the theory presented in the book leads to a very intuitive and neat way to design internal model-based regulators. A particular attention is given to propose an intriguing framework also for readers more interested to practical aspects. In many parts of the book the intention of the authors is clear to fill the gap between theory and practice, by providing practical design guidelines and by addressing implementation issues. In this respect the text is definitely recommended not only to academic researchers but also to industrial engineers working in the field of nonlinear control design.

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