Book Review

Uniform Output Regulation of Nonlinear Systems: A Convergent Dynamics Approach—A. Pavlov, N. van der Vouw, and H. Nijmeijer (Berlin, Germany: Birkhäuser, 2006). *Reviewed by Christopher I. Byrnes*

In this book, which represents an enhancement of the first author's 2004 Ph.D. doctoral dissertation written at Eindhoven University of Technology, the authors treat the problem of output regulation for a nonlinear control system. The problem of output regulation has its classical origin in the servomechanism problem and has been substantially generalized both to lumped (and distributed parameter) linear multivariable systems and lumped multivariable nonlinear systems. In all instances, the central problem of interest is the attenuation of disturbances and the asymptotic tracking of reference signals.

There are, of course, a number of approaches to, and alternatives in the formulation of, such problems. For example, one approach has been to solve the problem of exact tracking of a prescribed, *known* trajectory, and then to hope to find conditions for the stability of this trajectory. Another is to consider those reference signals/disturbances generated by an autonomous exogeneous system (exosystem). For example, consider the problem of tracking a sinusoidal trajectory with known frequency but perhaps unknown amplitude, and phase. In the context of exact tracking, one would need to know the amplitude and phase. If one considered the sinusoid to be the output of a harmonic oscillator, the amplitude and phase arise as initial conditions which, in the setup of an output regulation problem with error (rather than state) feedback, need not be prescribed.

The case where the frequency is unknown is perhaps the best introduction to the theory of robust output regulation, as well as to adaptive output regulation. There has been a great deal of attention paid in the literature, even in the 20th Century, to this important extension of the classical problem formulation. However, this is a very challenging open problem area from the point of view of both theory and practice. Despite the efforts of many researchers, as presented in this book or any of the recent surveys of the literature on this problem, solutions are usually obtained only in rather tightly controlled situations. For the case of nonlinear adaptive or robust output regulation, I would have to agree with A. S. Morse's assessment of the current state-of-the-art of adaptive control (which is nonlinear) as being similar to the classical theory of automatic control prior to the work of H. Bode. There is just very little currently known about the relative performance of proposed feedback schemes. From a researcher's point of view, this is reason to continue.

One of the knotty issues that confronts the launch of any serious research program aimed at "shaping the steady-state response" of a complex system is actually defining the steady-state response itself. In most books about nonlinear dynamics and control, it is simply not discussed. I applaud the authors for doing so in the book under review.

Why is this issue so knotty? For one thing, there is not just one undefined notion here, rather there are two, and neither of these twins are easy to define. In plain terms, every trajectory should be a combination of a transient and a steady-state response, but knowing their combination is not the same as knowing either of the twins. In short, at least one of these two concepts has to be defined precisely enough to know whether, in fact, one is shaping the steady-state response.

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Indeed, I believe that coming to grips with the notion of steady-state response involves one of the "grand tautologies" in the control of complex systems. There is a delicate, and even frustrating, balance between defining it rigorously and giving away the impact of your results in the hypotheses. This, together with the relative sophistication involved in making any reasonably coherent definition, explains why most books avoid it.

The authors' starting point is that the three fundamental issues that need to be addressed are *regularity* of solutions to the closed-loop system, *uniform convergence* of solutions (initialized in an appropriate set) and *asymptotic output zeroing*. The use of italics for these three properties are those of the authors. This is not only reasonable, but smart.

While no one would argue with some kind of *regularity*, it is worth pointing out that some of the output signals, arising say in the literature on exact tracking, provably tend to zero but are not the signals they seem since, in some cases, the corresponding closed-loop state trajectory escapes in finite time.

Until relatively recently, the importance of the *uniform convergence* of the error variables for a bounded set of initial conditions, was not appreciated in the nonlinear control literature. This is especially important in the nonequilbrium setting, which is what the output regulation problem is all about. A number of other important concepts and constructs are related to this observation and discussed in this book, from the authors' perspective. First, in the equilbrium case, exponential stability of an equilibrium for the unforced system guarantees uniform continuity of state trajectories and error variables for small initial conditions in the exosystem. This follows from an elementary Lyapunov-theoretic argument. Second, in the nonequilibrium case we also need to guarantee some kind of uniform convergence, both for the existence of a well-defined steady-state response and for a well-behaved *asymptotic output zeroing*. Indeed, in the case where the tracking error is the output-to-be-controlled, asymptotic output zeroing is one of the primary goals of output regulation.

These two observations are among the most important discoveries in the recent literature on nonlinear output regulation, including the book under review. Beginning with their own formulation of these basic ideas, which is a nontrivial undertaking, the authors develop a global approach to output regulation along familiar lines. The book also makes contact with the theory of synchronization of complex dynamics and includes an illustrative concrete example.

I think the book is extremely interesting, but I would note that the existing literature on nonlinear systems and control contains substantial alternative programs in nonlinear output regulation that have addressed these issues from different points of view. In each instance, just as in this book, researchers were striving for a formulation of the basic concepts that would strike a favorable balance within the grand tautology discussed above. My personal view on this is that many of these ideas were, at best, "in the air" when the thesis that underlies this book was being contemplated and that the authors worked them out from their own particular view, which I find to be stimulating.

A few comments are, however, in order on both the equilibrium and the nonequilibrium cases of nonlinear ouptuput regulation. In the equilibrium case, the authors suggest that most of the prior literature has dealt with linear exosystems and, more specifically, the harmonic oscillator. Actually, much of the early breakthroughs in nonlinear output regulation were focused on the use of invariant manifolds, specifically center manifold theory, for nonlinear control systems with nonlinear (neutrally stable) exosystems. In particular, these approaches explicitly used invariant manifolds to give a nonlinear version, albeit local, of the frequency response described in the book presently under review.

The nonequilibirum case considered in the present book begins with a discussion of the theory of convergent systems developed in the last half of the 20th century in the Russian literature. It is used extensively to develop the existence of a steady-state response and its relationship to uniform convergence, especially for forced systems. I wish, however, that the authors would have enhanced their treatment of forced systems, beginning with the forced van der Pol oscillator itself for historical reasons. One of the upshots of the intricate work on this topic by Cartwright and Littlewood was the formulation of the notion of dissipative systems by Levinson and its subsequent generalization in the Russian literature, as described in the book by Pliss listed in the bibliography of the book under review. In fact, a very special case of the dissipative periodic systems studied by Pliss is an input-to-state system driven by a periodic signal, a scenario which itself finds a prominent home in the present book.

To be fair, I haven't written a book that has this level of historical review in it either, but I believe it should be done. That being said, I found the book to be ambitious and rigorous, tackling some hard conceptual issues in a distinguishable way that the authors have made their own.